Measuring the Flow of Water in a Culvert

Victor Aguilar www.axiomaticeconomics.com/Flow_Water_Culvert.pdf

Let r be the radius of the culvert and x be the distance from the center of the culvert to the water line, which is positive if the culvert is less than half full. But it is easier to measure the height of the water, h, than the distance from the center of the culvert to the water's surface, x, so in the last step we substitute $r - h$ for x.

If you have an old tape measure that does not retract very well anymore, cut off eight feet of it and staple it to a board to measure the height of the water.

Culverts have grooves and lands similar to a rifled gun barrel except, instead of a spiral pattern, the grooves and lands are concentric. The height of the water and the diameter of the culvert should be measured to the inside of the lands. The radius is half the diameter.

The cubic volume of water flowing through the culvert is the cross sectional area times the length divided by the time it takes to float a stick through the culvert. So, in addition to your measuring stick, you will need a stop watch or a wrist watch that displays seconds.

The radius and length of the culvert are constants so, having measured them once, you can post this information on a nearby fence post.

The only measurement you have to take is dipping your stick in the water and noting how much of it is wetted to get h. Evaluate the formula shown below. $acos\frac{r-h}{r}$ is the arc-cosine of $\frac{r-h}{r}$. This requires a scientific calculator; $acos$ is written cos^{-1} and requires pushing the $SHIFT$ (on Casio calculators) or $2nd$ (on Texas Instruments calculators) button and then the COS button.

$$r^2 acos\frac{r-h}{r} - (r-h)\sqrt{2rh - h^2}$$

The derivation of this formula shown on the next page is an example of what is required of high school students studying for the Calculus AB exam.

$2\int_{x}^{r}\sqrt{r^2-t^2}\,dt$	Cross section of water
$2\int_{asin\frac{x}{r}}^{\frac{\pi}{2}}\sqrt{r^2-r^2sin^2(u)}rcos(u)du$	$u=asin\left(\frac{t}{r}\right)$ $t=rsin(u)$ $dt=rcos(u)du$
$2r^2\int_{asin\frac{x}{r}}^{\frac{\pi}{2}}\sqrt{1-sin^2(u)}cos(u)du$	Factor out r^2
$2r^2\int_{asin\frac{x}{r}}^{\frac{\pi}{2}}cos^2(u)du$	Pythagorean Theorem
$r^2\int_{asin\frac{x}{r}}^{\frac{\pi}{2}}du+sin(u)cos(u)$	Reduction Formula
$r^2[u+sin(u)cos(u)]_{asin\frac{x}{r}}^{\frac{\pi}{2}}$	Integrate
$r^2\left(\frac{\pi}{2}-asin\frac{x}{r}-\frac{x\sqrt{r^2-x^2}}{r^2}\right)$	Evaluate at integration limits; $cos\left(asin\frac{x}{r}\right)=\frac{\sqrt{r^2-x^2}}{r}$
$r^2\left(\frac{\pi}{2}-asin\frac{x}{r}\right)-x\sqrt{r^2-x^2}$	Factor r^2 into the last term
$r^2acos\frac{x}{r}-x\sqrt{r^2-x^2}$	Complementary angles
$r^2acos\frac{r-h}{r}-(r-h)\sqrt{2rh-h^2}$	$x=r-h$