THE

DIARIAN MISCELLANY:

CONSISTING OF

All the Useful and Entertaining Parts, both Mathematical and Poetical, extracted from the

LADIES' DIARY,

From the beginning of that work in the year 1704, down to the end of the year 1773.

With many additional

SOLUTIONS and IMPROVEMENTS.

In five Volumes.

VOL. II.

By CHA. HUTTON, F. R. S.
Professor of Mathematics in the Royal Military Academy.

LONDON:
Printed for G. ROBINSON and R. BALDWIN in Pater-noster Row, 1775.

W. W. Beman
qt.

THE MATHEMATICAL PARTS OF THE LADIES' DIARIES.

1732.

Of the Eclipses in 1732.

TO the inhabitants of our terraqueous globe there will happen five eclipses: Three times will the moon, in her wandering course, interpose and hide the splendid rays of the sun from our view; and twice will the earth, in its course, so fall in the line between the sun and moon, as to hinder her receiving the light she borrows from the sun, to enlighten the earth by reflection.

1. Moon eclipsed May 28, at 2 in the afternoon, invisible.

2. Sun eclipsed June 11, about noon; but by reason of the moon's south latitude and parallax, is invisible here.

3. Sun eclipsed November the 6th, at 4 in the afternoon; but so small, as not visible to the naked eye.

4. Moon eclipsed on Monday the 20th day of November, at three quarters after nine at night, total and visible.*

Computed by	Begin. h.	m.	Begin to.	da	End to.	da.	Midd		End		Digits	
Astronom. Carol. Coventry	viii	2	ix	0	x	36	ix	48	xi	33	21	17
Mr. Chattock, London	8	1	9	0	10	42	9	51	11	41	21	27
Mr. Leadbetter, London	8	13	9	11	10	46	9	58	11	44	20	48
Mr. Bulman, Lewisham	8	9	9	6	10	43	9	54	11	40	21	14
Mr. Bulman, Carlisle	7	58	8	55	10	32	9	43	11	29	21	14
Mr. Turner, Hull	8	37	9	35	11	13	10	24	12	11	21	0
Mr. Oats, Givenna	7	41	8	39	10	17	9	28	11	15		
Mr. Williams, Middleton	4	2	- - -		- - -		- - -		7	38	17	2
Mr. Brown, Bridgenorth	7	58	8	5	10	32	9	44	11	29	21	14
Mr. Paternoster, Hitchin	7	50	8	49	10	32	9	41	11	31	21	35

5. Sun eclipsed December the 6th, at 15 minutes after nine in the morning; visible, but so very small, that not above one three hundred and sixtieth part of his diameter will be obscured, and so not perceptible to a naked eye.

New

* This eclipse was observed thus:

At	By	Begin. h.	m.	s.	Immers. h.	m.	s.	Emers. h.	m.	End h.	m.	s.
London	Geo. Graham	8	1	30	8	59	30	10	38	11	37	0 a.t.
London	Mr. Hodgson	8	1	30						11	36	30 a.t.
York	John Turner	8	1	0	8	59	0	10	40	11	41	0 a.t.
Rome	Di. Revillas, Jo. Bottarius, & Eust. Manfredi	8	51	19	9	48	24			12	26	55 t.t.

New Questions.

I. QUESTION 163, *by Mr.* Sam. Ashby.

In verdant fields one summer's morning fair,
In walking forth to take the pleasant air,
Pleas'd with the harmony o'th' warbling notes
Of larks and nightingales' extended throats.
And pleasing zephyrs, with a gentle breeze,
Spread o'er the plains, did waft the verdant trees;
And Sol's refulgent rays join to complete
This lovely scene: Where I by chance did meet
A Geodecian, in a park, by th' way,
Was thither come, the same for to survey:
Whose form he a right-angled triangle found,
In which was made a walk exactly round,
And touch'd all sides of the said triangle;
In which round walk four other walks quadrangle
Were made; denoted by *A*, *B*, *C*, *D*,
Meeting in the round walk's periphery.
The area * of the whole triangle's known,
And each side † of the quadrilateral's shown;
By which the following he was to produce,
Base, perpendicular, and hypothenuse?
But finds his skill will not resolve this doubt,
So begs you'll lend your aid to help him out.

* *The area* = 55296 *square chains.*

† AB = 152·99
BC = 191·11
CD = 93·03
DA = 41·56

II. QUESTION 164, *by Mr.* John Turner.

Two men, *A* and *B*, buy a piece of ground in an unknown northern latitude: but it was observed that on a certain day in the year, also unknown, the sun's altitude upon the south part of the meridian, at the said place, was 42° 30′; and upon the north part of the meridian, his altitude above the horizon was 4° 30′: The limits of the ground were to be marked out by the shadow of the vertex of a tree 20 yards high, on that same day when the altitude of the sun on each part of the meadow was observed as above-mentioned. It is required hence to find the latitude of the place, and the ſun's declination; and alſo the ſhare of the ground belonging to each man; *A* being to have for his part the greateſt triangle that can be cut out of the ſaid conic ſection deſcribed by the ſhadow of the tree's top, and *B* to have the remainder.

III. QUESTION 165, *by Mr.* Tho. Grant.

Pray, gentlemen gaugers, be pleaſed to lend
Your aſſiſtance and aid to a brother and friend;
Who lately has met with a caſk in his round,
The content of which by him cannot be found
From any problem or theorem taught,
By thoſe who have on ſtereometry wrote.
A ſpheroidical fruſtum it ſeemeth to be,
Whoſe dimenſions are ſuch as hereunder * you ſee;
Hence you are deſir'd to ſhew its content
By a general rule, and how much of the length
Is on each ſide the greateſt bulge of the caſk;
Which done will reſolve him in all he does aſk.

* *Gr. head* 32·0. *Leſ. head* 27·0. *Bung* 36·0. *Length* 45·0.

IV. QUESTION 166, *by Mr.* Chr. Maſon.

A canon give, that will exhibit fair
All perfect numbers; and alſo declare
What thoſe from unit to ten millions are?

V. QUESTION 167, *by Mr.* Turner.

Let there be a triangle whoſe 3 ſides are given, viz. 415, 353, and 488: And upon the three angular points, as centers, let there be deſcribed three circles whoſe radii are 130, 80, and 70: Let a fourth circle be drawn, which ſhall touch theſe three circles. It is required to find its diameter?

VI. QUESTION 168, *by Mr.* Chr. Maſon.

I once ſupinely trifling time away,
With two old quondams who at dice would play.
Each ſtak'd his guinea, fifteen up the game;
And I by chance had juſt got ten o'th' ſame.
The other two had not ſuch luck to thrive;
The one being eight, the other only five:

When

When they propos'd no farther to advance,
But part the ſtakes, according to each chance.
And I well weighing gameſters fickle caſe,
With feign'd denial, did their choice embrace.
I now deſire ſome artiſt to unfold,
How much each gameſter is to have o'th' gold.

The Prize Question, *by Mr.* Rob. Fearnſide.

A whimſical merchant of late did import,
Than buſineſs more for diverſion and ſport,
Cylindric and conical poles not a few,
Whoſe dimenſions * in part, you have here in full view.
Now it happen'd, as with him one ev'ning I ſat,
By degrees did begin mathematical chat;
Till by ſome how or other this bargain at laſt,
Gave riſe to this queſtion, he ſtarted in haſt.
'The greateſt of thoſe ſort of poles I wou'd know,
'That's poſſible up this ſame chimney to go,
'Whoſe width, I remember, when meaſur'd, to be
'Juſt forty-eight inches at the mantle tree;
'And likewiſe between the ſaid tree and the floor,
'The diſtance was found to be twice as much more:
'The man who the eaſieſt method can ſhew,
'On demand twelve diaries may claim as his due.'
Half aſſur'd of ſucceſs, I reſolv'd to begin,
His queſtion to ſolve, and the diaries to win;
But I found after all, to my grief and vexation,
The *X*'s quite vaniſh'd out of my equation.
Therefore, ladies, the manner to ſolve it pray ſhew,
And when reading the diaries I'll think upon you.

* *The baſes of the cylindric poles were 12 inches, and the ſides of the recti-conical cones, were to their baſes as 4 to 1.*

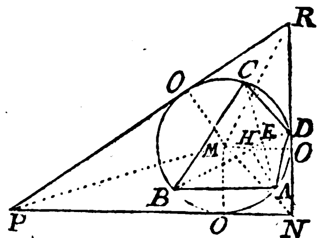
Questions

1733.

Questions answered.

I. QUESTION 163 *answered by Mr.* W. Grimmett.

DRAW the diagonals BD and CA; and from the center of the inscribed circle draw PM, RM, and NM: Let AH be perpendicular to BD, as also MO's to PN, PR, and RN.



Put $s = BC = 191{\cdot}11$, $c = AD = 41{\cdot}56$, $b = CD = 93{\cdot}03$, $d = BA = 152{\cdot}99$, and $a = BE$. Then from the similarity of triangles we have as, $d : a :: b : \frac{ab}{d} = CE$; and after the same way of reasoning DE will be had $= \frac{bca}{sd}$; and $AE = \frac{ca}{s}$. But $a + \frac{bca}{sd} = BD$, and $\frac{ab}{d} + \frac{ca}{s} = AC$; and then we shall have $\frac{baa}{d} + \frac{caa}{s} + \frac{bbcaa}{sdd} + \frac{bccaa}{ssd} = cs + db$; which reduced will be $a = \sqrt{\frac{ssscdd + d^3bss}{bdss + ddcs + bbcs + ccdb}}$ $= 167{\cdot}65 = BE$; and then BD will be found $= 189{\cdot}81$.

Again, from the circle and its inscribed triangle BAD, in which the perpendicular AH is let fall, it will be as $AH : DA :: BA$: the diameter of the circle $= 191{\cdot}6832$; and consequently MO the radius is $95{\cdot}8416$.

Put $b =$ area of the triangle PNR; $n = MO$; $x = RN$. Then $PN = \frac{2b}{x}$, and $\sqrt{xx + \frac{4bb}{xx}} = PR$; whence $nx + \frac{2bn}{x} + n\sqrt{xx + \frac{4bb}{xx}} = 2b$: Per reduction there comes

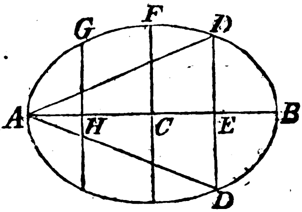
forth

forth $xx - \frac{bx}{n} - nx + 2b = 0$; which solved, x will be found $= 285\cdot7084 = NR$: Then $PN = 387\cdot079$, and $PR = 481\cdot1031$ chains. *Q.E.I.**

II. QUESTION 164 *answered by Mr.* Rob. Fearnside.

'Tis obvious that the declination of the sun is equal to half the sum of the meridian altitudes, which consequently is 23° 30′; and the latitude 71°.

Now the tree being supposed to be placed in *H*, 'tis evident, as the sun does not set, that its summit will describe the ellipsis *AGFDBDA*; therefore, by plain trigonometry, *AH* will be found $= 21\cdot826$, and $BH = 254\cdot124$; and (finding the altitude of the sun when due east or west) $GH = 43$, and consequently *FC* the semi-conjugate diameter $= 79\cdot5$.



Then put $AB = 2a$, $FC = b$, $CE = x$, and $DE = y$. Then, per conics, $aa - xx : yy :: aa : bb$; therefore $y = \frac{b}{a}\sqrt{aa - xx}$. Now $\overline{a+x} \times \frac{b}{a}\sqrt{aa-xx}$, = area of the triangle *DAB*, must be a maximum; which put into fluxions and ordered, x will be $= \frac{1}{2}a$, and the area of the greatest triangle will be $= 7124\cdot37 = 2$ a. 3 r. 32 p. = *A*'s share,
and $27335\cdot88 = 4$ 0 29 = *B*'s share.

Mr. *Grimmett* having discovered a new property of the ellipsis, after a solution to this question, concludes with this other following method.

Supposing a circle inscribed in the ellipsis, then it will be as the radius of the inscribed circle, is to the perpendicular height of the equilateral triangle inscribed therein; so is the semi-transverse of the ellipsis, to the perpendicular of the greatest

---

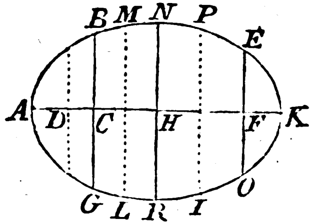
* I. QUESTION 163.

To inscribe a quadrilateral, whose sides are given, in a circle, may be seen in VIETA's *Opera Mathematica* p. 277, and in SIMPSON's *Select Exercises* pr. 35.

greateſt triangle inſcribed in the ellipſis = 206·9652. The anſwer will be 14277·7 = 2 a. 3 r. 32 p. = A's } ſhare.
20251·5 = 4 0 29 = B's }

Mr. *Charles Forreſt* has calculated the ſolution to this queſtion trigonometrically, with the ſun's altitude, azimuth, and length of the ſhadow at every hour of the day, and from thence according to the doctrine of conics has given the true ſolution as above.*

III. QUESTION 165 *anſwered by Mr.* Ri. Lovatt.



Given $\begin{cases} CF = 45 = d, \\ NH = 18 = b, \\ BC = 16 = n, \\ EF = 13{\cdot}5 = m. \end{cases}$

Put $a = AH$, $e = CH$, and $h = \frac{bb}{dd}$. Then, per conics, $aa : bb :: aa - ee : nn$; hence $\frac{aabb - aann}{bb} = ee$. Subſtitute $pp = \frac{bb - nn}{bb}$: Then $ap = e$; and hence $d - ap = HF$.

Again,

---

* II. QUESTION 164.

Having found the declination and latitude as in the firſt ſolution above, viz. by taking half the ſum of the greateſt and leaſt altitudes for the declination, and then by taking the complement of the difference between this declination and the greateſt altitude for the latitude of the place, which are general rules; next compute the altitude when due eaſt or weſt, and then ſay as radius : the height of the tree : : the cotangent of each of theſe three altitudes : each of the three lines HA, HB, HG.——All the reſt of the firſt ſolution above is very clear.

The truth of Mr. Grimmett's theorem above may appear thus: From Mr. Fearnſide's ſolution we find that the altitude of the greateſt triangle is 3-4ths of the tranſverſe axe; and by geometry we know that the altitude of an equilateral triangle is alſo 3-4ths of the diameter of its circumſcribed circle; wherefore as the diameter of any circle is to the altitude of its inſcribed equilateral triangle, ſo is the tranſverſe axe of an ellipſe to the altitude of its greateſt inſcribed triangle.——We may hence remark alſo that the equilateral is the greateſt triangle that can be inſcribed in a circle.

Again, $aa : bb :: aa - dd + 2dpa - ppaa : mm$; and hence $aamm = aabb - ddbb + 2dpbba - bbppaa$. Substitute $c = bb - mm - ppbb$; and $k = 2dpbb$; then $a = \sqrt{\frac{ddbb}{c} + \frac{\frac{1}{4}kk}{cc}} - \frac{\frac{1}{2}k}{c} = 40$. Hence $CH = 18.311$, and $HF = 26.68$.

To find the content; put $m = AH = 40$, and $n = CH = 18.311$; find 18311000000000000000 ordinates rightly applied betwixt C and H, which will give the equal cylinder; thus put $d = 1831100$, &c. then $\frac{*3mmd - \frac{3}{2}nn\frac{1}{2}n - nnd}{3d} \times \frac{1}{4}b$ = the square of the equal cylinder;

and $\frac{*\sqrt{3mmd\frac{3}{2}nn - \frac{1}{2}nn - \frac{1}{3}n - nnd}}{3d} \times 4b = 34.7198814$ $= ML =$ the diameter of the cylinder $= BNRG$; and by the same method the cylinder = to $NEOR$ is $= 33.2216007 = PI$; hence the content is $\left\{ \begin{array}{l} BNRG = 61.470027 \\ NEOR = 82.041786 \end{array} \right\} 143.5118$ ale gallons.

The same answered by Mr. J. Turner.

Put $\left\{ \begin{array}{ll} \text{bung diam.} & 36 = m \\ \text{greater head} & 32 = n \\ \text{lesser head} & 27 = s \\ \text{length} & 45 = 2t \end{array} \right\}$ and AC, the part wanting, $= 2x$. Then is $HF = t + x$, and $CH = t - x$.

As $tt + 2tx + xx : mm - ss :: tt - 2tx + xx : mm - nn$, by the property of the ellipsis, (as per Ward's Introduct. p. 448) therefore $ttmm + 2tmmx + mmxx - ttnn - 2tnnx - nnxx = ttmm - 2tmmx + mmxx - ttss + 2tssx - ssxx$; by reduction and transposition, $ssxx - nnxx + 4mmtx - 2tnnx - 2tssx = ttnn - ttss$.

Put $ss - nn = -b$, and $4mmt - 2tnn - 2tss = c$, and $ttnn - ttss = d$; then $-bxx + cx = d$; and, extracting the root, $x = \frac{c}{2b} - \sqrt{\frac{cc}{4bb} - \frac{d}{b}} = 4.10$ inches; then $2x = 8.20$ inches = the part wanting DC: from whence we find HF the distance from the greatest bulk of the cask to the lesser head to be $= 26.6$ inches, and CH to the greater head 18.4 inches. And the content is 143.9 ale gallons.

Mr. Grimmett, after two different solutions to this question, delivers this theorem:

Theorem. { The square root of each difference between the square of half the bung diameter and the square of half the diameter of each head, put into one sum: It will be as the sum is to either of those roots, so is the length of the cask to the distance of the respective head from the bung.*

IV. QUESTION 166 answered by Mr. Grimmett.

If from unity be taken how many numbers soever in double proportion continually, until the whole added together be a prime number; and if this whole be multiplied by the last term of the series which constitutes the prime, the product will be a perfect number. 36 Euclid 9.

From such a series it may be observed, that any term made less by unity, will be = the sum of all the preceding terms. Put therefore $a = 2$; and $x =$ its variable exponent (for in the first operation it will represent 1, in the next 2, and then 3, &c. till it be raised to a^{x+1} and being lessened by unity may be a prime number. Thus

x being

---

* III. QUESTION 165.

By the nature of the ellipse, $\sqrt{HN^2 - CB^2} : \sqrt{HN^2 - FE^2} :: CH : HF$; and, by composition, &c. $\sqrt{HN^2 - CB^2} + \sqrt{HN^2 - FE^2} : CF :: \begin{cases} \sqrt{HN^2 - CB^2} : CH \\ \sqrt{HN^2 - FE^2} : HF \end{cases}$, which therefore both become known; and this is Mr. Grimmett's theorem, mentioned above.

Again, by the nature of the ellipse, $\sqrt{HN^2 - CB^2} : CH :: HN : HA$ the semi-transverse; which being thus found, the contents of the two parts $BNRG$, $NEOR$, of the cask being computed separately by the common rules, their sum will be the whole content.——Or indeed their contents are easily computed without the transverse axe by rule 1 page 278 of my Mensuration.

N. B. The two expressions marked * in Mr. Lovatt's solution are wrong printed; but they are here given as they stand in the original, as it is not easy to distinguish what are the true expressions meant.

$$x \text{ being} = \left\{\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ \&c. \end{array}\right\} \overline{a^{x+1}-1} \times a^x = a^{2x+1} - a^x = \left\{\begin{array}{ll} 6 & \text{perf. num.} \\ 28 & \text{perf. num.} \\ — & \text{abounding} \\ 496 & \text{perf. num.} \\ — & \text{abounding} \\ — & \text{dimunitive} \\ & \&c. \end{array}\right.$$

Whence the perfect numbers are 6, 28, 120, 496, 2016, 8128, 32640, 130816, 523776, 2096128, and 8386560; all the perfect numbers required per question.

Mr. C. Mason *the proposer gives this rule:*

$\overline{1+2}\times 2 = 6$; $\overline{1+2+4}\times 4 = 28$; $\overline{1+2+4+8}\times 8 = 120$, &c.

Mr.

---

* IV. QUESTION 166.

By *Eucl.* IX. 36, $:1+2+2^2+2^3+2^4+$ &c. to $2^n : \times 2^n$ is a perfect number when the sum of the series is a prime number; but the sum of the geometrical series is $2^{n+1}-1$, therefore $\overline{2^{n+1}-1} \times 2^n$ is a perfect number when $2^{n+1}-1$ is a prime number. Taking $n=0$; then $2^{n+1}-1$ is $=1$ a prime, and $1 \times 2^0 = 1 \times 1 = 1$ the first perfect number: If $n=1$; then $2^{n+1}-1=3$ a prime, and $3 \times 2^1 = 6$ the next perfect number: If $n=2$: then $2^{n+1}-1=7$ a prime, and $7 \times 2^2 = 28$ the 3d perfect number: If $n=3$; then $2^{n+1}-1=15$ which is not a prime, and therefore $15 \times 2^3 = 120$ is not a perfect number: In like manner it will appear that no other greater odd number can be put for n so as to make the expression $\overline{2^{n+1}-1} \times 2^n$ a perfect number; n must therefore be always an even number for finding the other perfect numbers; but it cannot be any even number, as some have falsely asserted. Dr. Harris says that there are only ten perfect numbers between 1 and 1,000,000,000,000.

This rule of Euclid's only demonstrates that a number found by it will be a perfect number; but neither it nor any other that I know of, shew that there may not be other perfect numbers besides those found by this rule.

C 2

Mr. Sam. Ashby *answers thus:*

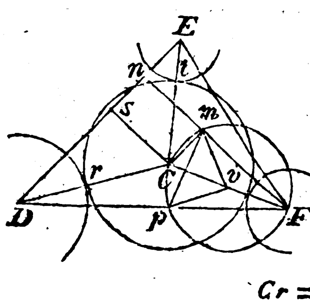
The canon. { If from any power of 2 be subtracted unity, and that remainder be a prime number, multiply it by half the said power, and that product will be a perfect number.

Mr. Robert Fearnside's *answer.*

Let $y^n x$ be the number sought; its aliquot parts will be $1+y+y^2+y^3$, &c. till the exponent becomes n; and $x+yx+y^2x+y^3x$, &c. till the exponent be likewise n; then, from the nature of a perfect number, $1+y+y^2+y^3$ + &c. $+x+yx+y^2x+y^3x+$ &c. $=y^n x$; and consequently $x=\frac{1+y+y^2+y^3 \text{ \&c.}}{y^n-1-y-y^2-y^3}$: Now, that x may be a whole number, 'tis requisite that y^n-1-y- &c. be $=1$, which only happens when y is $=2$; whence the canon required becomes $2^n x$. If $n=1$, x will be $=1+2$, and the first perfect number $=6$. If $n=2$, x will be $=1+2+4$, and the second perfect number will be $=28$. If $n=4$, the third perfect number $=496$. If $n=6$, the fourth perfect number is 8128. If $n=8$, the fifth number is 130816. If $n=10$, the sixth number will be 2096128; which are all the perfect numbers from unity to ten millions.

Woolfius in his *Elem. Math.* supposes n to be successively the numbers 1, 2, 3, 4, 5, &c. which will not hold when n is supposed = the odd numbers 3, 5, 7, 9, &c. and Mr. *Cunn*'s rule for finding a perfect number will not find all the above numbers; see p. 12 of his Decimals.

V. QUESTION 167 *answered by Mr.* Turner *the proposer.*



This problem is taken from lemma 6 book 1 of Sir Isaac Newton's Principia, where the geometrical construction may be seen.

Having the three sides of the triangle, the perpendicular Fn, and segments Dn and En, may be found.

Draw Cs perpendicular to DE, and Cm perpendicular to Fn; and put

$Cr =$

$Cr = Ct = Cv = a$, $Fn = 345{\cdot}8 = b$,
$Cm = sn = e$, $En = 70{\cdot}7 = c$,
$mn = Cs = u$, $Dn = 344{\cdot}3 = d$. Then is
$Dr = 130 = r$, $Ds = d - e$, and
$Fv = 70 = s$, $Es = c + e$; also
$Et = 80 = n$, $Fm = b - u$: Then

$$\left.\begin{array}{l}(1)\ rr + 2ra + aa = dd - 2de + ee + uu \\ (2)\ nn + 2na + aa = cc + 2ce + ee + uu \\ (3)\ ss + 2sa + aa = bb - 2bn + uu + ee\end{array}\right\}\text{per 47 Euc. 1.}$$

Subtract the third step from the first, and transpose the terms, and dividing by $2b$, we shall have

$$(4)\ u = \frac{rr - ss + 2ra - 2sa + bb - dd + 2de}{2b}.$$

Put $rr - ss + bb - dd = f$; and $2r - 2s = g$; then $u = \frac{f + ga + 2de}{2b}$; and $uu = \frac{ff + ggaa + 2fga + 4fde + 4dgae + 4ddee}{4bb}$

Subtract the second step from the first, and transposing makes $2de + 2ce = dd - cc - rr + nn + 2na - 2ra$.

Put $2d + 2c = h$; $dd - cc - rr + nn = k$; and $2n - 2r = -l$;
(5) Then $e = \frac{k - la}{h}$; and $ee = \frac{kk - 2kla + llaa}{hh}$.

In the first step substitute the value of uu found, then $rr + 2ra + aa = dd - 2de + ee + \frac{ff + 2fga + ggaa + 4fde + 4dgae + 4ddee}{4bb}$, or $4bbrr + 8bbra + 4bbaa - ggaa - 2fga - ff - 4bbdd = 4bbee + 4ddee + 4fde - 8bbde + 4dgae$: Put $4bb + 4dd = m$; $4fd - 8bbd = -n$; and $4dg = p$; then $4bbrr + 8bbra + 4bbaa - ggaa - 2fga - ff - 4bbdd = mee - ne + pae$; for e and ee put the values above found and bring it out of the fractions, will give this quadratic equation:

$$\left.\begin{array}{l}+4bbhh \\ +plh \\ -gghh \\ -mll\end{array}\right\} aa \left.\begin{array}{l}+8bbhhr \\ +2mkl \\ -2fghh \\ -nlh \\ -pkh\end{array}\right\} a = \left\{\begin{array}{l}4bbddhh \\ +ffhh \\ +mkk \\ -4bbrrhh \\ -nkh.\end{array}\right.$$

In numbers $aa + 195{\cdot}085a = 52778{\cdot}24$ when divided by the coefficients of the highest power; and, extracting the root, $a = 152{\cdot}048$ the radius of the circle touching the three circles given in position and magnitude; consequently the diameter is $= 304{\cdot}096$ the answer.

Mr. *George Brown*, after giving the true answer in a very concise way, says, since it is not limited how the three circles shall be placed on the angular points, it will admit of so many answers as the circles are to be varied.

Mr. Rich. Lovatt'*s answer to the same.*

Put $b = \frac{488^2 - 130^2 - 70^2}{488 \times 2}$; $d = \frac{130 - 70}{488}$; $n = \frac{353^2 - \overline{80 - 70}|^2}{353 \times 2}$; $m = \frac{80 - 70}{353}$; $p = \frac{DFE}{2}$; and $a = CE$. Then, per axiom 4, $b - ad = pE$; and $n - am = mF$; and CF will be the diameter of a circle, whose periphery will pass through mCF; and (20 Eucl. 3) the angle at the center, v, is double to the angle, F, at the periphery; also (5 Eucl. 1) the angle $vpm = vmp$. Put $h =$ s. $\angle vmp = 33° 34'$; $k =$ s. $\angle mvp = 112° 56'$; then $h : \frac{a}{2} :: k : \frac{ak}{2h} = mp$. And by that noble theorem which I mentioned in the Diary 1731, quest. 143, we have this analogy: $\overline{b - ad} \times \overline{n - am} : \frac{b - ad - n + am}{2} + \frac{ak}{2h} \times \frac{-b + ad + n - am}{2} + \frac{ak}{2h} :: 1 : pp$; and $pp \times \overline{b - ad} \times \overline{n - am}$

$$= \frac{b - ad - n + am}{2} + \frac{ak}{2h} \times \frac{-b + ad + n - am}{2} + \frac{ak}{2h}.$$

In numbers, $1682aa + 9.071a = 9473.58 + 1023.9$, and $a = \sqrt{63140.3} - 27 = 224.273$; hence $224.773 - 70 = 154.273 = Cr$, which doubled is the diameter 308.56 of the circle required.

VI. QUES-

---

* V. QUESTION 167.

This is one of the problems of APOLLONIUS on *Tangencies*, and is constructed by his restorers VIETA and our countryman the Rev. Mr. JOHN LAWSON, who has lately published an english restoration of this piece of APOLLONIUS's works, where it appears that the problem hath several cases according as the fourth circle is to touch the other three either all internal or all externally, or else some internally and the rest externally.

This problem has also been attended to by several other respectable persons, it being constructed by Sir ISSAC NEWTON in *lemma* 16 *lib.* 1 of the *Principia*, and in his *Universal Arith.* prob 47; by the MARQUIS DE L'HOSPITAL in his *Sections Conique* liv. 10 ex. 4 cor. 1; and by Mr. THO. SIMPSON at the end of his *Geometry.*

Con-

* VI. QUESTION 168 *answered by Mr.* Rob. Fearnside.

Let A, B, C, represent the three players; A wants 5 of being up, B 7, and C 10. Now it is plain the game will be ended in 20 throws at most; then $A+B+C$ must be raised to the 20th power; and as the players here are supposed equal, the coefficients of every term where the 5th power of A and upwards including the 20th is found, are to be added together, as also the coefficients where the 7th power of B and upwards is found, and the coefficients of the 10th power of C and upwards are to be added also together; these three totals will be in proportion to one another as the respective shares they are to have of the guineas.

He

Concerning this problem I shall also insert the following extract from the *Histoire des Mathematiques* par. M. MONTUCLA *Tom.* I. p. 263.

VIETA, in a dispute which he had with ADRIANUS ROMANUS, proposed to him this question: The solution which ROMANUS gave to it, though obvious, was very indifferent, viz. by determining the center of the circle sought in the point of intersection, of two hyperbolas; for as the problem is a plane one, it may be solved by plane geometry; by this VIETA solved it, and very elegantly: his solution is the same as that given in NEWTON's *Universal Arithmetic*. Another solution may also be seen in the 1st book of the *Principia* (this question being there necessary for some determinations in *Physical Astronomy*) wherein NEWTON, by a remarkable dexterity, reduces the two solid loci of ROMANUS to the intersection of two right lines.——Moreover, DESCARTES attempted to solve this problem by the help of the Algebraical Analysis, but without success; for of the two solutions which he derived from thence, he himself acknowledges (see Lett. Tom. III. let. 80, 81) that one furnished him with so complicated an expression, that he would not undertake to construct it in a month; whilst the other, though somewhat less complicated, was not so very simple, as to encourage him to set about a construction of it.——Lastly, the Princess ELIZABETH of *Bohemia*, who, it is well known, honoured DESCARTES with her correspondence, deigned to communicate a solution to this Philosopher; but as it is deduced from the algebraical calculus, it labours under the same inconveniences as that of DESCARTES.

* VI. QUESTION 168.

The method of solving questions of this kind, may be seen at page 43 or 192 of DE MOIVRE, or in some other books on *Chances*.

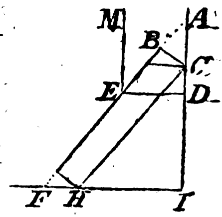
		l.	s.	d.
He who got 10 must have		2	3	10 +
He who got 8 must have		0	16	$1\frac{3}{4}$ +
He who got 5 must have		0	3	0 +

Mr. *John Ommanney*'s numbers are the same.

Mr. *James Hemmingway* and Mr. *Chr. Hale* also answered this question.

The PRIZE QUESTION *answered.*

Let FI represent the floor, and $ADEM$ the chimney; then put $BC = d$, $ED = b$, $DI = c$; $LB = x$. Now, per similar triangles, $x : d :: d : \frac{dd}{x} = BA$; and $d : \frac{dd}{x} ::$ $b : \frac{bd}{x} = AD$. After the same manner of reasoning we shall find $AC = \frac{d}{x}\sqrt{dd + xx}$; $CI = \frac{bd + cx}{x} - \frac{d}{x}\sqrt{dd + xx}$; and $CH = \frac{bd + cx}{dx}\sqrt{dd + xx} - \frac{dd}{x} - x$; which must be a maximum; consequently put into fluxions, &c. the following equation will come out; *i. e.*

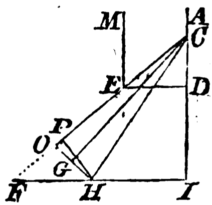


$$\left.\begin{matrix}cc\\-dd\end{matrix}\right\}x^6 + d^4x^4 - 2bcd^3x^3 + d^6x^2 + \left.\begin{matrix}bbd^6\\-d^6\end{matrix}\right\} = 0.$$

Brought into numbers and reduced, $x = 9.74$ inches; and consequently $CH = 175.2$ inches $= 14$ feet 7 inches 2 tenths, the length of the cylindric pole.

For the Conical Pole.

Let $ED = b$; $DI = c$; and $r = 4$; $CD = x$; $HO = 2y$: then $1 : r :: 2y : HC = 2ry$; $GC = y\sqrt{4rr - 1} = ny$ (putting $\sqrt{4rr - 1} = n$); as $2ry : ny :: 2y : PH = \frac{ny}{r}$; $PO = \frac{y}{r}$. Again, $x : \sqrt{bb + xx} ::$ $c + x : FC = \frac{c + x}{x}\sqrt{bb + xx}$; and $x : b :: \frac{ny}{r} : PF = \frac{bny}{rx}$. Consequently



bny

$\frac{bny}{rx} - \frac{y}{r} = \frac{c+x}{x}\sqrt{bb+xx} - 2yr$. Ergo (putting $2rr - 1 = m$) $y = \frac{cr+rx}{bn+mx}\sqrt{bb+xx}$. Now it is evident that $GC = \frac{cnr+nrx}{bn+mx}\sqrt{bb+xx}$ must be a maximum; which in fluxions, &c. the following equation will come out, viz. $x^3 + \frac{2bn}{m}x^2 + \frac{bcn}{m}x = b^2c - \frac{b^3n}{m}$; brought into numbers and reduced, gives $x = 44{\cdot}88$ inches, and $GC = 168{\cdot}48$ inches $=$ 14 feet 0 inches, 48, the length of the conical pole required.*

The lot of 10 diaries fell to *F. R. S.*

Of

---

* *The* PRIZE QUESTION.

It is true the process above will bring out the longest pole which can be put quite into or up the chimney, but some of the expressions used in it are very improper: thus the expression for *CH* in the former case, and *CG* in the latter, is not a maximum, but a minimum; for it has no maximum but infinity; and the thing to be found, though it be the longest pole that can be put quite up the chimney, is the minimum of *CH* or *CG*, that is the shortest pole which can rest with one end on *FI*, the other on *AI*, and its side touch the point *E*: for it is evident that whether way this line be moved from this narrowest or shortest position, its side will fall below the point *E*, and so it may be put up the chimney; but a longer cannot be put into the said shortest position, and therefore not up the chimney.

The former part of the process, for determining the length of the cylinder, may be brought out by a simple cubic equation thus: Put $AD = z$; also $DE = b$; and $DI = c$, as above. Then $AI = c + z$, and $z : b :: c + z : \frac{c+z}{z} \times b = IF$; hence $\overline{c+z}|^2 + \frac{\overline{c+z}|^2}{z^2} \times b^2 = AF^2$, which must be a minimum. This in fluxions, &c. we get $z^4 + cz^3 - b^2cz - b^2c^2 = 0$; the root of which is evidently $z = \sqrt[3]{b^2c} = AD$, from which the position is determined, and the length of the cylinder can be easily expressed in terms of b, c, and d its diameter; thus the length CH is $= \frac{c+bq}{\sqrt{1+q^2}} \times q - dq$, putting $q = \sqrt[3]{\frac{c}{b}}$.

Of the Eclipses in 1733.

To the inhabitants of our terraqueous globe, there will happen four eclipses: Twice will the moon in her wandering course interpose and hide the splendid rays of the sun from our view; and twice will the earth in its course so fall in the line between the sun and moon, as to hinder her receiving the light she borrows from the sun, to enlighten the earth by reflection.

1. Sun eclipsed on wednesday the 2d of May, at 6 in the evening, which will be a great and visible eclipse; and three-fourths of the sun's diameter will be darkened.*

Computed by		Begin. h.	m.	Mid. h.	m	End h.	m.	Dur. h.	m.	Digits d.	m
Astronom. Carolina,	Coventry	v	41	vi	35	vii	26	i	45	ix	45
Mr. Chattock,	London	5	44	6	38	7	29	1	46	9	6
Mr. Leadbetter,	London	5	42	6	36	7	27	1	45	9	20
Mr. Tho. Sparrow,	Notting.	5	40	6	37	7	31	1	5[illegible]	9	49
	Royston	5	49	6	45	7	40	1	50	9	51
Mr. Christ. Hale,	Derby	5	38	6	35	7	23	1	44	9	35
	London	5	43	6	38	7	27	1	43	9	19
Mr. Samuel Travis,	Utoxeter	5	39	6	34	7	28	1	49	9	20
Mr. Will. Brown,	Bridgenor.	5	37	6	31	7	22	1	45	9	46
	Gloucester	5	38	6	33	7	23	1	45	9	39
	Rome	6	53	7	39	8	24	1	21	8	12
	Edinburgh	5	30	6	24	7	17	1	47	10	31
	Paris	6	4	6	55	7	43	1	39	9	11
Mr. Will. Lovatt,	Mansfield	5	40	6	35	7	29	1	49	9	15
Mr. John Browne,	London	5	43	6	38	7	29	1	46	9	53
	Benhall	5	48	6	43	7	34	1	46	10	8
Mr. Tho. Wright,	Sunderland	5	29	6	25	7	17	1	47	9	47
Mr. John Bulman,	Lewish.	5	36	6	41	7	20	1	44	9	43
	Carlisle	5	24	6	30	7	10	1	46	10	1
Mr. Nicholas Oats,	Falmouth	5	3	6	56	6	52	1	49	8	4
Mr. John Turner,	Hull	5	18	6	13	7	6	1	48	10	12
Mr. Tho. Williams,	Middleton	5	41	6	36	7	27	1	45	9	35
Mr. Rich. Lovatt,	Derby	5	38	6	35	7	23	1	44	9	35

2. Moon

---

* This eclipse was observed thus:

At *Gottenburg* in *Sweden* by *D. Birgerus Vassenius.*

h.	m.	s.	
6	26	40	Before this was the beginning.
7	14	46	Total immersion.
7	16	54	Emersion.
8	5	50	End.

2. At

2. Moon eclipfed on thurfday the 17th of May, at 6 in the evening.

Computed by	Begin. h.	Begin. m.	Mid. h.	Mid. m.	End h.	End m.	Dur. h.	Dur. m.	Dig. h.	Dig. m.
Aftronom. Carolina, Coventry	5	51	7	21	8	50	2	58	8	22
Mr. Sparrow, Nottingham	5	47	7	17	8	46	2	59	8	27
Mr. Hale, Derby	5	26	6	57	8	28	3	2	8	3
Mr. Browne, Bridgenorth	5	47	7	16	8	46			8	23
Mr. Bulman, Lewifham	5	57	7	26	8	56	2	59	8	22

3. Sun eclipfed the 26th of October, at 5 in the evening, invifible.

4. Moon eclipfed the 10th of November, at 1 in the afternoon, invifible.

New

---

2. At *Wittemberg* in *Saxony* by *John Frid. Weidler.*

h.	m.	s.	
6	36	5	Beginning.
7	29	20	Eleven digits.
7	46	5	Sun fet eclipfed.

At	By	Begin. h. m. s.	Middle h. m. s.	End h. m. s.	Dig.
3. London	G. Graham	[illegible] 44 45	6 37 3	7 28 23 ap.t.	$9\frac{4}{5}$
4. Norton Court near Feverfham in Kent	Ste. Gray	[illegible] 49 15	6 40 [illegible]	7 32 30 ap.t.	$9\frac{7}{8}$
5. Otterden Place near Lenham in Kent	Granv Wheler Efq.	[illegible] 49 0		7 31 49 ap.t.	
6. Yeovil in Somerfetshire	J Milnet	5 34 0		7 14 30	

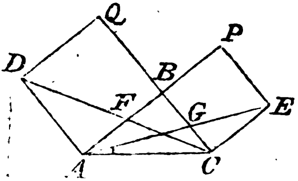
New Questions.

I. QUESTION 169, *by Mr.* Will. Grimmett.

In a certain dictionary, under the word *Conoid*, it is said, the solidity of an hyperbolic conoid is to its circumscribing cylinder as 3 to 10; and in an appendix of fluxions the same is also asserted; which is certainly false. Be pleased therefore to investigate the expression that does expound their ratio; and when you are in this way of thinking, suppose the generating hyperbola to become the plain of a west-declining dial, in the latitude of 50° north, and the focus to be the center of the same, in which, if you erect a wire perpendicular to the plain, the sun on its first shining on the plain, the 12th of May, will cast the shadow of the wire, so erected, exactly on the hour line of 8. Quere the declination of the plain.

II. QUESTION 170, *by Mr.* Sam. Ashby.

You that would learn the art and mystery
Of mathematics, learn geometry.
The first six books of Euclid are the best;
Which being known, you'll easily learn the rest.
And then, to put in practice what you know,
Observe a proposition here below;
On which, if you'll be pleas'd some time to spend,
You'll much oblige your mathematic friend.



If upon each leg *AB* and *BC*, including the right angle, be drawn a square *BD* and *BE*; and the lines *DC* and *EA*, which cut the said legs at *F* and *G*. I say, *BF* and *BG* are equal, and are each a mean proportional between the segments *AF* and *GG*; that is, as *AF* : *FB* :: *FB* : *GC*, &c. Quere the demonstration geometrically.

III. QUES-

III. Question 171, by Mr. Geo. Brown.

One morning fair bright Phœbus did display
His glorious rays over the northern sea.
There from a port in * fifty-one degrees,
Three ships set sail, their † course as here you see;
Then each ship chang'd her course, and did another get;
And when an equal distance run, they all did meet.
Now each ship's second course and distance run,
Likewise the same from whence they first did come,
Unto this place where now they lie,
With its latitude, is what you're to descry?

* *Of latitude.* † *S. E.* 33·8. *S. S. E.* 49. *S. S. W.* 35·5 *leagues.*

IV. Question 172, by Mr. Chr. Hale.

Suppose the product of two lines — *Inches.*
Be as the margin here defines; — 2332800
The third line then I fain would know,
That will the greatest area shew;
For that exactly will descry
The height of All Saints at Derby.

V. Question 173, by Mr. John Turner.

Two ships sail from a certain port to sea,
Unto two ports whose latitudes agree.
The first she sails between the south and east,
The other makes her way 'twixt south and west.
If both their courses you together join,
'Twill make degrees 112, and minutes twenty-nine:
The ship's departure which to the eastward went,
Is miles two hundred twenty-nine, and seven-tenths:
Their distances must this * proportion bear
Unto each other. Whence I pray declare
Each ship's true course, departure, distance run,
And latitude of the port where they begun?

* *As* 12 *to* 5, *westermost ship's distance run, being the greater.*
N. B. *They arrived in the latitude of* 35° 34' *north.*

VI. QUESTION 174, *by Mr.* Ri. Lovatt.

When mighty Newton the foundation laid
Of his mysterious art; none cou'd invade
Nor take from him the honour which was due;
Great Britain's sons will long his works pursue.
By curious theorems he the moon cou'd trace,
And her true motion give in every place;
The greatest areas he with ease cou'd shew,
It is from him alone the art we know; 3000
And to confirm the same, let us suppose 2000
The greatest area that we can inclose 1500
In four right lines, such as the margin shews. *a*
He that a theorem gives, shall have his name
Recorded in the ladies' book of fame.

Quere *a*, and the greatest area.

The PRIZE QUESTION, *by Mr.* Fearnside.

A young lady for some time a meadow has own'd,
In form of a right-angled triangle found,
Whose base I could measure, and found it to be
Chains ninety and five, links twenty and three:
But the other two sides were with water o'erflown,
So their lengths, tho' attempted, then could not be known.
Now a ditch issuing out from the * angle at th' base,
Made with the hypothenuse just fifteen degrees;
And in the cathetus a tree I espy'd,
Which in two equal parts did exactly divide
That part of the said perpendicular, or space
Included 'twixt where the ditch cross'd it, and base;
And if to the opposite corner you draw,
Or suppose a line drawn, from the tree, you must know,
It equally cuts the whole angle in two.
Now by will it was order'd, that who of this meadow
Could find the content, might marry the lady.
An admirer I've been, and there's nothing remains
But this to compensate my care and my pains.
Therefore if by these hints you aught can advance,
At the wedding, fair ladies, I invite you to dance.

* *i. e.* of the triangular meadow.

1734.

Questions answered.

I. QUESTION 169 answered.

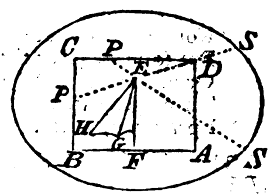
LET the abſciſſa of the hyperbola be $=x$, the ordinate $=y$, the parameter $=b$, and the tranſverſe $=a$; then the nature of the curve will be expreſſed by $yy=bx+\frac{bxx}{a}$; then by the doctrine of fluxions $\frac{pbxx}{4r}+\frac{pbxxx}{6ra}$ will expreſs the ſolidity of a conoid whoſe altitude is $=x$, and the radius of its baſe $=y$: And the ſolidity of its circumſcribing cylinder will be expreſſed by $\frac{pbxx}{2r}+\frac{pbxxx}{2ra}$: Conſequently their ratio will be expreſſed by $\frac{3}{2}a+x$ to $3a+3x$, which is agreeable to that deduced from the method of indiviſibles.

If we conſider ſuch a one whoſe altitude is equal to the tranſverſe axis of the generating hyperbola; the ratio will be expounded by that of 5 to 12.

When the ſun is in the plane of any dial, the ſhadow of the ſtyle, and that of a wire erected perpendicular to the plane in the center of that dial, will be coincident.

Demonſtration.

Let $ABCD$ repreſent the plane of a dial coincident with the plane of the paper; then the point E will be the perpendicular wire, EF the hour line of 12, EH the ſtyle, EG the ſubſtyle; SS a parallel the ſun deſcribes, which will here become an ellipſis. Now the ſun in ſome point of the orbit will be in the plane of the dial, ſuppoſe at S or S; then its plane by the



line drawn from the ſun through the center of the dial will

 give

give the shadow of the style on the plane, as SP or SP; for a line drawn from the sun through any other point of the style more remote from the plane will not fall on the plane. Again the shadow of a wire erected perpendicular on a plane, when the sun is in that plane will be coincident with a line drawn from the sun through the interfection of the wire with that plane (which is here said to be the center of that dial) consequently when the sun is in the plane, the shadow of a style (however inclined) and the shadow of the wire erected perpendicular in the center will be coincident. *Q. E. D.*

Therefore from the latitude of 50 N. the declination on the 12th of May and hour of 8, the azimuth will be found 99° 48′ from the north, which is the declination of the plane from the point.

Mr. *Fearnside* has given the same answer. Mr. *Turner*, Mr. *Smedley*, Mr. *Ommanney*, Mr. *Dunthorne*, Mr. *Quank*, Mr. *Coltourn*, and some others, make the declination 9° 9′, or 9° 14′, and the azimuth 80° 46′.

II. Question 170 *answered by Mr.* Rob. Fearnside.

'Tis plain, by similar triangles, that $CB+AB : AB :: BC : FB$. Again $CB+AB : BC :: AB : GB$. Permutando $CB+AB : AB :: BC : GB$. Ergo $FB = GB$. *See the fig. to the quest.*

For the other part of the demonstration; by similar triangles $AB\ (=AD) : AF :: BC : BF$; and $AB : BG :: BC\ (=CB) : CG$. Therefore, *ex æquo* $AF : BG :: BF : GC$. *Q.E.D.*

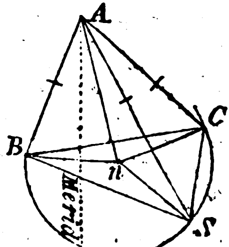
A Grubean Lady's answer to the same.

To oblige you, mathematic friend, Sam. Ashby,
I'll prove your wond'rous prop. but not too rashly.
First, to your scheme let's add the *Q* and *P*,
Not out of absolute necessity;
But to make clear the proof, as nose on face,
Or plain as pike staff; that is all the case.
Then, geometric friend, without more fuss,
I pray you, take the demonstration, thus.

The triangles APE and ABG, being similar, it is $AP : PE :: AB : BG$; (by Euclid 4. 6) that is $AB + BC : BC :: AB : BG$, because all the sides of a square are equal. In like manner the triangles CQD and CBF, being similar, it it is $QC : CB :: DQ : FB$; *i.e.* $AB + BC : BC :: AB : FB$. Therefore FB is $= BG$. Again, the triangles DAF and FBC, and the triangles ABG and ECG are similar; therefore (DA or) $AB : BC :: AF : FB$; and $AB : BC$ $(= CE) :: BG : GC$. Whence $AF : FB :: BG$ (or BF) $: GC$ (by Eucl. 11. 5). *Q.E.D.*

III. QUESTION 171 *answered by Mr.* J. Turner.

This is altogether solved by trigonometry. For, first, in the triangles BAS, CAS, there is given two sides and a contained angle, to find BS, and the $\angle BSA$, and CS and the $\angle CSA$; add these two angles together, and then you have in the $\triangle BSC$ two sides and a contained angle, to find BC: Now in the $\triangle BnC$, the angle at n is double the $\angle BSC$, and being isosceles, it is easy to find $Bn = Cn = Sn = 19{\cdot}44$ leagues, each ship's 2d distance run, from B, S, C, to meet at n. Again, in the $\triangle ABC$, find the $\angle$s ABC and ACB, to which add the $\angle$s CBn, BCn, we find that the 2d course of the first ship steered



S. S. W. from B to n is E. by N. 5° 26′ easterly,
2 ship S. E. C to n is W. S. W. 1 37 westerly,
3 ship S. S. E. S to n is N. W. 3 12 northerly.

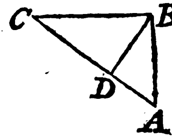
Lastly, in the $\triangle BAn$, there is given BA, Bn, and the angle contained; to find $An = 31{\cdot}4$ leagues, the distance from the port A to where they all meet: And by the $\angle BAn$ found, the course from A to n is S. by E. 34′ southerly. The difference of latitude of the ships when at n, $= 30{\cdot}7$ leagues.

The same is answered in this manner by Mr. *Fearnside*, Mr. *Skews*, Mr. *P. Sharp*, Mr. *Quant*; Mr. *Wooler*, Mr. *Colbourn*, Mr. *Hemmingway*, and Mr. *Oats*; which agree so near to the proposer's answer and one another, that I presume I need not to exhibit any of them at large.

IV. QUESTION 172 *answered.*

I have received a great many answers to this question, but they generally agree that it is unlimited, or else they do not rightly understand the proposer's meaning; but if that product be broke into two equal lines, making the legs of a right-angled triangle, the hypothenuse will be 180 feet. I shall here give you the proposer's own solution.

Put $dd = 2332800$, and $a = AC$. $a : d + d :: d - d : 0 = AD - DC$: And $\frac{a}{2} = AD$.



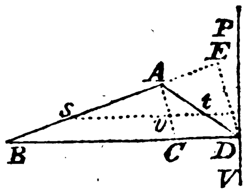
$\frac{4ddaa - aaaa}{16}$ = greateſt area ſquared.

In fluxions, $8dda\dot{a} - 4a^3\dot{a} = 0$: Reduced gives $a = 2160$ inches. The exact height of All-ſaints tower 60 yards.*

V. QUESTION 173 anſwered by Mr. Fearnſide.

Put $b = CD$, $d = \frac{12}{7}$, ſine of the ſum of the courſes $112° 29' = c$, coſine $= a$, $x = AE$.

Then $a : x :: 1 : AD = \frac{x}{a}$, and $DE = \frac{cx}{a}$, $BA = \frac{dx}{a}$, $CA = \sqrt{\frac{xx}{aa} - bb}$. Then $BD^2 = xx + \frac{2dxx}{a} + \frac{ddxx}{aa} + \frac{ccxx}{aa} =$ (putting $p^2 = 1 + \frac{2d}{a} + \frac{dd}{aa} + \frac{cc}{aa}$) $ppxx$, therefore $px = BD$, and $px - b = BC$: Per ſimilar triangles, $px : x + \frac{dx}{a} :: \frac{dx}{a} : px - b$; hence x will be found $= \frac{pbaa}{ppaa - ad - dd} = 134{\cdot}28$ miles. Conſequently $BA = 842{\cdot}6$ miles, $AD = 351{\cdot}15$ miles, $BC = 799{\cdot}79$ miles, $\angle ABC = 18° 12' 14''$, $BAC = 71° 37'$, $CAD = 40° 51'$, and latitude $31° 45'$.



The

---

* IV. QUESTION 172.

Since the product of any two ſides of a triangle drawn into the ſine of their included angle produces double the area; therefore when the product of the ſides is given, the area will be as the ſine of the angle; but the right angle has the greateſt ſine, therefore the triangle is right-angled when a maximum. And then into whatever two parts the given product is broken, the area will be ſtill the ſame; but then the hypothenuſe will vary, and is ſhorteſt when the triangle is iſoſceles, in which caſe it is $= \sqrt{2 \times 2332800} = 2160$ inches $= 60$ yards.

The same answered by Mr. Turner, *the proposer.*

Though this question may be solved by a simple quadratic equation, yet I always prefer conciseness in mathematics, and send you the geometric construction, and trigonometric calculation, which are vastly easier than the algebraic operation.

Construction.

Assume any 2 lines as *As*, *At*, in the ratio of 12 to 5, and let them contain the given angle *sAt*, produce the lines indefinitely, and let fall *Av* perpendicular to *st*, and at the the distance of 230 draw a parallel to *Av*, as *VP*, and from where it cuts *At* produced as at *D*, draw *DC* parallel to *ts*, and where it meets *As* produced will form the triangle required.

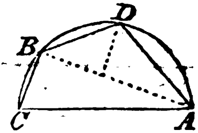
Solution.

As, *At*, being assumed = any numbers in the ratio of 12 to 5, and the angle contained given: Find *st*, and thence *vt* and *vA*. And then it will be as $vt : vA :: GD : CA$, from whence all the rest are easily known. *Miles.*

The course of one ship is S. 71° 38′ westerly, dist. run 843·7
the other S. 40 52 easterly, —— 351·5
Latitude of the port sailed from 40° north. Difference of latitude 266 miles.

VI. QUESTION 174 *answered by the proposer.*

Given $AD = 3000 = m$, $DB = 2000 = b$, $BC = 1500 = 2p$. Put $\overline{AD+DB} \times \overline{AD-DB} = d$, and $AB = x$. Then per 4 and 47 Eucl. I, $\sqrt{\dfrac{4m^2x^2 - x^4 - 2dx^2 - d^2}{4}}$ = the area *ABD*; and px = area *ABC*, which is right-angled when the trapezium becomes a maximum. The



sum in fluxions is $\dfrac{2m^2x\dot{x} - x^3\dot{x} - dx\dot{x}}{2\sqrt{4m^2x^2 - x^4 - 2dx^2 - d^2}} + p\dot{x} = 0$:
Substitute $n = 2m^2 - d$, then $x^4 - 2nx^2 + 4p^2x^2 = 16m^2p^2 - 8dp^2 - \dfrac{4n^2d^2}{x^2} - n^2$. This affected equation

will

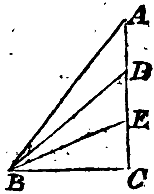
will be reduced to a quadratic $x^2 - 1187500 = 5217800x$ $= 4134\cdot34,3$ then $AC = 4398\cdot1$; and the greatest area $5921925\cdot2575$.

Or, when the area of the trapezium becomes a maximum, it will be inclosed in a semicircle, and the variable line AC will be the diameter. Therefore making $a = AC$, we have $a = 4398\cdot1$.*

The PRIZE QUESTION *answered.*

Given $BC = 95\cdot23$ chains $= b$, sine $\angle ABD = 15° = s$, $\angle ABE = EBC$, and $DE = EC$. Put $x = s.\angle EBC$; then $\sqrt{1-xx} : b :: x$

$CE = \dfrac{bx}{\sqrt{1-xx}}$; hence $\dfrac{2bx}{\sqrt{1-xx}} = DC$.



Then, per Euc. 47. I, $BD = b\sqrt{\dfrac{3x^2+1}{1-x^2}}$. Again the cosine of double the $\angle EBC$ = sine of the $\angle A$ will be found to be $= 1 - 2x^2$; then $1 - 2x^2 : s :: BD :$ $DA = \dfrac{bs}{1-2x^2}\sqrt{\dfrac{3x^2+1}{1-x^2}}$. Then by 3d prop. 6 Euclid, $BC : BA :: EC : AE$; which analytically expressed, and the equation ordered, becomes $4x^6 - 3s^2x^2 - ss = 0$. Reduced, the sine of the $\angle EBC$ will be found to be that of $34° 27' 47''$, and $CA = 247\cdot13$; consequently the area of the meadow will be $= 1176\cdot709$ acres.

Of

---

* VI. QUESTION 174.

Since the trapezium (or any other figure) will be a maximum when it is inscribed in a semicircle, the unknown side being the diameter, as is now generally known; there is no occasion for the use of fluxions in the operation, it being only to apply three given chords so as to fill up the semicircumference, as *Ward* has done in his 16th problem.

Of the Eclipses in 1734.

There will be but two eclipses this year, the one visible and the other invisible.

The first of the sun the 22d of April, at 10 in the morning.*

The other of the sun the 15th of October, at 7 at night.

New Questions.

I. Question 175, *by Mr.* Rob. Fearnside.

A lady of wit, youth, and beauty beside,
Remote from all cares, but of being a bride,
Surpriz'd her fond lover one morning in May,
And dispatch'd him for parson and licence away.
But how great her confusion, when Strephon brought news
That the parson a licence to grant did refuse!
Her age, which the lady nor lover cou'd tell,
Was the cause that this fatal disaster befel.
Therefore, ladies, their humble request is, you'll show.
The way how to do't from the * data below.
And this by a general method explain,
So that lovers may never be non-plus'd again.

* *Cycle of* ☉ 18, *golden number* 8, *roman indiction* 10, *that year the lady was born.*

II. Ques-

* This eclipse was observed at *Rome* by the Abbot *Didacus de Revillas* and *Andreas Celsius.*

h.	m.	s.	True time, p. mer.
22	24	35	The beginning a little over.
23	5	0	Middle, 2 digits.
23	52	1	End.

II. QUESTION 176, by Mr. John Gundy.

Whilst I was surveying for his Grace the Duke of Buccleugh, it was my chance to meet with a piece of land in the form of a rectangle, or long square; and the proportions were such, that if it had been two perches broader, and three longer, it would have been sixty-four perches larger than before. But, on the contrary, if it had been three perches broader, and two longer, it would then be sixty-eight perches larger than it was by my survey. Quere, What was the area of the said piece of land?

III. QUESTION 177, by Mr. John Turner.

The dimensions of a conical frustum are known
To be such as hereunder*; from whence I'd have shown
The greatest cylinder that can be inscrib'd therein,
Its base's diameter, and height, I do mean.
And if, for variety's sake, we again
Suppose the said frustum to be cut by a plane
Thro' the lesser diameter's extremity,
And parallel to the cone's axe; be so kind,
The solid content of each part for to find?

* *Given the greater diameter of the frustum* CD = 44 *inches*
lesser diameter — — AB = 20
perpendicular height — MN = 30

IV. QUESTION 178, by Mr. Chr. Mason.

I being lately in a timber yard,
Where blocks, and beams, and scantlings lay prepar'd;
There a young artist did my aid implore,
A piece to measure he ne'er met before.
He'd read most authors which on solids treat;
Yet this quaint solid did his skill defeat.
Four regular equal faces it did bear,
They bounded by isosceles triangles were.
Three feet each base or shortest side did count,
The legs or longest did to five amount.
I've given what is requisite to know,
The true content and properties pray shew?

V. QUESTION 179, *by Mr.* J. Bulman.

At a certain place in northern latitude, the ſun was obſerved to riſe exactly at 3 h. 58 m. and at 6 o'clock his altitude was taken the ſame morning, and found to be 15 deg. 20 min. his declination being then north. Required the latitude of the place where, and day of the year when, thoſe obſervations were made?

VI. QUESTION 180, *by Mr.* John Grundy.

A gentleman, who was a great lover of the mathematics, had a large eſtate, which lay in four ſeveral entire manors, of which he had drawn four ſeveral maps. Now he, by his often looking over the contents of theſe maps, found the quantities of acres belonging to each of theſe, would make four numbers in continued proportion; whereof the ſum of the two middlemoſt numbers is 1152, and that of the two extremes 1728. He lying on his death-bed, called his four ſons to him, ſaying, My dear children, my glaſs being almoſt run, and the eſtate I have to leave amongſt you was for the moſt part gained by my own induſtry, and ſo entirely at my own diſpoſal, I ſhall leave it amongſt you, with this proviſo, that he that anſwers the aboveſaid queſtion firſt, ſhall have the largeſt manor for his ſhare; and ſo the reſt of you the others, according to your birthright. It is demanded the quantity of acres that each manor does contain?

The PRIZE QUESTION, *by Mr.* Maſon.

A worthy wight, who does in wealth abound,
And, to a wiſh, with earthly bleſſings crown'd;
Free from oppreſſion, and perfidious gain,
True Briton-like, doth liberty maintain.
On a fair ſite a ſtately fabric rais'd,
Whoſe view a Wren or Vanburgh wou'd have prais'd.
High on the north fine rural groves ariſe,
Shelt'ring from blaſts and fierce inclement ſkies.
There glades, and grots, and groteſque works appear,
And bounteous green ſtill flouriſh round the year.
The dropping rocks their trickling tears repeat,
Weeping for joy they've ſuch a ſafe retreat.
The drops unite, and into ſtreams do grow;
In peaceful murm'rings thro' their channels flow.

Until one source does all the rest invade,
And then in haste doth form a large cascade,
Which rolls itself into a reservoir;
Then art's requir'd where nature work'd before.
Near on the east the new-made soils produce,
The choicest ligums that's for kitchen use;
The melonry calls for the gard'ner's care,
For heat, for moisture, and sometimes for air.
As seasons change, with changes he is stor'd,
For to supply his bounteous master's board.
Next, on the west, Pomona claims her seat,
Most free from blights, tho' not the most from heat.
The vernal scenes ravish th' beholder's eye,
And fragrant blossoms, figur'd variously,
Diffuse their sweets; their vying wafts contend,
Till gentle Zeph'rus does the contest end.
The dancing foliage with immantlings play,
And laden branches their choice fruits display.
Yet wanteth still, the edifice to grace,
A large parterre the southern front to face.
Then soon an artist of undoubted skill,
Was there produc'd the same for to fulfil:
His order was, ten acres he shou'd take,
And on three aspects a canal to make.
The earth dug thence must on the surface lie,
To mix the soil, and raise it one foot high.
Yet no dimensions, but proportion, giv'n,
The depth to breadth, as one is unto seven.
And in the midst an oval fountain place,
Where groups of Tritons must the center grace.
The depth three feet, the area in roods a score,
And the plus earth converted as before.
The diameters shou'd such proportion bear,
As the garden's length and breadth (or very near.)
The shorter space 'tween the canal and fount,
To feet, as in the margin will amount. 238
Now the dimensions you're desir'd to show,
Both of canal, the fount, and garden too?

Questions

1735.

Questions answered.

* I. QUESTION 175 *answered by Mr.* Rich. Dunthorne.

THE solution to this question is in Keil's Astronomy, lect. 29 p. 379 and 380, by finding three numbers $285x$, $420y$, and $532z$; so that the first divided by 28 leaves the cycle of the sun a remainder, the second by 19 leaves the golden number, the third by 15 leaves the indiction. Then if the sum of these numbers be divided by 7980, the remainder will be the year of the Julian period required. Or,

The cycle ☉	18	mult. by 4845	= 87210	The sum of
Golden numb.	8	—— 4200	= 33600	the products
Indiction	10	—— 6916	= 69160	is 189970.

Which divided by 7980, there remains 6430, the Julian period; from which subtract 4713, the Julian period at our Saviour's birth, remains 1717, the year required in answer to the question.

Answered

---

* I. QUESTION 175.

The solar cycle is a period of 28 years, the lunar of 19, and the indiction a period of 15. The year before the christian æra was the 9th of the solar cycle, the 1st of the lunar, and the 3d of the indiction cycle. Wherefore 9, 1, and 3 being severally added to any year x of Christ, and the sums divided respectively by 28, 19, and 15, the remainders will shew the several years of the cycles for that year. But, in the present case proposed, the remainders are 18, 8, and 10; hence then $\frac{x+9-18}{28}$, $\frac{x+1-8}{19}$, and $\frac{x+3-10}{15}$ must be integers; or $\frac{x-9}{28}$, $\frac{x-7}{19}$, and $\frac{x-7}{15}$ integers.

Answered by Mr. John Ommanney.

Dr. Keil in his Astron. Lectures, p. 380, says, if 4845 be multiplied by the cycle of the sun, and 4200 by the golden number, and 6916 by the Roman indiction, and the sum of their products divided by 7980, the remainder (neglecting the quotient) will be the year of the Julian period; from which subtract 4713, there will remain the year of the christian æra. And in this question the answer is 1717, as above.

II. QUES-

Put the first of these equal to the integer m, that is $\frac{x-9}{28} = m$; then $x = 28m + 9$ a value of x answering the first condition. Write this in the 2d, then $\frac{28m + 9 - 7}{19}$ or $\frac{28m + 2}{19} = m + \frac{9m + 2}{19} =$ an integer; therefore $\frac{9m + 2}{19} = \frac{18m + 4}{19} = m - \frac{m - 4}{19} =$ an integer; therefore $\frac{m - 4}{19} = n$ an integer; hence $m = 19n + 4$; which substituted in the value of x, we have $x = 28 \times \overline{19n + 4} + 9 = 532n + 121$ a value of x answer the two first conditions. This value of x being written in the 3d original integer, we have $\frac{532n + 121 - 7}{15}$ or $\frac{532n + 114}{15} = 35n + 7 + \frac{7n + 9}{15} =$ an integer; hence $\frac{7n + 9}{15} = \frac{14n + 18}{15} = n + 1 - \frac{n - 3}{15} =$ an integer; therefore $\frac{n - 3}{15} = p$ an integer; consequently $n = 15p + 3$: this written in the last value of x, it becomes $x = 532 \times \overline{15p + 3} + 121 = 7980p + 1717$ a general expression for the year answering all the three conditions, in which p may be either nothing or any whole number. In the present case it is evident that the value of p must be nothing, and then $x = 1717$, the year of the lady's birth.

It is evident that such a combination cannot happen again till the year $9697 = 7980 + 1717$, when the value of p is 1; and that the successive years of its happening are found by the continual addition of 7980.

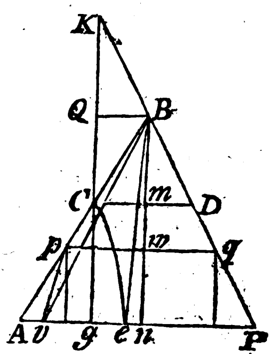
II. QUESTION 176 *answer'd by* Eumenes Pamphilus.

Let $x =$ length, $z =$ breadth. Then, per quest.
$xz + 3z + 2x + 6 = xz + 64$, and
$xz + 2x + 3z + 6 = xz + 68$. Their difference is
$x - z = 4$, hence $x - 4 = z$; which being substituted for z, there comes out, by the 1st equation,
$xx + x - 6 = xx - 4x + 64$, or $5x = 70$. Hence
$x = \frac{70}{5} = 14$ perches the length, and
$z = x - 4 = 10$ perches the breadth, also
$xz = 140$ perches, or 3 roods 20 perches the content.

Mr. *Turner*'s answer is in substance the same.

III. QUESTION 177 *answered by the proposer*.

In order to find the height of the whole cone Bm. Put $Bm = x$, $Cm = b$, $mn = c$, $An = d$. Then, by similar triangles, as $x : b :: x + c : d$. Hence $bx + bc = dx$, and $x = \frac{bc}{d - b} = 25$; consequently $Bn = 55$ inches. Again, put $p = Bn$, $b = AP$, $d = \cdot 7854$, $x = nw$ the height of the greatest cylinder; as $p : b :: p - x : \frac{bp - bx}{p} = pq$. But the solidity of the cylinder $= \frac{bbxx - 2bbpx + bbpp}{pp} \times dx$;



in fluxions, it is $3dbbxx\dot{x} - 4dbbpx\dot{x} + dbbpp\dot{x} = 0$. Divide all by $dbb\dot{x}$, and $3xx - 4px + pp = 0$; Ergo $x = \frac{1}{3}p = 18\cdot 3333$ inches, the cylinder's height; and the diameter of its base $29\cdot 3333$. Again, from $Ag = 12$, and $Pg = 32$, being given, $gv = ge$ is found $= (\sqrt{12 \times 32})\ 19\cdot 6$ inches, and $ve = 39\cdot 2$: the area of the segment Aev (by page 404 of Ward, or much easier by a table of segments) is found to be $335\cdot 88$ square inches. Which multiplied by $\frac{1}{3}Bn = 18\cdot 333$, gives the solidity of the pyramid $ABevA = 6157\cdot 8$ cubic inches.

Now the base of the pyramid $CveBC$ is an hyperbola, whose area is thus found: $ng = 10$ is the hyperbola's conjugate semi-diameter $= b$, $ve = 39\cdot2 = g$ its bounding ordinate, $Cg = 30$ the abscissa, CQ the transverse semi diameter, $= 25$, $Qg = p = 55$, $AP = d = 44$. The hyperbolic logarithm of $\frac{d+g}{2b}$ or 4·16 is 1·4255142 $= S$. I say the area of the hyperbola is $= \frac{pg}{2} - \frac{2pbbS}{d} = 721\cdot62$ square inch. which is a contraction of Dr. Wallis's quadrature of the hyperbola, p. 328 of his Algebra. This multiplied by $\frac{1}{3}ng$, the perpendicular altitude of the pyramid, or 3·333, gives 2405·40 its solidity. Subtract this pyramid $CveBC$ from the pyramid $ABevA$, the remainder is the content of the ungula or cuneus $AveCA = 3752\cdot4$ cubic inches, which was required. And lastly, if you subtract this from the content of the conic frustum $ACDP$, which is easily found $=$ 25258·46, the remainder is the solidity of the other part or ungula $PevCP = 21506\cdot06$ inches.

Answered by Mr. Geo. Brown,

Who projects the scheme, and all its parts exactly the same as above, and carries the process through the whole, from whence I shall only collect such parts as are essential to the answer.

The area of the segment $Aev = 335\cdot915$ square inches,
The solidity of the inclined pyr. $ABg = 6158\cdot45$ cubic inch.
The area of the hyperbola $= 721\cdot45$,
Solidity of the inclined pyramid $CBg = 2404\cdot95$,
Which taken from ACg,
Leaves the solidity of the hoof $ACve = 3753\cdot49 = 2\cdot1721$ feet.
Which taken from the given frustum of the cone 14·6171,
Leaves the content of the part $PDCveP = 12\cdot445$.
The height of the greatest cylinder $wn = 18\cdot333$ } inches.
And its base's diameter — — 29·333 }

IV. Question 178 *answered by* Juvenis Mathematicus.

This body seems to agree with Euclid's definition of a pyramid. Per 47 Eucl. 1, $cc - \frac{bb}{4} = \frac{4cc - bb}{4} =$ to the perpendicular of one of the isosceles triangles; and if the body be conceived to be divided into two equal parts by a tri-

angular

angular plane, two ſides of which are the perpendiculars above, and the 3d $= b = 3$. Then by the 14th prop. of Keil's Plain Trigonometry, $\sqrt{\frac{4cc-bb}{4}} : \sqrt{\frac{4cc-bb}{4}}+b :: \sqrt{\frac{4cc-bb}{4}}-b$: the difference of the ſegments of the baſe. The rectangle of the means divided by the firſt extream $= \sqrt{cc-\frac{bb}{4}} - \frac{bb}{\sqrt{\frac{4cc-bb}{4}}} =$ difference of the ſegments. Hence then $\sqrt{\frac{4cc-bb}{4}} - \frac{bb}{2\sqrt{\frac{4cc-bb}{4}}} =$ the greater ſegment, and $\frac{bb}{2\sqrt{\frac{4cc-bb}{4}}} =$ the leſs ſegment; alſo $bb - \frac{b^4}{4cc-bb} =$ the ſquare of the perpendicular height of this triangle. But $\frac{1}{2}b\sqrt{\frac{4cc-bb}{4}} =$ area triangle or baſe of the ſolid body. Wherefore $\frac{1}{3}\sqrt{bb-\frac{b^4}{4cc-bb}} \times \frac{1}{2}b\sqrt{\frac{4cc-bb}{4}} = 6{\cdot}791536746428$ cubic feet, the ſolid content required.

The ſame anſwered by Mr. Ed. Golding.

A general Rule to find the Solidity. From the ſquare of one of the legs, abate the ſquare of half the baſe; and multiply the ſquare root of the remainder by the ſquare of the baſe: The laſt product divide by 6, and the quotient ſhall be the content of the propoſed ſolid.

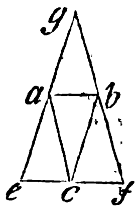
For the perpendicular Height. The ſquare of the ſide or leg $5 = 25$; ſubtract twice the ſquare of $\frac{1}{2}$ the baſe $1{\cdot}5 = 4{\cdot}5$; there remains $20{\cdot}5$ whoſe ſquare root $4{\cdot}5277$ is the perpendicular height.

For the Solidity. Multiply the ſquare of the baſe $3 = 9$, by the perpendicular $4{\cdot}5777$, and the product $40{\cdot}7493$ is the ſolidity of the circumſcribing priſm. As $6 : 1 :: 40{\cdot}7493 : 6{\cdot}79155$ the ſolid feet in the alabrum, by which name I call this ſolid till a fitter name be impoſed. And to prove it but $\frac{1}{6}$ of the prism. Suppose the first figure, being a square prism, be 3 feet square at the ends, and 6 feet high, the content will be 54. Cut off from the prism (1.) the 2 prismatic wedges *a* and *b*; which both together contain 9 upon 3 = 27 feet; leaving the wedge *c*, which suppose to be the second figure; from it cut off the two pyramids *e* and *f*, which both together contain 9 upon 2 = 18. Leaving the the alabrum only behind: The two wedges = 27, and two pyramids = 18, make 45, which taken from 54, remains the alabrum 9 = $\frac{1}{6}$ of the prism. *Q. E. D.*

The same answered by Mr. J. Bulman.

From the square of the given side of the isosceles triangle = 25, subtract the square of half the given base = 2·25, the remainder 22·75 is the square of the perpendicular to the triangle: from which subtract again the square of half the base, the square root of the remainder 4·527692 = the length of the circumscribing square prism: whose solidity is 40·749228 feet, $\frac{1}{6}$ of which is 6·791538 feet, the solidity of the body required.

This solid may easily be cut from a prism having square bases. Or if a piece of stiff paper or pasteboard be cut according to the following figure, and folded in the lines *a b*, *b c*, and *c a*, so as *c f* and *c e* coincide or make but one line, and *g* touches the joined points *e* and *f*, it will perfectly represent the body proposed in the question.



Several other true answers I might exhibit, were it not that I have in some measure exceeded my limits; and that the rather, because several have pronounced this question unintelligible and unlimited; such I hope will be convinced of their error by these answers; and not hastily judge and condemn others, that at first don't appear obvious to them.

V. QUES-

V. QUESTION 179 *answered by Mr.* N. Oats.

Put a = ſine of the altitude at 6; d = ſine of the aſcen. diff. s = ſine of the latitude; $\sqrt{1-ss}$ = its coſine. Then, per ſpherics, $s : 1 :: a : \frac{a}{s}$ = ſine decl. and its coſine = $\sqrt{1-\frac{aa}{ss}} = \frac{\sqrt{ss-aa}}{s}$. Again $\frac{\sqrt{ss-aa}}{s} : \frac{a}{s} :: 1 : \frac{as}{\sqrt{ss-aa}}$ = tang. declin. $\sqrt{1-ss} : s :: 1 : \frac{s}{\sqrt{1-ss}}$ = tang. lat. $1 : \frac{s}{\sqrt{1-ss}} :: \frac{as}{\sqrt{ss-aa}} : d$; ergo $\frac{ass}{\sqrt{ss-aa}} = d\sqrt{1-ss}$. Reduced, it is $s^4 - aass + aa - ss + \frac{aass}{dd} = 0$; hence s = ·835728 = the ſine lat. = 56° 41′ 32″. And the declination = 18° 26′ 45″, anſwering to May 2, or Aug. 4.

Mr. *John Turner* has given a curious ſolution to this queſtion, both trigonometrically and analytically; where he makes the ſun's declination north 18° 28′, and the latitude of the place north 56° 40′.

VI. QUESTION 180 *anſwered by Mr.* Beacham.

This queſtion is compoſed out of Sir Iſaac Newton's Arith. or Ronayne's Algebra. The firſt ſhare is 1536, the ſecond 768, the third 384, and the fourth 192 acres.

Mr. *John Corbett* has given a true algebraic ſolution to this queſtion.

The

The PRIZE QUESTION *answered by Mr.* J. Doubt.

Given 10 acres $= 435600$ square feet $= k$; 20 roods $= 5445$ square feet $= g$; the distance of fount to the canal $= 476 = b$: The area of a circle whose diameter is $1 = \cdot 7854 = m$: Put the transverse diameter of the fount $= x$, and the conjugate $= y$: then $\frac{g}{xm} = y$; $y : x :: b : \frac{bxxm}{g} = a$: and also $\overline{a+x} \times \overline{b+y} = \frac{bbxxmm + 2bxgm + gg}{gm} = k$: and so $bxm + g = \sqrt{kgm}$. Hence will be found $x = 100\cdot 8844$, $y = 68\cdot 7199$. The length of the garden $799\cdot 677$ feet; breadth $544\cdot 719$ feet; and then $435600 - 5445 - 16335 = 413820$ the solid content of the canal, which let be $= s$, and $799\cdot 6771 \times 2 + 544\cdot 719 = 2144\cdot 074$ (the extent of the canal) $= l$. Put breadth $= a$, the depth $= e$; then per question, $7 : 1 :: a : \frac{a}{7} = e$: and $\frac{laa}{7} = s$. Therefore $a = \sqrt{\frac{7s}{l}} = 36\cdot 7566$ feet; and $e = 5\cdot 2509$ feet.

Some have taken this question to be unlimited; indeed the words [three aspects] do not absolutely determine whether two sides and one end, or two ends and one side; but one would rather take it in the former. But that it should be an oblong is plain enough from those words, that the *oval should be in proportion to the garden's length and breadth.* However the author's meaning has been well understood by several persons, as by the near agreement of some of their answers here below do appear.

	Length	*Bread.*	*Transv.*	*Conju.*	*Bred. Can.*	*Depth*
The *proposer*	799·9	544·4	100·9	68·6	32·7	4·68
Mr. *John Turner*	799·4	544·8	101·1	68·8	38·3	5·47
F. R. S.	799·0	544·6	100·9	68·6	32·6	5·01
Mr. *John Chorley*	794·7	548·1	96·1	72·1	38·3	5·48

The prize of 10 diaries was won by Mr. *John Turner* of Hull.

Of

Of the Eclipses in 1735.

There will happen four eclipses this year: twice will the moon's dark body be interposed between the sun and the earth, and hinder the sun's rays from falling on the terraqueous globe; and twice will the earth come in the direct line between the sun and moon, and prevent the moon from receiving her borrowed light from the sun.

1. Moon eclipsed March 27, at 11 forenoon, invisible.

2. Sun eclipsed April 11, at 11 at night, invisible.

3. Moon eclipsed September 21, after 1 morning, visible.

4. Sun eclipsed October 5, at 2 morning, invisible.

That of the moon, September 21, only visible, as follows.

Computed by	Begin. h. m.	Mid. h. m.	End h. m.	Dur. h. m.	Dig. h. m.
By Astronomia for Coventry	12 23	1 39	2 56	2 33	6 2
Mr. Chattock for London	11 52	1 21	2 50	2 57	7 12
Mr. Leadbetter for London	0 16	1 33	2 50	2 34	5 37
Mr. Dunthorne for Ramsey	0 29	1 46	3 3	2 33	6 3
Mr. Hampson, Leigh, Lancash.	11 39	1 0	2 22	2 43	6 29
Mr. Sparrow, Nottingham, by Flamstead's	11 52	1 14	2 35	2 43	6 18
Mr. Sparrow, Nottingham, Astron. Carol.	12 20	1 36	2 52	2 32	5 59
Mr. J. Wooler for Whitby	11 47	1 8	2 29	2 42	6 4
Mr. J. Thomas, Penzance	11 49	1 6	2 19	2 30	5 35
Mr. Williams, Oxfordshire	0 12	1 30	2 46	2 34	5 40
Mr. C. Forrest, Newcastle	11 55	1 21	2 48	2 53	6 38
Mr. J. Hilton for London	0 16	1 33	2 50	2 34	6 37
Mr. Brown for Bridgnorth	0 18	1 35	2 52	2 33	6 2
Mr. J. Bulman	0 29	1 46	3 3	2 34	6 3

New Questions.

I. QUESTION 181, *by Mr.* John Turner.

In the triangle ABC there is given the side $AB = 65$ feet, and the side $BC = 74$; and in this triangle there is an equilateral triangle inscribed as SWH, whose side SH is parallel to the base AC: And lastly, in the equilateral triangle there is a circle inscribed, whose area is known to be $= 307{\cdot}9$ square feet. It is required to find the side of the equilateral, and the base of the external triangle?

See the fig. to the solution.

II. QUES-

II. Question 182, *by Mr.* Chr. Mason.

On a rich globe where bounteous nature smil'd
And fragrant sweets the sense of man beguil'd;
Near a fair grove where warbling birds did sing,
With joyous notes, to the beloved spring;
I thither went with orders to survey
A field, with four unequal sides there lay,
And a foot-path lay 'cross the longest way.
Lest we shou'd trespass on the smiling plain,
With our rude feet, and dragging of the chain,
I took my station strait along the way,
And in the same completed my survey.
One hundred rods it stretch'd from end to end;
Th' obtuse angles it also did subtend;
Which said angles they are plac'd * below,
And their two perpendiculars also;
That is, if I more plain to you must say,
They from those angles fall upon the way.
Ye douty brethren, that do drag the chain,
And you who with your poles do 'thwart the plain,
And those that stalk the Laions by Spanish strides,
With each your art unfold the unknown sides.

* *The obtuse angles are 108° and 118°, and perpendiculars 9 and 7 chains.*

III. Question 183, *by Mr.* Tho. Fearn.

A general disposing his army into a square battle, finds he has 284 soldiers over-and-above; but increasing each side with one man, he wanted 25 to fill up the square. Quere the number of soldiers.

IV. Question 184, *by* Eumenes Pamphilus.

Poets, like me, will often strain and soar,
To make that dubious which was plain before;
And merely for to make our fustian chime,
We'll metamorphoze reason into rhime:
And mathematics too, you often hear,
In rhimes are flat and nauseous to the ear,
And sometimes dubious: Therefore might I chuse,
We'd often have 'em writ in plainer prose.

There

There are three numbers in continued proportion; the product of the three multiplied into one another is 512, and the sum of the extremes 34. Three-fourths of the greater extreme being called years, and the other fourth weeks, will exactly shew my age; the mean, the month, and the less extreme, the day of the month I was born: Whence you are desired to shew the year, month, and day on which I was brought to light? Nov. 27, 1733.

V. QUESTION 185, *by Mr.* Rob. Fearnside.

A parabolic curve, * whose length, in feet,
Is five times ten more ten times five complete.
The greatest area that can be inclos'd
By this curve, and an ordinate suppos'd
Unto its greatest axe rightly apply'd,
Vouchsafe, ye fair, with candor to decide?

* *Of the 2d kind, whose equation is* $ax^2 = y^3$.

VI. QUESTION 186, *by Mr.* Ed. Hauxley.

The use of a meridian line in astronomy, geography, dialling, &c. is very great, and on its exactness all depends: whence infinite pains have been taken by divers astronomers to have it to the last precision. M. Cassini has distinguished himself by a meridian line drawn on a copper-plate on the pavement of the church of St Petronia at Bologna in Italy, the largest and most accurate in the world. In the arched roof of the church, at a great height above the pavement, is a little hole, through which the sun's image, when in the meridian, falling upon the line, marks his progress all the year. When finished, Mr Cassini, in a public writing, informed the mathematicians of Europe of a new oracle of Apollo, or the sun, established in a temple, which might be consulted with entire confidence as to all difficulties in astronomy.

Bologna, by some tables, lies in the latitude of 44° 8′ north; and Dr. Burnet, late Bishop of Salisbury, in his Letters many years ago, has given us an account of this meridian line, from whence the height of the hole in a perpendicular above the pavement, and the length of that copper meridian from that perpendicular, under the said hole, to its utmost extent, where the sun's image marks the tropic of Capricorn, in one sum makes 285 feet 10 inches. From hence the height of the hole, or nodus above the floor, and the length of the meridian line, and the distances of the tropics from the equinoxes may be severally known! and are here required?

The

The PRIZE QUESTION, *communicated by Mr.* Samuel Ashby, *in a Letter to* Tho. Grubbian.

Ladies, since you your leisure time employ
In matters of geometry,
And have so curiously disclos'd
The questions that have been propos'd;
Farther to try your skill in lines,
Your mathematic friend consigns.
Desires next year, that you'll send back,
His quere solv'd i'th' almanac.

If in any plain triangle, as ABC, you draw lines from each angle through any point E within the triangle, till they cut the opposite sides, as Ca, Ab, and Bc. The rectangles of the alternate segments at those sides will be equal. *See the fig. to the solution.*

viz. $Ab \times Ca \times Bc = bC \times aB \times cA$.

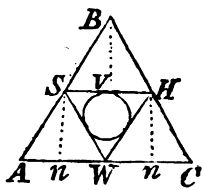
The demonstration of this curious proposition is required.

1736.

Questions answered.

I. QUESTION 181 *answered by Mr.* Christ. Mason.

FIRST, per Euclid, circles are in proportion as the squares of their diameters, therefore the diameter of the circle will be found 19·79976. And, per plain trigonometry, as s. 30° : diameter :: s. 60° : 34·28 the side of the circumscribing trigon $= SH = HW = WS$. The square root of $\frac{3}{4}$ of the square of the side is the perpendicular $VW = 29·69$.



Let $b = AB = 65$; $c = BC = 74$; $d = SH = 34·28$; $p = VW = 29·69$; $a = BV$ sought. Then 4 Eucl. 6, as $a : d :: a + p : \frac{dd + dp}{a} = AC$;

$= AC$: And per 47 Eucl. 1, $\sqrt{bb - aa + 2pa + pp} = AW$; also $\sqrt{cc - aa + 2pa + pp} = WC$; and $AW + WC = \frac{da + dp}{a}$: which, properly reduced, gives $a = 31.42$; and $a + p = BW = 60.735$; $AW = 23.15$; $WC = 42.28$; therefore $AC = 65.43$.

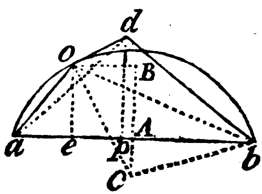
The same answered by Eumenes Pamphilus.

As 3.1415926 &c. : 1 :: 307.9 : 98.00763 = sq. radius. Whose root 9.89988 = radius; and 19.79976 = diameter. Then, by Ward's Proportion, p. 336, as .28867513 : 2 :: 9.899 : 34.29419 the side of the equilateral triangle required.

Now let $BA = a = 65$; $BC = b = 74$; $SH = c = 34.29419$; $x = Ac$; and let Sn and Hn be perpendiculars let fall from the points S and H upon the base AC. Then will $Sn = Hn =$ three times the radius of the inscribed circle $= 29.69964 = d$; then, per 2 Eucl. 6, $x : c :: a : \frac{ac}{x} = BS$; and $x : c :: b : \frac{bc}{x} = BH$; $a - \frac{ac}{x} = SA$; and $b - \frac{bc}{x} = HC$; then per 47 E. 1, $\sqrt{aa - \frac{2aac}{x} + \frac{aacc}{xx} - dd} = An$; and $\sqrt{bb - \frac{2bbc}{x} + \frac{bbcc}{xx} - dd} = nC$; consequently $x = c + \sqrt{aa - \frac{2aac}{x} + \frac{aacc}{xx} - dd} + \sqrt{bb - \frac{2bbc}{x} + \frac{bbcc}{xx} - dd}$; and by reduction $x = 67.743589 = AC$ the base required.

II. QUESTION 182 *answered by the Proposer Mr.* Mason.

The obtuse angles given are $adb = 108°$ and $aob = 118°$; the perpendiculars $pd = 9$ chains, and $eo = 7$ chains; and the base $ab = 25$ chains. Required bd, do, and oa; the other three sides of the trapezium? Which will be found trigonometrically, viz. $ad = 16{\cdot}61$; $db = 14{\cdot}23$; $bo = 17{\cdot}71$; $ao = 11{\cdot}15$; $do = 5{\cdot}76$.*



III. QUES-

---

* II. QUESTION 182.

In this question are concerned two triangles aob, adb, on the same given base, and whose perpendiculars and vertical angles are also given; to find their sides and the distances of their vertexes.

Construction.

On the given base describe two segments of circles capable of containing the given vertical angles; then at distances equal to the two perpendiculars draw two parallels to the base, and they will cut the respective circles in the required vertexes o and d of the two triangles. As is too evident to need a demonstration.

Calculation.

Let C be the center of the circle passing through one of the vertexes; and CAB perpendicular and oB parallel to ab, and the other lines as per figure. Then $ACb = 62°$ = the supplement of the given $\angle aob$, and ACo is = the difference of the $\angle$s (oab, oba) at the base. But s. $\angle ACb$: cos. $\angle ACb$ :: Ab ($\frac{1}{2}ab$) : AC; hence $CB = AC + oe$; then (since Ab and CB are the sines of the $\angle$s ACb and Cob to the same radius Cb or Co) Ab : CB :: s. $\angle ACb$: s. $\angle CoB$ = the complement of the difference of the $\angle$s oab, oba, at the base. Hence their sum and difference being known, the angles themselves become known; and thence the sides ao, bo.——In like manner are found the sides ad, bd. And thence od.

III. QUESTION 183 *anſwered by* Ruſſellus.

Put $a =$ ſide of the firſt ſquare; $b =$ the overplus (284); and $c = 25$ the number wanting to fill up the ſecond ſquare. Then $aa + b = aa + 2a + 1 - c =$ the number of ſoldiers per queſt. $b = 2a + 1 - c$; $b + c - 1 = 2a$; and $\frac{b + c - 1}{2} = a = 154$; $aa + b = 24000 =$ the number of ſoldiers.

Nearly in the ſame manner it is anſwered by *Eumenes Pamphilus.*

Mr. *Fearne* the propoſer obſerves that in any queſtion of this kind, the two numbers given muſt be the one even, the other odd.

IV. QUESTION 184 *anſwered by* Ed. Golding.

The three numbers in continual proportion are 2, 8, 32: for $2 \times 8 \times 32 = 512$, and $2 + 32 = 34$; which exactly agrees with the propoſal: three-fourths of the greater extreme is 24 for years; and one-fourth is 8 weeks; ſo he was 24 years 8 weeks old; and he was born October 2, 1709.

The ſame anſwered by Mr. John Turner.

Put a, e, and u for the three numbers ſought; $b = 512$; $c = 34$. Then, by the queſtion, $aeu = b$; $a + u = c$; and $au = ee$: by the firſt $au = \frac{b}{e}$ (in the 3d ſtep) $= ee$; hence $eee = b$; and $e = \sqrt[3]{b} = 8$ the mean. And from hence u is found $= 32$, and $a = 2$; ſo he was 24 years 8 months and 8 weeks old.

Anſwered by Mr. Maſon.

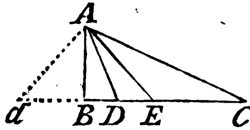
Let a, e, and u be the three numbers ſought, and $aeu = z = 512$; $a + u = s = 34$ per queſt. Then $au = ee$ per property, hence $\frac{z}{e} = ee$, therefore $z = eee$; and $\sqrt[3]{z} = 8 = e$; conſequently $a = 2$; $u = 32$; then 24 years and 8 weeks muſt be his age, the mean muſt repreſent October, and the leſſer extreme the ſecond day thereof.

V. QUEST. 185 *answered by Mr.* Fearnside *the Proposer.*

Putting the equation $ax^2 = y^3$ of the curve into fluxions, we have $2ax\dot{x} = 3y^2\dot{y}$; hence $\dot{x} = \frac{3y^2\dot{y}}{2ax} = \frac{3\dot{y}}{2}\sqrt{\frac{y}{a}}$. Then the fluxion of the area $= y\dot{x}$ is $\frac{3y\dot{y}}{2}\sqrt{\frac{y}{a}}$, whose fluent $\frac{3yy}{5}\sqrt{\frac{y}{a}}$ is the area a maximum. Again, the fluxion of the curve $\sqrt{\dot{x}^2 + \dot{y}^2}$ will become $\dot{y}\sqrt{\frac{9y+4a}{4a}}$, whose correct fluent $\frac{\left(\frac{9y+4a}{a}\right)^{\frac{3}{2}} - 8}{27} \times a$ is $= 50 = c$. Then the value of either a or y being found from this equation, and written for it in the above area or maximum, the maximum will then contain only one of the letters a or y, and whose fluxion being made equal to 0, that letter will be determined, and thence all the rest.

VI. QUESTION 186 *answered by Mr.* Ed. Golding.

There is given $AB + BC = 285$ feet 10 inches; required AB, BC, BE, and BD. I here use 23° 30′ for the greatest declination: then 90° — 44° 8′ = 45° 52′ the elevation of the equinoctial; and the tropics and equinoctial will have the same angles in respect to each other. Of all which BD, BE, BC are tangents to the radius AB. Produce the line CB till $Bd = BA$; then, per question, $Cd = 285$ feet 10 inches = 3430 inches. Then the angle dAB = angle d = 45°, and (CAB =) 68° 31′ + 45° = 113° 31′ = CAd.



As s. CAd : Cd : : s. d : CA = 2649·24 inches.

Rad. : AC : : s. CAB : CB = 2453·95 = 204 feet 5 inch. 95 par.

Rad. : AC : : s. ACB : AB = 976·05 = 81 feet 4 inch. 05 par.

Sum	3430	= 285	10	0

And

And proceeding in the same manner shall find

	Inches		Feet	Inch.	Parts
BD = — — —	380·7423	=	31	8	7423
♋ ♎ *DE* — — —	588·8556	=	49	0	8556
♈ ♑ *EB* — — —	1484·3521	=	123	8	3521
The sum = *BC* = —	2453·95	=	204	5	95
To which add *AB* =	976·05	=	81	4	05
Total — — —	3430·	=	285	10	0

	AB	*CE*	*DE*	*CD*	*BD*	*CB*
By *F. R. S.*	83·33	121·19	49·54	171·19	31·31	202·50
Mr. *C. Mason*	82·83	122·16	49·33	171·58	31·42	203·00
Mr. *J. Turner*	83·58	121·35	49·42	170·77	31·47	202·25
Mr. *E. Hauxley*	83·58	121·35	49·42	170·77	31·47	202·[illegible]
Mr. *J. Hampson*		122·75	46·25	172·00		203·5[illegible]

N. B. No notice is taken of the distance of Bologna's longitude in time from the ☉'s entering the first scruple of the equator or tropics, nor of the ☉'s refraction, in the answers to this question, as being too curious.*

The

---

* VI. QUESTION 186.

The above calculation of this question is not very accurate with regard to the angles; however any one may easily make it as accurate as he pleases by the same method, or rather by this following.

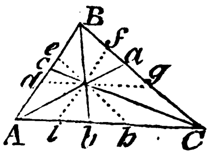
In the above figure, we have given all the angles formed at the point *A*, and the sum of the legs *AB*, *BC*; to find the rest. For *A* represents the hole, *BC* the meridian, *AB* the perpendicular on it, *AE* a ray from the sun in the equinoctial, and *AD*, *AC* rays from him when in the two tropics: therefore the ∠ *BAE* = the lat. = 44° 8′; to and from which adding and subtracting the declination (23° 29′) = ∠ *EAC* = ∠ *EAD*, the sum (67° 37′) will be the ∠ *BAC*, and the difference (20° 39′) the ∠ *BAD*.

But, by plane trigonometry, 1 (rad.) : *t* (tang. ∠ *BAC*) : : *AB* : *BC*; then, by composition, &c. $1 + t$: *AB* + *BC* : : $\begin{cases} 1 : AB, \\ t : BC. \end{cases}$

Which being known, then 1 : *AB* : : $\begin{cases} \text{tang. } \angle BAE : BE, \\ \text{tang. } \angle BAD : BD. \end{cases}$

The PRIZE QUESTION *anſwered by Mr.* C. Maſon.

Per 31 Eucl. 1, draw lines parallel to the given ſides thro' O the given point, as dg, eh, fi. Then, per 2 Eucl. 6, will be the following proportions, viz. $dO : Og :: Ab : bC$; and $iO : Of :: Ac : cB$; alſo $eO : Oh :: Ba : aC$; from which will proceed the following compound proportions, viz. $dO \times iO \times eO : Og \times Of \times Oh :: Ab \times Ac \times Ba : bC \times cB \times aC$; and, per 4 Eucl. 6, $hO : eO :: iO : de$; and $Og : Od :: Of : de$; Ergo $\frac{eO \times iO}{hO} = \frac{dO \times Of}{Og}$; which, reduced, gives $dO \times fO \times hO = eO \times iO \times gO$; pert. alterna. & comp. ergo $Ac \times Ba \times bC = cB \times aC \times bA$. *Q.E.D.*

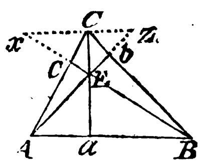


Mr. *J. Turner*'s anſwer is omitted, as being nearly the ſame.

Errata. The letter O is omitted at the point of interſection.

Another Demonſtration by Mr. Ed. Enotts.

Draw xCz parallel to AB, and produce Ab and Bc till they meet with the line xz. The $\triangle AEa$ is ſimilar to CEz; and xCE to aEB; and bzC to AbB; alſo xCc to AcB: therefore $xC : CE :: Ba : aE$; and $CE : Cz :: aE : aA$; alſo $xC : Cz :: Ba : Aa$; but $xC : Cz$ is compounded of $xC : AB$ and of $AB : Cz$; but $xC : AB :: Cc : cA$, and $AB : Cz :: Bb : bC$; therefore $xC : Cz$ $(:: Ba : Aa) :: Cc : cA$ and $Bb : bC$; that is $Ba : Aa :: Cc \times Bb : cA \times bC$; therefore $Ba \times Ac \times Cb = Aa \times Cc \times Bb$. *Q.E.D.*



The prize of 10 diaries was won by Mr. *Natt. Percivall.*

Of

Of the Eclipſes in 1736.

Within the ſphere of the earth's orbit will happen ſix eclipſes this year. Four times will the moon, in her wandering courſe, interpoſe, and hide the ſplendor of the ſun from falling on the earth or its atmoſphere. And twice will the earth in its courſe ſo fall in the line between the ſun and moon, as to hinder her receiving the light ſhe borrows from the ſun, to enlighten the earth by reflection.

1. Sun eclipſed March 1, at 2h. 36m. afternoon, but the moon's latitude prevents its being viſible to us.

2. Moon eclipſed viſible and total on monday March 15, near midnight. Several calculations of which are as follow.

Computed by	Beg. h. m.	Mid.	End	Dur.	Dig. d. m.
By Aſtron. Car. at Coventry	10 10	11 55	1 4	3 29	21 56
Mr. Chattock, London	10 11	12 0	1 48	3 36	22 14
Mr. Leadbetter, London	10 6	11 51	1 36	3 29	21 45
Mr. Hauxley, Kirkleatham	10 2	11 58	1 33		21 56
	9 48	11 33	1 19	3 31	21 55
Mr. R. Dunthorne, Ramſey	10 17	12 2	1 47	3 20	21 57
Mr. J. Hampſon, Leigh	10 4	11 50	1 35	3 31	22 2
Mr. S. Prichard, London	9 11	11 55	2 42	5 31	21 54
Mr. J. Bulman, London	10 16	12 1	1 16		
Edinburgh	10 4	11 49	1 34		
Dublin	9 48	11 33	1 6	3 29	21 57
Carliſle	10 5	11 50	1 55		
Deptford	10 17	12 1	1 46		
Mr. G. Maſon, Newcaſtle	9 42	11 28	1 14	3 32	21 55
Edinburgh	9 48	11 34	1 20	3 32	21 55
Mr. J. Wilſon, Morpeth	10 1	11 46	1 31	3 29	21 46
Mr. T. Williams, Oxfordſhire	10 0	11 47	1 31	3 31	21 47
Mr. W. Brown, Bridgnorth	10 6	11 55	1 36	3 29	21 57
Mr. J. Thomas	9 44	11 30	1 17	3 32	21 38
Mr. W. Baylis, Oxfordſhire	10 1	11 47	1 31	3 30	22 0
Mr. T. Sparrow, Nottingham	10 3	11 48	1 33	3 20	21 45
Mr. C. Forreſt, Newcaſtle	9 41	11 30	1 18	3 37	22 19
Mr. J. Hilton, London	10 12	11 57	1 41		21 42
Mr. G. Forſter, Branſpath	9 58	11 43	1 30	3 32	22 1
Mr. T. Cowper, Wellingbrough	10 20	0 6	1 51	3 31	21 34

3. Sun

3. Sun eclipsed March 31, at 7 morning, invisible.

4. Sun eclipsed August 25, at 9 morning, invisible.

5. Moon eclipsed, total and visible, Sept. 9, at 3 morning.

Computed by	Begin. h. m.	Mid.	End	Dur.	Dig.
Astronomia Car. Coventry	1 5	2 59	4 53	3 48	21 2
J. Chattock, London	12 44	2 50	4 56	4 12	22 1
C. Leadbetter, London	1 7	3 2	4 57	3 49	20 32
E. Hauxley, { Yorkshire	12 25	2 20	4 14		20 30
{ Streets J.	1 15		5 4	3 49	21 2
R. Dunthorne, Ramsey	1 7	3 6	5 4	3 57	21 3
J. Hampson, Lancashire	1 14	3 14	5 14	3 59	21 12
S. Prichard, London	1 22	3 17	5 12	3 50	21 13
J. Bulman	3 22	5 15	7 8	3 46	18 38
G. Mason, Newcastle	1 9	3 4	4 58	3 49	21 2
Edinburgh	1 15	3 9	5 4	3 49	21 2
T. Williams, Oxfordshire	1 4	2 58	4 53	3 48	20 33
W. Brown, Bridgnorth	1 2	2 56	4 50	3 48	21 2
J. Thomas, Penzance	0 50	2 43	4 36	3 46	20 39
W. Baylis, Mixbury	1 2	2 58	4 52	3 49	20 47
Mr. T. Sparrow, Nottingham, from Flamstead's Tables	0 53	2 51	4 49	3 56	21 10
C. Forrest, Newcastle	1 10	3 8	5 5	3 54	21 0
J. Hilton, London	1 7	2 53	4 39		19 54
C. Forster, Branspath	1 4	2 59	4 58	3 53	20 46
T. Cowper, Wellingborough	0 59	2 56	4 54	3 55	21 14

6. Sun

The 2d Eclipse was observed thus:

At	By	Beginning	Tot. Im.	Emersion	End
		h. m. s.	h. m. s.	h. m. s.	h. m. s.
Fleet Str. London	G. Graham	10 13 0	11 11 0	12 49 0	13 47 0
Ibid.	Mr. Celsius		11 10 0		13 46 0
Cov. Gar. London	Dr. Bevis	10 11 40	11 10 0	12 47 56	13 46 25t.t.
Greenwich	Dr. Halley	10 13 37	11 9 42		
Yeovil, Somerset	John Milner	10 6 0	11 4 30	12 43 30	13 39 15t.t.

6. Sun eclipsed September 23, at 5 evening, part visible.*

Computed by	Beg.	Mid.	End	Dur.	Dig.
Astron. Carolina, Coventry	4 56	5 28	6 9	1 22	3 41
J. Chattock, { London	4 57	5 37	6 15	1 18	3 10
J. Chattock, { Coventry	4 48	5 29	6 8	1 20	3 18
Mr. Ra. Hulse, Sandbank	4 44	5 36	6 4	1 27	3 40
R. Dunthorne, Ramsey	4 53	5 34	6 31	1 20	3 36
J. Hampson, Leigh	5 5	5 49			4 0
T. Sparrow, Nottingham	4 50	5 32	6 16	1 26	4 0
Anonymous, Leicester	4 39	5 23	6 3	1 24	3 44
J. Dorking, Suffolk	4 55	5 37	6 18	1 22	3 45
G. Mason, Newcastle	4 45	5 27	6 8	1 22	3 40

Mr. *J. Dunthorne* gives the following account of the Transit of Mercury over the Sun, October 31, 1736, for Ramsey, calculated by Astronomia Carolina.†

Equal

The 5th Eclipse was observed thus:

At	By	Beginning	Tot. Im.	Emers.	End
		h. m. s.	h. m. s.	h. m.	h. m.
Fleet Str. London	G. Graham & Ja. Short	12 58 0	14 3 45		a.t.
Cov. Gar. London	Dr. Bevis	12 56 50	14 2 25		a.t.
Wittemberg	J. F. Weidler	13 50 0	14 53 0	16 44	
Hudson's Bay	C. C. Middleton		8 43 0	10 29	11 37 t. t.

* The beginning of this 6th Eclipse Dr. Bevis observed, in London, at 4 h. 45 m. 31 s. app. time. The other observations could not be made for clouds.

† This Transit was observed by several persons at different places; but, by reason of clouds, the observations were very imperfect, so that all that are worth recording here are as follows.

At	By	Beginning	Tot. Im.	Emerf.	End
		h. m. s.	h. m. s.	m. s.	m. s.
Bologna	E. Manfredi	22 7 56	22 11 12	50 50	54 36 app. t.
Greenwich	Dr. Bevis & Dr. Halley				8 33

	d.	h.	m.	s.
Equal time of the true conjunction in the ecliptic	31	0	8	53
True longitude of the sun ♏ — — —		19	23	14
of Mercury in the ecliptic ♏		19	23	14
Inclination of Mercury north — — —			31	1
Hourly motion of Mercury from the sun —			12	49
Equal time of the nearest approach — —		23	47	52
Equat. ad 3'58 the apparent time — —		23	51	50
Nearest distance of centers 14' 9" semidiam. sun			16	6
The beginning or ingress of Mercury's center		22	44	33
End or Emersion of Mercury's center — —			59	7

New Questions.

I. QUESTION 187, *by Mr.* Tho. Simpson.

By reading your di'ry a croud of strange notions
Crept into my head, of your rules, laws, and motions;
Your extravagant fancies my senses confound;
Can the unwieldy earth at the sun caper round?
But you say, she's an atom, each star a huge sun,
And attendant worlds with their moons round 'em run.
Such a tott'ring strange whirligig you've set's upon,
We wonder ere now we're not shak'd off and gone:
If what eyes ne'er saw you so soon can disclose,
Then pray solve this question: The earth, we'll suppose,
Round her axis in thirty-eight minutes to roll;
Shou'd we, who're degrees thirty-eight from the pole,
Be hurl'd thro' the air; where should we descend?
How long wou'd it be ere our circuit did end?
How far from the center in six hours time
Wou'd they be, who live in the midst o'th' hot * clime?
Kind artist, be pleas'd these things to let's know?
We'd rather believe you, than e'er find them so.

N. B. We suppose the earth sole actor, and to continue inviolate, and that we shall acquire the same velocity as the place of our residence. 52 deg. lat.

* *The equator.*

II. QUES.

II. QUESTION 188, *by Mr.* Eumenes Pamphilus.

I pray be ſo kind as the third ſide to ſhow
Of a three-corner'd field, by theſe data below.
Two hundred ninety-ſix poles, and no more,
Is one ſide; another two hundred and four.
The content of it likewiſe, I know, has been found
One hundred and thirty-eight acres of ground;
The ſought ſide, to free you from errors, I know,
Is longer than either of the given two.
If to ſolve me this queſtion, your aid you will lend,
You'll highly oblige a kind brother and friend.

III. QUESTION 189 *by Mr.* Rob. Beighton.

Solomon, we are aſſured in holy writ, was a man of the moſt extenſive judgment, and the wiſeſt of all mankind in his days. How great a philoſopher he was, may be gathered from his writings; and amongſt them are ſome indications that he was acquainted with the circulation of the blood, (not rightly deſcribed to us before Dr. Harvey wrote of it in 1628) the compoſition and frame of the human bodies, as well as others. His ſkill in architecture is ſufficiently evident from the account we have of his building that moſt magnificent, beautiful, and perfect pile, the temple, and its furniture: but I cannot learn from the divine writings, or from Jeſephus's fabulous hiſtory, how perſons can form an idea in what orders, and in what manner exactly the workmanſhip was performed, as to give us exact draughts and models thereof: whether of the Corinthian or Ionic orders, with their members and moldings, ſuch as we have had tranſmitted down to us from the Greeks and Romans, and copy at this day. Solomon, no doubt, would have rejoiced to have added to him one bleſſing which we have enjoyed, the acquaintance of (the glory of the age) Sir Iſaac Newton, who, we may ſay, has exceeded all men ſince Solomon's time. It may perhaps be thought a preſumption too nice and curious, to examine any of Solomon's great works mathematically: but as nothing is intended, nor can be ſaid to leſſen his wiſdom or ſtupendous work. I ſhall only ſuppoſe a queſtion was taken from that deſcription the inſpired writer has given us of his molten ſea, 1 Kings vii. 23. 'And he 'made a molten ſea ten cubits wide from brim to brim, 'round in compaſs, and five cubits high, and a line of thirty 'cubits did compaſs it about.' It is evident it could not be a circle;

a circle; for then the line that compaſſed it muſt have been more than thirty cubits, viz. 31·415926535g. Therefore we may ſuppoſe it to be elliptical, (or that the workmen were not very curious in their meaſures, or not ſkill'd in geometry) and the tranſverſe diameter ten cubits, and its periphery thirty: What then would be the leaſt triangle that could circumſcribe the ſame? and how analytically to inveſtigate the ſolution?

N. B. A cubit is equal to 21·888 inches.

IV. Question 190, *by Mr.* John Bulman.

There is a caſk, ſuppoſed the fruſtum of a parabolic conoid, or a caſk of the 2d variety. The bung diameter is 38·4 inches; the head diameters are unequal, the greater 33·5, the leſſer 28·8; the length of the caſk 54·27. Required the content of the caſk in ale gallons, the diſtance of each head diameter from the bung; it being ſuppoſed that there had been a decay in the caſk, and cut off, and a new and larger head put in at one end; and to let us know the diameter, length, and content of the greateſt cylinder that can be inſcribed therein, the circumference of each baſe touching that of the caſk around?

V. Question 191, *by Mr.* John Turner.

Within a maſon's yard one day,
A ſtone of ſize immenſe there lay;
The form of which (it was agreed on)
To be a parallelopipedon.
The depth, and breadth, and length, I here declare
In true arithmetic progreſſion are:
The ſtone's content below is * ſhown,
With what is needful to be known,
In order to inveſtigate
All the dimenſions ſeparate.

* *The ſolidity is* = 5184 *cubic feet, and the common difference of the depth, breadth, and length is equal to one forty-eighth part of the rectangle of the depth into the length. The depth of the ſtone being the leaſt dimenſion, and the length thereof the greateſt.*

VI. Question 192, *by Mr.* C. Maſon.

Find three ſuch cube numbers, whoſe ſum may be both a ſquare and cube number; and if that ſum be ſquared, to be a cube; alſo if cubed, ſhall be a ſquare.

The

The PRIZE QUESTION, *by Mr.* Tho. Simpson.

Young Strephon, long bless'd with his charming fair,
In happy consort liv'd devoid of care;
Till cruel fate call'd the fair nymph away
From his kind arms, to cross the raging sea;
Where horrid tempests, in thick darkness, roar,
And toss'd his dearest to an unknown shore.
He mourns, is restless, wanders day and night,
In ev'ry clime to find his dear delight.
Her lovely aspect his wing'd soul inspires;
He longs, sighs, wishes, melts in soft desires.
Propitious Venus pities his sad moan,
Glides shining down from her etherial throne,
And smiling says, (in a majestic tone)
'Thrice ten degrees north of th' equator lies
'This place, from which as Sol due east doth rise,
'Set out; and keep him always in your face;
'Move not too fast, but such an equal pace,
'To be eight miles more south at four hours end,
'And you'll arrive to th' arms of your dear friend.'
Thus said, she vanish'd from his wond'ring sight:
But still the swain is in a mournful plight,
Unless, fair nymphs, you'll vouchsafe to explain,
And shew the place, that must the fair regain.

i. e. *the distance required to move in an hour.*

A Geographical Paradox by Mr. C. Mason.

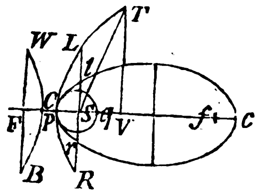
If Bristol from London be exactly due west;
Observe what I say; you may think it a jest,
That London, from Bristol, I can make appear,
From what I've observ'd, due east will not bear.
This seeming absurdity pray reconcile,
When Bristol's from London one hundred mile.
And in your next diary make it appear,
What rhomb 'tis that London from Bristol does bear.

1737.

Questions answered.

* I. QUESTION 187 *answered by Mr.* J. Turner.

THE circumference of the earth being 25080 miles, at the equator, if it revolves round its axis in 38 minutes, each point of the equator will go through 11 miles each second, with this velocity the body is projected from P. Let $SP = 1$: then the velocity of a body moving in a circle Pq, at the distance of one semidiameter from the earth's center, is such, as would make it go 4·92 miles each second uniformly; and according to Mr. Abr. de Moivre, in Philos. Transact. Abridg'd, p. 5. vol. 4.



Putting $R = 11$, $Q = 4·92$; $RR - 2QQ : RR :: SP : PF$. i.e. $72·6 : 121 :: 1 : 1·666$, and the point F will be the other focus; and because $FC = PS = 1$, therefore $CP = ·666$ the transverse axis of the section, which is an hyperbola.—— $QQ : RR :: 2SP : LR$; that is, $24·2 : 121 :: 2 : 10$; hence the latus-rectum of the hyperbola is 10 semidiameters of the earth.

Again,

* I. QUESTION 187.

The above computations of Mr. Turner's are rightly made as far as they are carried, but he has left some part of the question undetermined, which we shall here supply.

The body projected from the latitude of 52° will describe an ellipse whose focus is the center of the earth, its transverse axe = 19·13 (semidiameters of the earth), and its parameter 3·791, as determined above. So that the ellipse, no where touching the earth but in the place from whence the body is projected, the body will

Again, each point in the parallel of 52 degrees goes through 6·773 miles each second, put this $= R$; $2QQ - RR : RR :: SP : Pf$; that is, 2·53 : 45·87 :: 1 : 18·13, and the point f will be the other focus: and because $fc = PS = 1$, therefore $cP = 19{\cdot}13$, the transverse axis of the section, which in this case is an ellipsis, and $QQ : RR :: 2SP : lr = 3{\cdot}791$ its latus-rectum.

Merones

---

will revolve about the earth without falling upon it again or touching it except in the place from whence it was projected, supposing the earth's revolution to cease at the instant when the body is projected; but if the revolution be continued, the body will touch some other part of the same parallel of latitude, or will return to the vertex of its orbit again when some other point of the said parallel passes under it in revolving; and which point will be thus found: Putting $s = 16\frac{1}{12}$ feet, $v = 6{\cdot}773 \times 5280$ feet velocity in the vertex, and $f = SP = 3992 \times 5280$ feet; then the periodic time, or time of one revolution, is known to be $3{\cdot}1416 \times \frac{4sff}{\overline{4fs - vv}|^{\frac{3}{2}}}$ $= 132277$ seconds; this being divided by (38', or) 2280″, the quotient 58 shews the number of compleat revolutions the earth makes in the same time, and the remainder 37 shews what part of another revolution is made; wherefore as 2280 : 37 :: 360° : 5° 50′ = the difference of longitude required from the point of projection.

Again, with regard to the body projected from the equator, the path will be an hyperbola whose focus is also the earth's center, its transverse axe $\frac{2}{3}$, and latus-rectum 10, as determined above; and consequently its conjugate axe $= 2\sqrt{\frac{5}{3}}$. Then to find at what distance from the focus it will be in 6 hours $= 21600$ seconds: Since the body describes equal areas in equal times, and the rate of description being $\frac{11 \times 1}{2 \times 3992} = \frac{11}{7984}$ square semidiameters of the earth per second, therefore $\frac{21600 \times 11}{7984}$ will be the area described in 6 hours, or the area included by the curve, and the focal distances from the vertex and the other end of the curve, viz. the space SPT, and in which $PS = 1$.

Merones says that the body in 6 hours will be 8 of the earth's diameters from the center. Mr. *Lowe* says the equator will go 10·9 miles, the parallel of 52°, 6·7; and in 6 hours the moving body would be 1486709 miles from the earth's center. Mr. *Abr. Lord* 246321, Mr. *Geo. Brown* 204660, and Mr. *Hauxley* 400589 miles. I have not the ingenious author's solution by me, to the intricate and difficult question, so shall say no more of it till I can procure it.

II. QUES-

Now if the ordinate TV be drawn, and PV be put $= x$, the transverse $PC = \frac{2}{3} = t$, the conjugate $2\sqrt{\frac{5}{3}} = c$, and the area $PTS = \frac{21600 \times 11}{7984} = A$; then $SV = x - 1$, $TV = \frac{c\sqrt{tx + xx}}{t}$, and the $\triangle STV = \frac{x-1}{2} \times \frac{c\sqrt{tx+xx}}{t}$; consequently the hyperbolic segment will be expressed by $A + \frac{x-1}{2} \times \frac{c\sqrt{tx+xx}}{t}$. But by Rule IV. p. 376 Mensuration, the area of the same segment is expressed by $\frac{21\sqrt{tx + \frac{5}{7}xx} + 4\sqrt{tx}}{75} \times \frac{4cx}{t}$; these two expressions being made equal to each other, and reduced, we have $168x\sqrt{tx + \frac{5}{7}xx} + 32x\sqrt{tx} = \frac{150At}{c} + \overline{x-1} \times 75\sqrt{tx+xx}$; or, by restoring the values of t and c, it will be $168x\sqrt{2x + 2\frac{1}{7}xx} + 32x\sqrt{2x} = 30A\sqrt{5} + \overline{x-1} \times 75\sqrt{2+3xx}$; and the root x is easily found $= 3·083 = PV$. Then $ST = \sqrt{TV^2 + VS^2} = \sqrt{cc \times \frac{tx + xx}{tt} + \overline{x-1}^2} = 4x + 1 = 13\frac{1}{3}$ semidiameters $= 53227$ miles, the distance required.

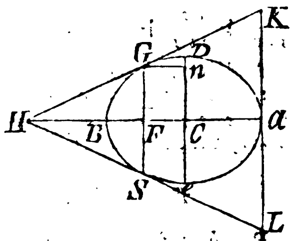
* II. QUESTION 188 *answered by the Proposer.*

Let $x =$ the side sought, then will $\frac{x}{2} + 250 =$ half the sum of the sides; each side being subtracted from it, and the three remainders multiplied into it, will give the square of the content, and will produce $16154\,xx - \frac{x^4}{16} - 132250000 = 487526400$ poles; which, reduced, gives $x = 460$ poles, the side required.

Mr. *Turner*'s solution is omitted, as being on the same principles.

* III. QUESTION 189 *answered by Mr.* Turner.

Let $Ba = 10$ cubits $= b$; $x =$ the conjugate axis De; the periphery $= 30$ cubits $= d$; therefore a quadrant $BGD = 7{\cdot}5$.



The gentlemen of the Weekly Oracle have given us a theorem, by which nearly to find the periphery of an ellipsis, viz. To twice the square root of the sum of the squares of the two principal diameters, add one-third of the conjugate diameter, and it will give the circumference within less than one-hundredth part of the whole. Hence $2\sqrt{bb + xx} + \frac{1}{3}x = d$; the root $x = 9{\cdot}056$ cubits $= 198{\cdot}22$ inches.

But

---

* II. QUESTION 188.

This question may be easier solved thus:

Let the two given sides be a, b, and the given area A. Then $\frac{2A}{ab} =$ the sine of the included angle, whose cosine call c; then, by trigonometry, $\sqrt{aa + bb - 2abc}$ or $\sqrt{aa + bb - 2\sqrt{aabb - 4AA}} =$ the side required.

But by fluxions, put $BC = a = 5$; $DC = c$; $b = 7{\cdot}5$; $x = Gn = FG$: Then, by the property of the ellipsis, $FG = y = \frac{c}{a}\sqrt{aa - xx}$, and $\sqrt{\dot{x}^2 + \dot{y}^2} = \frac{\dot{x}}{a}\sqrt{\frac{a^4 - aaxx + ccxx}{aa - xx}}$. Throw this into an infinite series, and the fluent will be $x + \frac{c\ x^3}{6a^4} + \frac{c^2x^5}{10a^6} - \frac{c^4x^5}{40a^8} + \frac{c^2x^7}{14a^8} - \frac{c^4x^7}{28a^{10}} + \frac{c^6x^7}{112a^{12}} + \frac{c^2x^9}{18a^{10}}$, &c. which will be equal to the length of the arch GD; and supposing x to flow till it becomes $x = a$, then $a + \frac{c^2a^3}{6a^4} + \frac{c^2a^5}{10a^6} - \frac{c^4a^5}{40a^8}$, &c. $= b = a + \frac{c^2}{6a} + \frac{c^2}{10a} - \frac{c^4}{40a^3}$, &c.

Now I find the powers of c above c^2 are very small, and may be set aside at present, and the law of the progression in c^2 being visible, we have $a + \frac{cc}{6a} + \frac{cc}{10a} + \frac{cc}{14a} + \frac{cc}{18a} + \frac{cc}{22a} + \frac{cc}{26a}$ &c. $= b$.

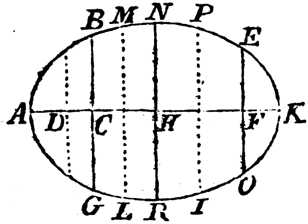
And by reduction $c^2 = 21{\cdot}17$, from which subtract the negative powers of c^4 which are equal to about ${\cdot}52$. And then $c = \sqrt{20{\cdot}05} = 4{\cdot}544$, and the conjugate $= 9{\cdot}088$ cubits, nearly the same as before.

For the least circumscribing triangle.

Put $b = Ba = 10$; $c = De = 9{\cdot}056$; $z = BF$; $aF = b - z$; then the subtangent $HF = \frac{bz - z^2}{\frac{1}{2}b - z}$; and by the property of the ellipsis, $b^2 : c^2 :: bz - zz : GF^2$; therefore $2GF = \sqrt{\frac{4ccbz - 4cczz}{bb}}$. Now the Δs HGS, HKL are similar, whence $HF : (GS) \sqrt{\frac{4ccbz - 4cczz}{bb}} :: (HA) \frac{bb - bz}{b - 2z} : (KL) = \frac{b}{2z} \times \sqrt{\frac{4cchz - 4cczz}{bb}}$: but $\frac{1}{2}KL \times Ha =$ the area of the $\Delta\ HKL$ which is to be a minimum, i. e. $\frac{b}{4z} \times \frac{bb - bz}{b - 2z}\sqrt{\frac{4cchz - 4cczz}{bb}} =$ area; which reduced is $= \frac{b - z}{b - 2z}\sqrt{\frac{b - z}{z}} =$ minimum; this in in fluxions is $\frac{b\dot{z}}{\overline{b-2z}|^2}\sqrt{\frac{b-z}{z}} - \frac{b-z}{b-2z} \times \frac{b\dot{z}}{2z^2\sqrt{\frac{b-z}{z}}}$ $=0$; which equation, reduced, gives $z=\frac{1}{4}b=2\cdot5$. From whence the area of the $\Delta\, HKL$ is $=117\cdot6$ cubits $=39\cdot135$ square feet.*

IV. Question 190 *answered by the Proposer.*

Put $x=DC=$ the difference of the distances of the two heads from the bung; $a=NH=19\cdot2$; $b=BC=16\cdot75$; $c=EF=14\cdot4$; $d=GF=54\cdot27$; then $\cdot5d+\cdot5x=DH=HF$; and $\cdot5d-\cdot5x=CH$: Per property of the curve, $aa-cc : \cdot5x+\cdot5d :: aa : \frac{\cdot5aax+\cdot5aad}{aa-cc}=AH$, and $\frac{aax+\cdot5ccd-5ccx}{aa-cc}=AC$



* III. Question 189.

Put $a=10=$ transverse, $z=$ conjugate, and $c=30=$ circumference. Then, by Rule VI. p. 233 Mensuration, $\frac{a+z}{4}\times p + \frac{1}{2}p\sqrt{\frac{aa+zz}{2}}=c$, where p is $=3\cdot1416$; hence $z=a-\frac{4c}{p}+4\sqrt{\frac{2c}{p}-a\times\frac{c}{p}}=9\cdot0876=$ the conjugate axe very true.

Again, for the least triangle; since $CF : FB :: aF : FH$ by the nature of the ellipse, and $aF=FH$ in all curves, $\therefore CF=FB=\frac{1}{4}aB$. Then $aH=2aF=\frac{1}{2}aB$, and $aK=2FG=\frac{2DC}{aB}\sqrt{aF\times FB}=\frac{2DC}{aB}\sqrt{\frac{1}{4}aB\times\frac{1}{4}aB}=\frac{DC\sqrt{3}}{2}$; consequently $Ha\times aK=\frac{1}{4}DC\times aB\sqrt{3}=118\cdot05=$ the least triangle HKL required.

The proposer also solved this question, but the solution is omitted as it was so very falsely printed.

$= AC$; and per prop. $\frac{aax + \cdot5ccd - \cdot5ccx}{aa - cc} : bb :: \frac{\cdot5aax + \cdot5aad}{aa - cc} : aa$; then, per 16 Eucl 6, $a^4x - \cdot5aaccd - \cdot5aaccx = \cdot5aabbx + \cdot5aabbd$; hence $x = \frac{bbd - ccd}{2aa - bb - cc} = DC = 15\cdot93174$. Hence the leſſer head from the bung $= FH = 35\cdot10087$ inches, and $CH = 19\cdot1691$.

The content of $\left\{\begin{matrix} NEQR \\ NBGR \end{matrix}\right\} = \left\{\begin{matrix} 112\cdot6191 \\ 69\cdot3193 \end{matrix}\right\}$ ale gallons, or $\left\{\begin{matrix} 137\cdot483 \\ 84\cdot623 \end{matrix}\right\}$ wine gallons.

The whole content 181·9384 ale gallons or 222·106 wine gallons. The diameter of the greateſt cylinder inſcribed $= BG = 33\cdot5$, length 38·33, and its content 119·8304 ale gallons or 146·2842 wine gallons.

Mr. *R. Dunthorne*, in a curious and conciſe manner, has wrought this queſtion, the content being the ſame as above, but he makes the diameter 30·88, and length 46·587 inches, for the greateſt inſcribed cylinder.

Mr. *Turner* ſays the queſtion refers to p. 444 of Ward's Math. and the ſecond variety is the parabolic ſpindle, and therefore makes the content greater, viz. 195·28 ale gallons; and the cylinder's length 45·5, the diameter 23·4 inches.

Mr. *Pilgrim* obſerves on this queſtion, That in all fruſtums of a parabolic conoid, as have their abſciſſa to their leaſt diameter more than half of the abſciſſa to the greater, can be inſcribed no greater cylinder than that to the fruſtum's leaſt diameter and height. Hence the diameter $= 30\cdot223$, length $= 49\cdot699$, content in ale gallons $= 126\cdot436$.

V. QUESTION 191 *anſwered by Mr.* Dunthorne.

Put x, y, and $z =$ the depth, breadth, and length of the ſtone, $b = 5184$, and $c = 48$: then $\frac{b}{y} = xz$, and $\frac{b}{cy} = \frac{xz}{c} =$ the common difference per queſtion. Whence $y - \frac{b}{cy} = x$; and $y + \frac{b}{cy} = z$; conſequently $y^3 - \frac{bb}{ccy} = b$; reduced $y^4 - by = \frac{bb}{cc}$. Solved, $y = 18$; whence $x = 12$, and $z = 24$.

Mr. *Joſ. Reſſer*, Mr. *Robert Heath*, and Mr. *Paul Sharp*, have anſwered this queſtion in a very methodical manner.

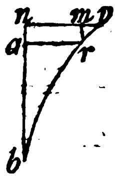
VI. QUES-

VI. QUESTION 192 *answered by Mr.* J. Hill.

The three numbers are $\frac{x^6}{8}$, $\frac{8x^6}{27}$, and $\frac{125x^6}{216}$; x being any number at pleasure: Hence if $x = 216$, we have three whole numbers for the answer.

The PRIZE QUESTION *answered by Mr.* Tho. Simpson *the Proposer.*

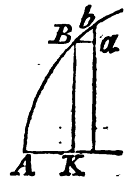
Let the radius be $= 1$; then the sine of the latitude $= \frac{1}{2}$; for the versed sine of the hour from 6 put x; then the sine of the sun's azimuth will be $= \frac{1 - x}{\sqrt{1 - \frac{3}{2}x + \frac{3}{4}xx}}$; now whilst x flows $\dot{x}$ the arch of time flows $\frac{\dot{x}}{\sqrt{2x - xx}}$; which (because the man's motion is equal) must be in a constant ratio to the lineola (rv) = the distance moved by the man in the same time:



therefore by putting (v) for the said ratio, we have $(\dot{rv}) = \frac{v\dot{x}}{\sqrt{2x - xx}}$; and because the angle (mrv) is the complement of the sun's azimuth, it is as radius : sine of (mrv) :: $(\dot{vr}) : (\dot{mv}) = \frac{v\dot{x}}{2\sqrt{1 - \frac{3x}{2} + \frac{3x^2}{4}}}$: the fluent of which is $= \frac{x}{2} + \frac{3x^2}{8} + \frac{5x^3}{32} + \frac{27x^4}{512}$, &c. $\times v$, and when x becomes $\frac{1}{2}$, is the distance given $= \cdot 308v = 8$, therefore $v = \frac{8}{\cdot 308}$. But the fluent of $(\dot{vr})$ is the arch of 4 hours; $60° \times v = 1{\cdot}0472v = 27{\cdot}2$ = the distance moved in 4 hours; therefore 6·8 miles is the required distance.

The same answered by Merones.

Let AB be the line he describes, put a = the space moved in an hour, r = radius = 1; s = sine, c = cosine, t = tangent, f = secant, q = arch of 60°; $AB = z$, $AK = x$; y = the sine of the hour from 6; u = hour arch from 6: then will $\frac{ry}{\sqrt{rr-yy}}$ = tangent of hour. Let $\dot{z}$ and $\dot{u}$ be passed over in the same small part of time, then by uniform motion $q : 4a :: \dot{u} : \dot{z} = \frac{4a\dot{u}}{q}$; and by the nature of the circle $\sqrt{rr-yy} : r :: \dot{y} : \frac{r\dot{y}}{\sqrt{rr-yy}} = \dot{u}$. And by trigonom. $r : s :: \frac{ry}{\sqrt{rr-yy}} : \frac{sy}{\sqrt{rr-yy}}$ = tang. azim. and $\frac{ty}{\sqrt{ss-yy}}$ = s. azim. from 6. The angle Bba = ⊙ azimuth from 6. And $r : \frac{ty}{\sqrt{ss-yy}} :: (Bb)\,\dot{z} : (Ba)\,\dot{x} = \frac{ty\dot{z}}{r\sqrt{ss-yy}}$, &c. whence $a = \frac{qx}{4t \times 5329} = \frac{8q}{4t \times 5329}$ = 6·8072 miles required.



The prize of 10 Diaries was won by *Merones*.

The Geographical Paradox answered.

Under the artic pole we can look no way but south; as soon as you change the pole from being your zenith, you have vertical circles, and consequently eastings and westings.—— No two places in an oblique sphere can have their vertical circles perpendicular to the plains of both meridians, consequently no two places can be due E. and W. of each other, except they both lie under the equator. London will bear from Bristol, from E. to N. 1 deg. 18 min. vide Gordon's Geogr. paragr. 42 prob. 43.

Mr.

Mr. Rob. Heath's *Anſwer to the ſame.*

To determine the bearing of two places from each other, there is commonly given the diſtance of each place from the pole, and their difference of longitude, (or angle at the pole) to find the bearing angles. Whence it is evident, that when the diſtances from the pole are equal, the bearing angles are equal. When each place is 90 degrees from the pole, each bearing angle is 90: conſequently thoſe places muſt bear in a contrary direction, viz. E. and W. of each other, which can only be when both places are under the equinoctial. For if the diſtances from the pole are equal, and not equal to 90 degrees, then one place bears as far from the weſt of the other, as that bears from the eaſt of it; and if the diſtances are unequal (as of Briſtol and London) the bearing angles muſt be unequal, and conſequently thoſe places can't bear contrary.——The ſolution.

Given the diſtance from the pole to Briſtol equal to 38 deg. 30 min. from the pole to London equal to 38 deg. 28 min. the angle at London equal to 90 deg. the angle at Briſtol will be found by a ſingle proportion equal to 87 deg. 49 min. which is 2 deg. 11 min. E. to N. London bears off Briſtol.

Of the Eclipſes in 1737.

Within the ſphere of the earth's orbit will happen four eclipſes this year. Twice will the moon in her wandering courſe interpoſe, and hide the ſplendour of the ſun from falling on the earth, or its atmoſphere: And twice will the earth in its courſe ſo fall in a line between the ſun and moon, as to hinder her receiving the light ſhe borrows from the ſun, to enlighten the earth by reflection. The firſt is the greateſt eclipſe of the ſun that has happened here ſince the year 1724, or that will be before 1748, on Friday the 18th of February, at three in the afternoon. The ſeveral calculations for divers places follow.

Computed by	Beg. h.m.	Mid.	End	Dur.	Dig.
By Aſtron. Carolina, Coventry	2 25	3 46	5 0	2 35	10 5
Mr. Chattock, London	2 33	3 58	5 13	2 39	10 4
Mr. Chattock, Coventry	2 25	3 50	5 6	2 41	10 5
Mr. Leadbetter, London	2 1	3 24	4 42	2 41	9 45
Mr. T. Williams, Oxon	1 56	3 18	4 38	2 32	9 48
Mr. T. Withnall, Namptwich	2 3	3 27	4 44	2 40	10 32
Mr. Sam. Evans, Cornwall	2 50	3 40	4 30	1 40	10 37

Mr.

		Beg. h.m.	Mid.	End	Dur.	Dig.
Mr. A. Lord at Leicester, by	Shackerley's Tab.	2 11	3 37	4 59	2 43	11 17
	Harm. Cœlest.	2 15	3 39	5 1	2 46	10 4
	Leadbetter's	1 32	2 59	4 21	2 49	11 23
	Newton's	1 47	3 16	4 33	2 46	11 12
Mr. T. Cowper, Wellingborrow		2 11	3 34	4 53	2 41	11 8
Mr. E. Hauxley,	Kirkleatham	1 31	2 59	4 21	2 50	10 24
	York	1 56	3 24	4 38	2 42	9 45
	Newcastle	1 55	3 23	4 37		
Mr. W. Mobbs, Mixbury Oxon		2 12	3 35	4 53	2 41	9 58
Mr. Forsters, Branspath		1 53	2 17	4 40	2 47	10 30
Mr. C. Forrest, Newcastle		2 21	3 45	5 4	2 42	10 23
Mr. Tho. Wade, Leicester		1 30	2 57	4 19	2 49	11 18
Philostratus, Leeds		1 47	3 10	4 32	2 45	11 27
Mr. Wm. Brown,	Cleobury	2 21	3 41	4 56	2 35	10 11
	Barbadoes	8 5	9 39	11 23	3 18	6 48
Mr. T. Sparrow,	Nottingham	2 21	3 41	4 55	3 34	10 16
	Hitchin	2 26	3 46	5 0	3 34	9 56
Mr. Rob. Cooke, Newcastle		1 41	3 7	4 34	2 53	10 50
Mr. J. Wilson, Morpeth		2 13	3 37	4 50	2 37	10 13
Mr. Peter Barons, Landsend		1 23	2 50	4 3	2 40	8 15
Mr. J. Bulman,	London	2 29	3 51	5 5	2 36	10 0
	Carlisle	2 22	3 44	5 3	2 41	10 23
	Deptford	2 30	3 53	5 7	2 37	10 0
Mr. J. Dorking,	Coventry	2 25	3 46	5 0	2 35	10 5
	London	2 27	3 49	5 4	2 37	10 9
	Norwich	2 28	3 50	5 6	2 38	10 11
	Wrentham	2 29	3 52	5 10	2 41	10 11

2. Eclipse

The above Eclipse was observed thus:

At	By	Begin. h. m. s.	End h. m. s.
Fleet Street, Lond.	Geo. Graham	2 25 3	— — — ap. t.
Greenwich	Dr. Bevis and Dr. Halley	2 25 39	5 3 29 ap. t.
Cambridge	Cha. Mason	2 38 30	5 16 2 tr. t.
Rome	D. de Ravillas	3 43 2	— — —
Bologna		3 33 34	— — —
Edinburgh	Col. Mac Laurin	2 5 55	4 44 51 ap. t.
Ibid.	Hon. Sir J. Clerk, Bart.	2 5 36	4 44 50 ap. t.

This eclipse was observed to be Annular from about Morpeth in Northumberland to beyond Inverness in North Britain.

2. Eclipſe of the moon March 5, at noon, inviſible.
3. Sun eclipſed Auguſt 15, at 1 morning. inviſible.
4. Moon eclipſed on Monday Auguſt 29, at 8 morning.

Computed by		Beg.	Mid.	End	Dur.	Dig.
By Aſtron. Carolina, Coventry		2 11	3 20	4 28	2 17	4 30
Mr. Chattock, London		2 9	3 34	4 59	2 50	6 25
Mr. Leadbetter, London		2 34	3 44	4 54	2 20	4 34
Mr. Sam. Evans, Cornwall		3 51	4 55	6 0		5 2
Mr. A. Lord for Leiceſter, by	Shackerley's Tab.	2 27	3 51	5 15	2 48	6 3
	Harm. Cœleſt.	2 25	3 52	5 19	2 55	6 24
	Leadbetter's	2 8	3 22	4 36	2 28	5 1
	Newton's	2 11	3 39	4 47	2 36	5 33
Mr. T. Cowper, Wellingborough		2 44	3 55	5 6	2 22	4 41
Mr. W. Mobbs, Mixbury		2 30	3 40	4 50	2 20	4 25
Mr. T. G. Forſter, Branſpath		2 29	3 40	4 47	2 17	4 51
Mr. Tho. Wade, Leiceſter		2 0	3 15	4 30	2 30	5 10
Mr. Wm. Brown,	London	2 17	3 25	4 34		
	Cleobury	2 7	3 15	4 24	4 30	2 16
	Dublin	1 49	2 57	4 6		
	Rome	3 9	4 17	5 29		
Endymion, London		1 57	3 32	5 7	3 10	6 17
Mr. T. Sparrow,	Nottingham	2 7	3 16	4 25	2 18	4 36
	Hitchin	2 10	3 19	4 28	2 18	4 30
Mr. J. Wilſon, Morpeth		2 29	3 40	4 51	2 22	4 41
Mr. E. Hauxley,	Kirkleatham	2 30	3 40	4 50		
	York	2 29	3 39	4 49	2 20	4 34
	Newcaſtle	2 28	3 38	4 48		
	Hexham	2 27	3 37	4 47		
Mr. J. Bulman,	London	2 31	3 40	4 50		
	Carliſle	2 20	3 39	4 39	2 19	4 32
	Deptford	2 32	3 41	4 51		
Mr. Mark Raſton	London	2 8	3 25	4 42	2 34	5 7
	Yarmouth	2 14	3 31	4 48	2 34	5 7
	Durham	2 4	3 21	4 38	2 34	5 7
	Rotterdam	2 24	3 41	5 58		

New

A COMET

Was alſo obſerved this year by ſeveral perſons, particularly by *J. Bradley*, Sav. Prof. Aſtron. in the months of January, February, and March; and who from his obſervations, on the ſuppoſition of a Parabolic Orbit, computes theſe Elements:

In its Perihelion Jan. 19, 8 h. 20 min. Temp. Equat. Lond.

Inclination of its path to the Ecliptic —		18°	20′	45″
Place of the Deſcending Node — —	♉	16	20	—
Place of the Perihelion — — —	♒	25	55	—
Diſt. Perihel. from the Deſcending Node		80	27	—

Log. Perihel. Diſt. from the Sun 9·347960
Log. of the Diurnal Motion — 0·938188

New Questions.

I. QUESTION 193, *by* Merones.

Ye bright ſons of art, that a rule did impart
In the * Diary 1735, to obtain
The ſolid content of a conic ſegment,
Cut off by a vertical plane.†
Since ye're ſo expert, proceed in like ſort,
To compute us the ſurface convex
Of a ſegment the leſſer, and all ſuch to meaſure
A general theorem annex.

* *Queſtion 177, propoſed in the Diary 1734, and anſwered in 1735.*
† *Perpendicular to the baſe.*

II. QUESTION 194, *by Mr.* J. Turner.

Ye diarian train, whoſe genius and parts,
In the various branches of the liberal arts,
Have ſo long been experienc'd; pray, lend us a while
Your aſſiſtance, two brethren to reconcile.
The caſe it ſtands thus: Their father, at's death,
An elliptical field did unto them bequeath,
Whoſe axis tranſverſe is juſt ſixty chains,
The conjugate forty preciſely contains.
Within this encloſure a pond there was made,
Whoſe juſt ſituation may thus be diſplay'd:
If you from the vertex chains fifteen direct
On the axis tranſverſus do count, and erect
A ſemiordinate there, as it's eaſy to do,
Then i'th' middle o'th' ſame the pond you may view:
Thro' this pond (for the father ſo order'd the matter)
A fence it muſt paſs, but of ſuch a nature,
As to be, of all other, the ſhorteſt that can
Be drawn thro' the ſaid point, and terminate in
The elliptic periphery at both ends, which will
Cut the mead in two parts. Exert now your ſkill,
To find the length of the fence, and alſo what ground
Each brother muſt have, which from thence may be found.
Methinks, with the firſt, C. Maſon, I hear
Say, this will do; prepare more for next year.

III. QUESTION 195 *by Mr.* Rich. Lycett.

There is an oak tree (the fruſtum of a cone) whoſe length is ten yards, the diameter at the top one foot, at the bottom

three

three feet; and an ivy twisteth round it in the manner of a spiral screw, that each twist is ten inches distant; three-sevenths of the ivy is eat into the tree; the diameter of the ivy is at the bottom one foot, at the top three inches. Quere the length of the ivy, and the content of both?

IV. Question 196, *by Mr.* Robert Heath.

Three ships, *A*, *B*, and *C*, sailed from a certain port in north latitude, until they arrived at three different ports, all lying under the equinoctial; *A* sailed on a direct course, between the south and the west 175·62 leagues; *C* sailed 133 leagues between the south and east; *B* sailed a course between *A* and *C* 102 leagues, making the angle or rhumb with *A*, equal the angle that *C* made with the equinoctial. Hence it is required to find the port sailed from, each ship's course, and distance from each other, and their respective ports? and to solve it by an equation not higher than a quadratic.

V. Question 197, *by Mr.* Ant. Thacker.

Let $AB = 1000$, $BC = 2000$, $CD = 3000$, $DE = 4000$, and AE variable; shew how to find the greatest area that can possibly be included by these five lines. *See the fig. to the solution.*

VI. Question 198, *by Mr.* Chr. Mason.

On Albion's Austral bounds, on Sussex strand,
A range of rocks defend th' adjacent land
From vile invaders, and insulting waves;
Indulgent nature, whom she loves she saves.
Erst Seven Cliffs, but now three Charles's call'd,
The fatal place where once the Dutch were maul'd.
The furrow'd front with visage ghastly pale,
Frowns at the billows of each boist'rous gale;
Friendly informs afar, of Beachy Head,
A shoal of rocks to mariners a dread.
O ghastly sight! a speedy death to touch,
Full oft experienc'd by the found'ring Dutch,
Who vent'rously look o'er the bending brow;
Pigmy-like they seem to those below.
Seafaring fowl of num'rous sorts here throng,
Both for their refuge, and to brood their young;
And when surpriz'd, each have their diff'rent cry,
Altho' in discord, yet in harmony.
On the broad shoulders of these cliffs there lie
The fairest downs e'er fac'd the azure sky;

Where a rich carpet o'er the same is spread,
And num'rous flocks thereon are yearly fed;
Whose silver fleece, and sweeter flesh exceed
Bansted, or Bagshot, or fam'd Coleswold breed.
Near, on the east, there is a grateful soil,
Which well rewards the tiller's care and toil.
Fair smiling meads, where once a briny flood;
Fine glad'ing fields, where a fair city stood,
East Bourn now call'd, whilom Anderida.
But mouldring time the hardest flints decay;
Here's scarce one mark where the old ruins lay;
The footsteps dim, the hist'ry dark to trace,
The sea, the downs, the wield concur that it's the place.
But not to trifle with so old a tale,
Here's what will more the reader's ear regale;
This fertile place in plenty doth produce
All the substantials fit for human use.
Fowl, fish, and fruit, the seasons still supply,
Their luxury, not want, to gratify.
If from the top of this stupend'ous height,
You dextrously let fall a pond'rous weight,
The time observ'd, one-eleventh of that ('tis plain)
The sound requires, for to return again.
Now from the data draw your consequence,
You'll find the height of this great eminence.

The PRIZE QUESTION *by Mr.* Turner.

Could I (unskill'd in verse) such lines indite
As would the fair ones please, I'd oft'ner write,
And unto B--gh--n (as I now do here)
Send a prize question for th' ensuing year:
If he approves, ten diaries are the claim
Of him or her, who truly solves the same.
☞ In the annext triangle *BAD*,
The sides are given, as below you see;*
Upon the angular point *A* as center,
A semicircle, whose diameter
Is thirty feet exact, described there.
Now, 'tis required to draw two lines, that each
From both the given points *B*, *D*, may reach,
To some one point in the circumference
Of the said circle (put *F* for pretence) *See the fig. to*
And so, as that the angle *BFD* *the solution.*
The greatest that is possible, may be.
To find this, various methods may suffice;
But he who purposes to win the prize,
Must prove himself skill'd in arts mathematic,
And solve it by equation call'd quadratic.

* $BA = 40$, $AD = 53$, $DB = 55$ feet.

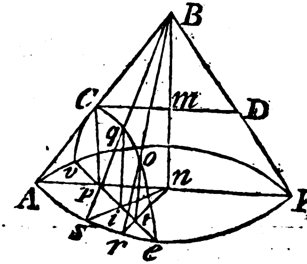
Ques-

1738.

Questions answered.

I. QUESTION 193 answered by the Proposer.

THrough the axis Bn, and any two points infinitely near each other as r, s, draw two planes Brn, Bsn, cutting the base in the lines rn, sn, and the plane of the hyperbola in the lines ot, qi, parallel to the axis Bn.



The ratio of the Δs nti, Boq, to each other, is compounded of the ratios of Δnti to Δnrs, and Δnrs to ΔBrs, and of the ΔBrs to ΔBoq, that is, of the ratio of nt^2 to nr^2, and of nr to Br, and of (Br^2 to Bo^2, or of) nr^2 to nt^2. Whence $\Delta nti : \Delta Boq :: nr : Br$. And since $\Delta nrs : \Delta Brs :: nr : Br$. Therefore the $\Delta nrs - \Delta nti : \Delta Brs - \Delta Boq :: nr : Br$. That is, area $trsi$: area $orsq :: nr : Br$. And since this always holds for any small correspondent parts of the circle and the conic surface, it follows that the wholes are in the same ratio. Hence this theorem,

As the radius of the base : side of the cone :: segment of the base : surface of the segment of the cone.

And the area of the segment $eAvCe$ will be found to be 904·38 square inches. The answer.

Mr. Ri. Dunthorne's *Answer to the same.*

The curve surfaces of cones may be considered as made up of an infinite number of infinitely narrow annuli, whose radii are in arithmetical progression, and the cosines of their segments equal.

Let $r =$ radius, $d = 3{\cdot}14159$ &c. and $b =$ the cosine of any indefinite segment, then by the arithmetic of infinites, $dr - 2b - \frac{b^3}{3rr} - \frac{3b^5}{20r^4} - \frac{5b^7}{56r^6} - \frac{35b^9}{576r^8}$ — &c. = the arch of that segment.

Put $R = An$, $B = Cm$, $A = Ap$, $C = AC$; and let $e =$ the infinitely small breadth of one annulus. Then will $dre - 2Be - \frac{B^3e}{3rr} - \frac{3B^5e}{20r^4} - \frac{5B^7e}{56r^6} - \frac{35B^9e}{576r^8}$ — &c. = superficies of any indefinite annular segment, constituting the curve surface of such conic segment.

Therefore such curve surface will be composed of an infinite number of such series, having B constant, and r increasing in arithmetic progression, from B to its greatest R, and $\frac{C}{e} =$ the number of terms. Consequently

$$\frac{dR^2C}{2A} - \frac{dB^2C}{2A} - 2BC + \frac{B^3C}{3RA} - \frac{B^3C}{3BA} + \frac{B^5C}{20R^3A} - \frac{B^5C}{20B^3A} + \frac{B^7C}{56R^5A} - \frac{B^7C}{56B^5A} + \frac{5B^9C}{576R^7A} - \frac{5B^9C}{576B^7A} + \&c. =$$

the curve of such conic segment, which from the numbers in question 177, gives 905 square inches, the answer nearly the same as the proposer's above.

If we suppose $r =$ radius, and $a =$ versed sine of any indefinite segment of a circle; we shall, by the arithmetic of of infinites, have

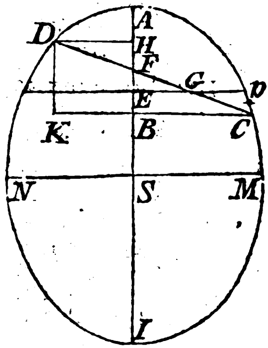
$$2\tfrac{1}{2} + \frac{4R^{\frac{1}{2}}A^{\frac{1}{2}}C}{3} + \frac{7A^{\frac{3}{2}}C}{15B} - \frac{71A^{\frac{5}{2}}C}{840B^{\frac{3}{2}}} + \frac{319A^{\frac{5}{2}}C}{10080B^{\frac{5}{2}}} - \frac{5419A^{\frac{9}{2}}C}{354816B^{\frac{7}{2}}} + \&c. =$$

the curve superficies of of such conic segment, in numbers above.

II. QUES-

II. QUESTION 194 *answered by the Author* J. T.

Let CD be the line sought. Put $AS = 30 = t$, $MS = 20 = n$, $AE = 15 = a$, $GE = 8\cdot66 = b$; $EF = z$, $SB = x$, $AB = t - x$, $BC = y$, $FB = t - x - a + z = m - x + z$.



As $b : z :: y : \frac{yz}{b} = FB$. And, by the property of the ellipse, $tt : nn :: tt - xx$. $\frac{ttnn - nnxx}{tt} = yy$. Ergo $x = \sqrt{\frac{ttnn - ttyy}{nn}}$. Put $r = tt$, $s = \frac{tt}{nn}$. Then $x = \sqrt{r - syy}$. Or, putting for x its value, and transposing, $\frac{mb + bz - yz}{b} = \sqrt{r - syy}$; and, squaring the parts, $mmbb + 2mbbz - 2mbzy + bbzz - 2bzzy + zzyy = bbr - bbsyy$. Put $q = bbs + zz$, $2p = 2mbb + 2bzz$; $g = bbr - mmbb - bbzz - 2mbbz$. Then we have $qyy - 2py = g$. And extracting the root, $y = \frac{p}{q} \mp \sqrt{\frac{g}{q} + \frac{pp}{qq}}$. Hence it is evident that $\sqrt{\frac{4g}{q} + \frac{4pp}{qq}}$ is equal to the difference of the double value of y; that is, of the lines $+CB$ and $-DH$, or CK. Lastly, $GE^2 : GF^2 :: CK^2 : CD^2$ which is to be a maximum; which in symbols will be, when the value of q, p, and g are restored,

$$= \frac{bb + zz + 4b^4rs - 4mmb^4s - 4b^4szz - 8mb^4sz + 4bbrzz}{b^6ss + 2b^4szz + bbz^4}$$

And, dividing by bb, and substituting letters for the known quantities, $\frac{h - kz + lz^2 - fz^3 + \varphi z^4}{a + dzz + z^4}$, in fluxions, $= 0$. It is evident that this will only produce an equation of the sixth power, by which the value of z comes out $= EF$.

Mr. *Lord* finds the length of the fence 32'03 chains; One share 161 a. 14 p. The other 27 a. 1 r. 20 p.

Metones,

Merones, by a curious and short process, finds *CD* the fence = 33·84; the greater segment 1561·45; the lesser 323·511.

From a geometrical construction I find the numbers nearly as follow: *EF* = 2·65 chains; *GE* = 8·6; *SB* = 12·12; *BE* = 2·88; *CD* the shortest fence 33·25; *CB* = 18·15; *Ep* = 17·2; *AB* = 18·0; *FB* = 5·38. And the content of the segment *DApCD* = 28 a. 3 r. 38 p. The greater *CMINDC* = 159 2 1, the shares; and the whole 188 1 39.

III. QUESTION 195 *answered by the Proposer*.

The center of the ivy will be found to lie without the periphery of the oak at the bottom ·055, and at top ·015 parts of a foot; which doubled and added to the diameters, at bottom 3·11, at top 1·03. Then the frustum's length on the outside is 30·01, and the angle with the base is 88° 0′ 52″. The circumference at bottom 9·77, and at top 3·235 feet. Then a line being drawn through the extremity of the top diameter and parallel to the central line is a mean arith. proportional; which muliplied by the number of twists 36·02 gives 234·2, which let be one leg of a triangle, 30·01 the other, with the angle between them 88° 0′ 52″. The other side will be found 235·3 feet the length of the ivy: Hence the content of the ivy is 80·87 feet, three-sevenths of which is 34·6; the content of the tree with the ivy growing into it is = 102·1; consequently 102·1 — 34·6 = 67·5 feet, the real content of the tree.

Merones's *Answer to the same*.

In the cone compleated, let s = the length of the side, c = the circumference at the base, d = the distance of the spiral threads, z = the spiral line, and x = the distance from the vertex to top of the first helix or turn.

Then $d : c :: \dot{x} : \frac{c\dot{x}}{d}$, and $s : \frac{cx}{d} :: \dot{x} : \frac{cx\dot{x}}{ds}$ = a small space the ivy moves round. And let the hypothenuse line the ivy moves round in be $= \dot{z}$; then $\dot{z} = \sqrt{\dot{x}^2 + \frac{ccx^2\dot{x}^2}{d^2s^2}}$ $= \frac{c\dot{x}}{ds}\sqrt{\frac{ddss}{cc} + xx}$. And, finding the fluent, z = 239·746 the length of the spiral ivy. Hence the content of the oak = 102·102; of the ivy = 81·6925 feet.

IV. QUES-

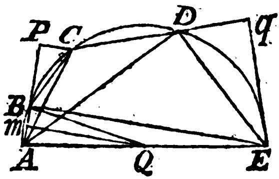
* IV. QUESTION 196 *answered by the Proposer.*

As 102 : 175·62 : : 133 : 228·99 leagues: The port sailed from is in 5° 5′ north latitude.

$$\text{The course of} \left\{\begin{array}{lrrl} A & 54^\circ & 38' & \text{west.} \\ B & 4 & 47 & \text{west.} \\ C & 40 & 10 & \text{east.} \end{array}\right\} \text{diff. long.} \left\{\begin{array}{rr} 7^\circ & 9' \\ 0 & 25 \\ 4 & 17 \end{array}\right\} \text{Req.}$$

V. QUESTION 197 *answered by Mr.* Rob. Heath.

Let $AB = 1000 = b$, $BC = 2000 = c$, $CD = 3000 = d$, $DE = 4000 = f$; and let $a = AE$ the diameter, radius $= 1$.



Then as

$a : 1 :: b : \frac{b}{a} =$ s. $\angle BEA$;

$a : 1 :: f : \frac{f}{a} =$ s. $\angle DAE$.

Now as $b : \frac{b}{a} :: c : \frac{c}{a} =$

s. $\angle BAC = \angle BEC$ by the nature of a circle.

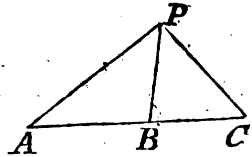
And as $f : \frac{f}{a} :: d : \frac{d}{a} =$ s. $\angle CED = \angle CAD$.

The

---

* IV. QUESTION 196.

The reason of the proportion, in the above original solution, for finding the distance (228·99) between the extreme ports, will clearly appear from the annexed figure, in which P represents the port sailed from, and A, B, C the ports arrived at by the respective ships. For, since the $\angle C = \angle APB$ by the question, and the $\angle A$ common to the two $\triangle$s APB, APC, therefore the third angles are equal, and those triangles equiangular. Hence then PB (102) : PA (175·62) : : PC (133) : $AC = 228·99$. And in like manner as $AC : AP :: AP : AB$. Hence the perpendicular and the angles are easily found.



Hence also the problem will be easily constructed. For it is only taking AC a fourth proportional to the three given lines.

The $\angle BEA + \angle CEB = \angle AEC$ whose sine $= \frac{b}{a}\sqrt{\frac{aa-cc}{aa}} + \frac{c}{a}\sqrt{\frac{aa-bb}{aa}}$; and the $\angle CAD + \angle DAE = \angle CAE$ whose sine $= \frac{d}{a}\sqrt{\frac{aa-ff}{aa}} + \frac{f}{a}\sqrt{\frac{aa-dd}{aa}}$; then say as $1 : a :: \frac{b}{a}\sqrt{\frac{aa-cc}{aa}} + \frac{c}{a}\sqrt{\frac{aa-bb}{aa}} : b\sqrt{\frac{aa-cc}{aa}} + c\sqrt{\frac{aa-bb}{aa}} = AC$. And as $1 : a :: \frac{d}{a}\sqrt{\frac{aa-ff}{aa}} + \frac{f}{a}\sqrt{\frac{aa-dd}{aa}} : d\sqrt{\frac{aa-ff}{aa}} + f\sqrt{\frac{aa-dd}{aa}} = CE$.

But $AC^2 + CE^2 = AE^2$ by 31 Eucl. 3, which is in algebraic terms,

$$\left.\begin{array}{l} dd - \frac{2ddff}{aa} + ff + 2df\sqrt{\frac{a^4 - a^2dd - a^2ff + ddff}{a^4}} \\ bb - \frac{2bbcc}{aa} + cc + 2bc\sqrt{\frac{a^4 - a^2bb - a^2cc + bbcc}{a^4}} \end{array}\right\} = aa.$$

Whence by reduction and the converging series a is found $= 6646{\cdot}316$, &c.

But the diameter AE or radius AQ may be more expeditiously found by a table of natural sines (which is an invention of my own) by finding out four chords, in the proportion of the chords given AB, BC, CD, DE. By a few trials, I find the angle AQm, under half the chord $= 8^\circ\ 39'$ the next less to a minute; and $8^\circ\ 40'$ the next greater. The operation is thus:

	°	′	″	‴	Nat. sines	°	′	″	‴	Nat. sines
Take	8	39	0	0	= 1503·981	8	40	0	0	=
angles	17	30	19	33	= 2×1503·981	17	32	23	57	= 2×
corre-	26	49	13	11	= 3×1503·981	26	52	32	40	= 3×
spond.	36	59	2	30	= 4×1503·981	37	3	59	42	= 4× 1506·857
	89	57	35	14		90	8	56	19	
Comp.	0	2	24	46 diff.	Sub.	89	57	35	14	dif. 11 21 5

Say, as $11'\ 21''\ 5'''$ to $60''$, so is $2'\ 24''\ 46'''$ to $12''\ 45'''$ to be added to the next lesser angle: Whence $8^\circ\ 39'\ 12''\ 45'''$ is the true angle AQm; consequently $17^\circ\ 18'\ 25''\ 30''' =$ angle AQB; and since AQB is an isosceles triangle, the angle $QAB = ABQ = 81^\circ\ 20'\ 47''\ 15'''$. By trigonometry, say $17^\circ\ 18'$ &c. (2974·929 N. S.) : 1000 :: $81^\circ\ 20'$ &c. (9886·161) : 3323·158 $= AQ$. Whence $AE = 6646{\cdot}316$ the diameter required.

Consequently the greatest area of the polygon $ABCDEA$ is $= 14567{\cdot}640$ &c. very near the truth.

The

The same answered by Merones.

In the last figure produce the lines AB, CD, and let fall perpendiculars on them from C and E.

When the area is the greatest possible, 'tis plain the angles ACE, ABE, and ADE must be right angles, therefore the figure $ABCDE$ is to be inscribed in a semicircle whose diameter is AE. Let AB be called a, $BC=b$, $CD=c$, $DE=d$, $BP=x$, $AE=u$. The Δs ACE, BCP, DEq are similar; therefore $b:x::u:\frac{xu}{b}=EC$. And $b:\sqrt{bb-xx}$ $::d:\frac{d}{b}\sqrt{bb-xx}=Dq$. And by Eucl. II. 12, $\frac{xxuu}{bb}=$ $cc+dd+\frac{2cd}{b}\sqrt{bb-xx}=CE^2$. And $aa+bb+2ax$ $=AC^2$. And by Eucl. I. 47, $AC^2+CE^2=AE^2$. Or $uu=aa+bb+cc+dd+2ax+\frac{2cd}{b}\sqrt{bb-xx}=$ $ss+2ax+\frac{2cd}{b}\sqrt{bb-xx}$, putting ss for the known quantities. Hence we have $ss+2ax+\frac{2cd}{b}\sqrt{bb-xx}$ $=\frac{bbcc+bbdd+2bcd\sqrt{bb-xx}}{xx}$. Which reduced gives $x=1795{\cdot}0355$; and $AE=6646{\cdot}31856$; and the greatest area $14567{\cdot}718\frac{1}{4}$.

Mr. *Walter Trott* gives the diameter 6646·3; area 1456740. Mr. *Robinson* (not considering it in a semicircle) 14247150·54: and Mr. *Bird* but 14202859. Mr. *Tatton* 13692100. *Arithmeticus* 14567·64. Mr. *Colburn* says 'tis 5920875, agreeable to M. Oracle. Mr. *Dorking* 14567642. Mr. *Forster* 14567638. Mr. *Lovatt* also solved it. Mr. *Thacker*'s number is 14567·560, and he shews that the figure must be in a circle.

VI. QUESTION 198, *by Mr.* Mason.

Let a be the time a heavy body requires in falling from the top of the cliff; then $\frac{a}{11}=$ the time that sound requires to move that space; $b=16$ feet; $c=968$ feet.

Then

Then $baa = \frac{ca}{11}$ per queft. and $11baa = ca$, which divide by a, and it is $11ba = c$; therefore $a = \frac{c}{11b} = 5.5$ seconds, the time fought. Then $5.5 \times 5.5 \times b = 484$ feet, the height fought. But if $b = 16\frac{1}{12}$ the anfwer is 481·417.

Nat. Percival. If found move 1142 feet in a fecond, as by late authors; then the height fought is 611·434 feet.

Col. Dagger's *Anfwer.*

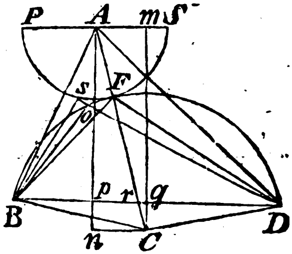
The defcent of a weight,* and motion of † found,
If agreed in one fecond of time to be found;
By the numbers annex'd, as the moderns allow,
Then the anfwer to Mafon hereunder we fhew.
And the height of the rock will be found to appear,
Two hundred and twenty two yards very near.
The procefs let Chriftopher fhew if he pleafe,
I'll not fcare the ladies with *A*'s and with *B*'s.

* Weight falls 16 feet 1 inch. † Sound moves 1142 feet.

Mr. *Powle* ufing the laft data gives the height 670·148 feet.
Mr. *Mitchell* (found's mot. 1140) anfwers 667·8 feet.
But taking the modern data, moft of my correfpondents give the true anfwer 670 feet = 223 yards 1 foot.

The PRIZE QUESTION *anfwered by Mr.* Rob. Heath.

In the triangle ABD, there is given $AB = 40 = b$, $AD = 53 = c$, $BD = 55 = 2d$, $PS = 30 = 2f$.



Make PS parallel to BD; draw mC perpendicular. In the center C defcribe a circle, to touch the given one at F, thro' the points B, D; the point of contact F, from which lines muft be drawn to B and D to conftitute the greateft angle, which is a maximum. Draw CA, CD and Ap the perpendicular of the triangle. In order to inveftigate the center C, and the lines BF and DF, find the perpendicular $Ap = 36.4342 = mq = b$; $qp = Am = 10.991 = g$; and let $qC = a$.

Then

Then per 47 Eucl. 1, $\sqrt{aa+dd}=CD=CF$; $\sqrt{aa+dd}+f=AC$; and $\sqrt{gg+bb+2ba+aa}=AC=\sqrt{aa+dd}+f$. Whence $gg+bb+dd-ff+2ba=2f\sqrt{aa+dd}$ by involution and transposition, or $k+2ba=2f\sqrt{aa+dd}$ by substitution. Whence $aa+\frac{kk}{bb-ff}a=\frac{4ffdd-kk}{4bb-4ff}$. And $a=5\cdot10647$: consequently $AC=42\cdot97$. Now the $\triangle ACm$ and Apr are similar, $mC:AC::Ap:Ar=37\cdot687$; $AC-Ar=rC=5\cdot283$; whence $Fr=22\cdot687$; $Br=26\cdot139$; $\angle CAm=75°\ 11'$; $\angle rBF=47°\ 9\frac{1}{2}'$; $BF=22\cdot91$; $DF=41\cdot02$; $\angle BFD=100°\ 30'$ a maximum.*

The same answered by the Proposer.

In the last figure draw An perpendicular to BD, and Cn perpendicular to An, and Cq perpendicular to BD. Put $Bq=27\cdot5=m$, $AF=15=b$, $Ap=36\cdot4=c$, $pq=nC=11\cdot1=d$; and $CB=CF=x$, $Cq=np=y$. Then is $CA=b+x$, and $An=c+y$.

And by 47 Eucl. 1, $\begin{cases} xx=yy+mm, \\ bb+2bx+xx=cc+2cy+yy+dd. \end{cases}$

Substitute for xx its value $yy+mm$, and then $bb+2b\sqrt{yy+mm}+yy+mm=cc+2cy+yy+dd$. Put $n=cc+dd-bb-mm$, then $2b\sqrt{yy+mm}=n+2cy$; and, involving, $4bbyy+4bbmm=nn+4ncy+4ccyy$. Lastly put $2p=\frac{4nc}{4cc-4bb}$, and $q=\frac{4bbmm-nn}{4cc-4bb}$; then $yy+2py=q$. And, extracting the root, $y=\sqrt{pp+q}-p=5\cdot1$; from whence the angle $BFD=100°\ 30'$.

Answered

---

* PRIZE QUESTION.

This problem is very easily constructed in all its cases, viz. when the angle is to be either a maximum, or a minimum, or a given quantity. For when it must be a given quantity, it is only describing on the given base BD a segment of a circle to contain the given angle, which segment will cut the given circle in two points, either of which will do. When the angle is to be a maximum or a minimum, the circle BFD must touch the given circle PFS respectively below or above the vertex A; which is very well done in the 12th prob. of LAWSON'S APOLLONIUS on TANGENCIES.

Answered by Merones.

Draw the lines Ds, Bs, Bo; then the angle BFD or $BoD = BsD + sBo$; whence BFD is greater than BsD. To find the radius, let $Bq = b$, $pq = c$, $Ap = p$, $AF = r$, $Cq = x$. Then will $BC = CF$, or $\sqrt{bb + xx} = \sqrt{cc + pp + 2px + xx} - r$. Thence $cc + pp + 2px = bb + rr + 2r\sqrt{bb + xx}$. And (putting $ss = cc + pp - bb - rr$) $ss + 2px = 2r\sqrt{bb + xx}$. And lastly $\frac{4pp}{4rr}xx + 4pssx = 4bbrr - s^4$. Whence $x = 6{\cdot}38716$, and $FC = 29{\cdot}19$, and the $\angle BFD = 100^\circ\ 32'$. *Q. E. I.*

The prize of 10 Diaries was won by Mr. *Ri. Gibbons.*

Of the Eclipses in 1738.

There will happen but two eclipses this year.

The first of the sun on the 7th of February, at six at night, the sun being then set, and invisible to us.

The second on friday the 4th of August, in the forenoon; the several calculations for divers places are these following,

Calculated

Calculated by	Beg.	Mid.	End	Dur.	Dig.
From Aſtron. Car. Coventry	10 7	11 7	12 8	. 4	3 35
By Mr. Chattock, { London	9 48	10 55	12 2	2 14	4 13
Scien. { Coventry	9 42	10 46	11 52	2 9	4 0
Mr. Leadbetter, London	9 57	11 2	12 8	2 10	4 8
Mr. Hulſe, Coventry	10 8	11 7	12 8	2 0	4 0
— Fr. Leadbetter's Tab. Lond.	9 57	10 41	11 24	2 2	4 6
Mr. Bamfield, Devonſhire	9 56	10 57	11 58	2 2	3 40
Mr. Cooper, { Northampton	9 50	10 51	11 53	2 3	3 34
Mr. Cooper, { York	9 54	10 51	11 48	1 53	3 4
Mr. Cooper, { London	9 53	10 56	12 0	2 7	3 45
Mr. Hughs, Flint	9 50	10 45	11 40	1 50	3 48
Mr. Sparrow, Bury S. Edmond's	10 9	11 9	12 11	2 2	3 45
Mr. Lord, Leiceſter	9 55	11 1	12 0	2 5	3 31
Mr. Barons, St. Ives, Cornwall	9 48	10 53	11 59	2 11	4 18
Mr. Wilſon, Morpeth	10 1	11 0	12 0	1 59	3 30
Mr. Facer, Wattlington	10 6	11 7	12 8	1 58	3 48
Wr. Williams, Middleton Sto.	9 50	11 1	12 6	2 16	4 10
Mr. Hampſon, Leigh, Lancaſh.	11 0	12 1	1 2	2 2	4 30
Mr. Wade, { Leiceſter	10 18	11 20	12 0	2 6	3 16
Mr. Wade, { Jeruſalem	2 2	3 15	4 4	2 12	4 42
Mr. Wade, { Gibraltar	9 20	10 2	12 34	3 14	8 42
Mr. Dorking, { London	10 9	11 10	12 12	2 3	3 36
Mr. Dorking, { Coventry	10 7	11 7	12 8	2 1	3 35
Mr. Dorking, { Norwich	10 11	11 12	12 13	2 2	3 35
Mr. Dorking, { Yarmouth	10 13	11 16	12 19	2 6	3 37
Mr. Dorking, { Yoxford	10 11	11 13	12 14	2 3	3 36
Mr. Forſter, Branſpath	9 59	11 0	12 5	2 6	3 11
Mr. Brown, { Cleobury	10 1	11 1	12 2	2 1	3 37
Mr. Brown, { London	10 13	11 15	12 17	2 3	3 45
Mr. J. Bulman, { London	10 12	11 13	12 18	2 6	3 37
Mr. J. Bulman, { Carliſle	10 2	11 0	12 6	2 4	3 0
Mr. J. Bulman, { Deptford	10 12	11 13	12 18	2 6	3 38
Mr. Withnalt, Namptwich	9 48	10 45	11 45	1 57	3 5

New

---

* This Eclipſe was obſerved thus:

At	By	Beginning	End
		h. m. s.	h. m. s.
Fleet Street, London	Mr. Graham & Mr. Short	9 59 20	11 59 36 a. m.
Upſal	And. Celſius	12 18 52	12 42 12 tr. time
Bologna	Euſt. Manfredi	10 51 25	1 18 1 tr. time

New Questions.

I. QUESTION 199, *by Mr.* Tho. Bird.

A ſpark having gain'd a young damſel's conſent,
He unto her father ſubmiſſively went,
Deſiring the favour to make her his bride:
Unto which petition her father reply'd,
'I have a nice garden, whoſe beautiful frame
'The form of a rhombus exactly does claim:
'In which is a ſquare inſcrib'd ſo by art,
'To touch the four fences alike on each part.
'Each ſide of this garden, ſo pleaſant and fair,
'Eight chains and three-fourths of a chain does declare;
'And alſo the ſquare in the middle inclos'd,
'Each ſide of ſix chains is exactly compos'd.
'Now tell us the garden's true area from hence,
'And I with your project of love will diſpenſe.
'But if your beſt judgment defective appear,
'In marriage you never muſt compaſs your dear.'
Now therefore kind ladies, come lend us your aid,
And ſhew Cupid's captive to compaſs the maid!
Produce the equation quadratic with care,
Nor ſuffer poor Strephon to bleed in deſpair.
For if your aſſiſtance you long do neglect,
His hanging or drowning you ſoon may expect.

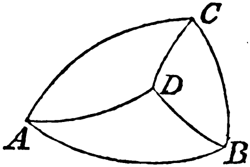
II. QUESTION 200, *by Mr.* Ri. Lovatt.

It was the ninth of June my friend and I
One evening late went forth to ſpy
Thoſe heav'nly bodies which to us appear,
Like blazing tapers, wand'ring in the air.
Then ſtraightway we a ſuppoſition made,
Thro' boundleſs æther, and it was convey'd
Round the celeſtial orbs as ſwift as thought,
Viewing thoſe wond'rous works by nature wrought.
Three ſtars among the reſt, their rays of light
ſeem'd to enlighten the dull ſhades of night;
Their diſtance from each other as we took,
Inſerted are i'th' margin of this book.
Their angles alſo, at the zenith made,
From thence we did their altitudes invade.
But here we ſtopp'd, no farther could advance,
Our theorems fail'd us, and our works were chance.

The

The altitudes therefore fair ladies give
In the next Diary: and your fame ſhall live
While the terraqueous globe her orb ſhall keep,
Or Adam in the ſilent grave ſhall ſleep.

Given $AC = 67^\circ\ 28'$, $BC = 60^\circ\ 0'$, $AB = 40^\circ\ 0'$.
The angles $ADC = 148^\circ\ 6'$, $CDB = 121^\circ\ 54'$.
Required AD, CD, and BD?



III. Question 201, *by Mr.* Chriſ. Maſon.

Suppoſe a ſhip ſet ſail from the latitude 51° north, and ſhape a north-weſt courſe, and ſail without interruption, Where will it at laſt arrive, and how many leagues run?

IV Question 202, *by Mr.* Tho. Cooper.

In a northern clime, two * hills ſublime
  Attract the diſtant ſight;
Th' exceſs of one, i'th' margin's † ſhown,
  Above the other's height.
If from the ſummit of each mount
  A line extended be,
The length thereof you muſt account
  Juſt ſixty miles and three;
Their ſite is ſuch, This line will touch,
  And reach the earth's ſurface;
And the contact, it is of fact,
  I'th' intermediate ſpace.
I pray diſcloſe the height of thoſe?
  And alſo at what time
The radiant ſun, on each of them,
  Will firſt begin to ‡ ſhine.

* *The higheſt hill is in lat. 65 deg. the other in 64 deg. 30 min.*
† *Diff. perp. height 119 yards; ear. ra. 6980000.*
‡ *On the winter ſolſtice.*

V. QUESTION 203, *by* J. B. S.

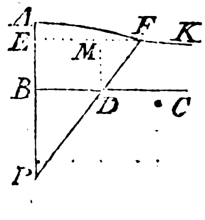
A ball of lead hanging from the top of a hall by a ſtring, drawn over a pulley, which is 20 foot long between the center of the ball and pulley, is ſet a ſwinging: The moment it begins to ſwing, a perſon, holding the other end of the ſtring, begins to pull it, and draws up the ball, and continues ſo to do, at an uniform ratio of 5 feet in a minute, until he has pull'd the ball quite up to the pulley. Quere, How many oſcillations will the ball make before it reach the top?

VI. QUESTION 204, *by Mr.* Maſon.

Soon after a hard gale of wind, ſome perſons ſtrolling along ſhore, upon the look-out, found a large caſk juſt driving aſhore, which proved a piece of old Jamaica rum: They ſoon boarded it, and racked off forty-one gallons, and filled up the caſk again with water, and acquainted ſome more of their party with their ſucceſs, who went and racked off the ſame quantity of the mixture, and filled it up again with water: In like manner it was ſerved ſo twice more; and at laſt, by the proof, there was found $25\frac{214}{729}$ gallons of rum remaining, the reſt water. How much did the whole piece contain, and how many gallons of rum was drawn out at each evacution?

The PRIZE QUESTION, *by Mr.* Ri. Dunthorne.

Let *AFK* be the conchoid of Nicomedes, which is continually approaching nearer to the line *BC*, yet if continued *ad infinitum* could never meet; which at firſt ſetting out to generate the line *AFK*, the diſtance *BA* is 16 inches; and the diſtance *BP* is 24 inches from the pole *P* to the aſymptote *BC*. It is required to find the diſtance of *F*, the point of inflection of the curve, from the line *BC*?



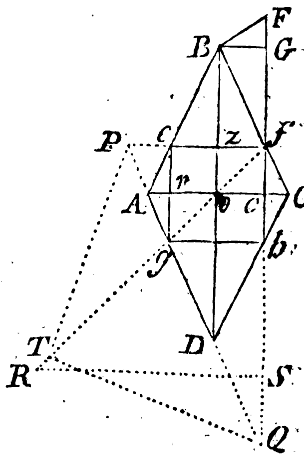
Queſtions

1739.

Questions answered.

* I. QUESTION 199 answered by Mr. Rob. Heath.

LET b = the sides $AB = BC$, &c. = 8·75 chains, $2c$ = $ef = fh$, &c. = 6 chains; then it is evident $ez = zf = 3$ chains = c: Let $Bz = x$. By similar $\triangle$s, $x : c :: x + c : \frac{x+c}{x} \times c = Ao$. But $Ao^2 + Bo^2 = AB^2$ by 47 Eucl. 1, which is in algebraic terms $\frac{xx + 2xc + cc}{xx} \times cc + xx + 2xc + cc = bb$. By reduction $\overline{xx + cc}|^2 + 2cx \times \overline{xx + cc} = bbxx$: by comp. $\square$ and extraction $xx + cc + xc = x\sqrt{bb + cc} = 9·25x$; whence the equation becomes $xx - 6·25x = -cc$. By comp. $\square$ and extraction a second time, $x = \pm\sqrt{9·765625 - cc} + 3·125 = 4$; or 2·25 (according to the ambiguity of the quest.) whence the area of the rhombus or garden $ABCD$ = 73·5 chains = 7 a. 1 r. 16 p.



The same answered by Mr. Hen. Travis *of Wirksworth.*

Erect the line BE perpendicular to CB, continue the line bf till it cuts BE at F, let fall the perpendicular BG, which is equal to $zf = a$; call fF, x; and fB, y; then will $cf = b - y$: Now it is evident that the rectangled triangles, Ccf, FBf, and FGB are similar, and that Ccf and FGB are equal, because $cf = GB$; hence per 47 Eucl. 1, $fB^2 + BF^2 = fF^2$; that is, $yy + bb - 2by + yy = xx = bb - 2by + 2yy$; and $fc : fC :: fB : fF$, or $a : b - y :: y : x$, $\because ax = by - yy$, and $-2ax = -2by + 2yy$. $bb - 2by + 2yy = xx$. Now if for $-2by + 2yy$ we put $2ax$, its value found, we have $bb - 2ax = xx$, or $xx + 2ax = bb$. Hence $xx + 2ax + aa = bb + aa$; $\because x + a = \sqrt{bb + aa} = 9{\cdot}25$; $ax = by - yy$, or $yy - by = -ax$, and $yy - 8{\cdot}75y = -18{\cdot}75$; then $yy - 8{\cdot}75y + 4{\cdot}375^2 = -18{\cdot}75 + 4{\cdot}375^2$; $y - 4{\cdot}375 = \sqrt{-18{\cdot}75 + 4{\cdot}375^2}$; $\because y = 5$. And the area of the rhombus $= 7$ a. 1 r. 16 p.*

II. QUES-

---

* I. QUESTION 199.

Produce the side AD of the rhombus both ways to meet the opposite side fe, fh, of the square, produced, in P and Q; then will PQ be $= 2AD$: For, $ef = 2ro$, and $Pe = 2Ar$ (because $ge = 2gr$), $\because Pf = 2Ao$, and by sim. $\triangle$s, $PQ = 2AD$. The problem is therefore reduced to this, To apply a given line PQ (or $2AD$) between two lines fe, fh, given in position, and to pass through a given point g; which is one of the problems of APOLLONIUS *Concerning Inclinations*, and which may be thus easily constructed.

Construction.

In fg produced take $fR = 2AD$ the given line, and draw $RS \parallel gh$; take also gT such that the rectangle fTg be $= RS^2$; then will the center T and radius RS cross fe and fh in P and Q, which will be the points required.

Demonstration.

Since $fTg = RS^2 = TP^2$, $\because Tf : TP :: TP : Tg$, and consequently the $\triangle$s TPf, TPg are similar by Eucl. VI. 6, $\because$ the $\angle TPg =$ the $\angle TfP = \angle RfS$ or $\angle fRS$; in like manner the $\angle TQP = \angle RfS$; consequently the $\triangle TPQ$ is similar to the $\triangle fRS$; but RS or $Sf = TP$ or TQ, $\because$ also $PQ = fR = 2AD$.

II. QUESTION 200, *by Mr.* Robert Heath.

Given sides $\left\{\begin{array}{l} AC = 67^\circ\ 28' = a \\ BC = 60\ \ 0 = b \\ AB = 40\ \ 0 = c \end{array}\right\}$ Nat. s. | Given angles $\left\{\begin{array}{l} ADC = 148^\circ\ 6' = d; \\ CDB = 121\ \ 54 = e; \\ ADB = 90\ \ 0 = 1. \end{array}\right.$

Whence by trigonometry is found the angles $CAB = 69^\circ\ 39'$, $ABC = 90^\circ\ 00'$, $ACB\ 44^\circ\ 7'$. Let $x = BD$. Then by spheric trigonom. $c : 1 :: x : \frac{x}{c} = s. \angle DAB$.

Let $m =$ sine angle CAB, and n its cosine.

Then $m\sqrt{\frac{cc - xx}{cc}} - \frac{nx}{c} = s. \angle CAD$. Say $d : a ::$ $m\sqrt{\frac{cc - xx}{cc}} - \frac{nx}{c} : \frac{am}{dc}\sqrt{cc - xx} - \frac{nax}{cd} = DC$. Now the sine $\angle ABD = \sqrt{\frac{cc - xx}{cc - ccxx}}$: Then since $\angle ABC$ = a right angle, therefore the complement of $\angle ABD =$ $\frac{x}{c}\sqrt{\frac{1 - cc}{1 - xx}} = \angle DBC$. By spherics, $e : b :: \frac{x}{c}\sqrt{\frac{1 - cc}{1 - xx}}$ $: \frac{bx}{ce}\sqrt{\frac{1 - cc}{1 - xx}} = DC = \frac{am}{dc}\sqrt{cc - xx} - \frac{nax}{cd}$. Which equation reduced, and turned into numbers, will be

$$2{\cdot}549528\sqrt{{\cdot}4131758987 - xx} - {\cdot}945629x = \frac{1{\cdot}21569285 2x}{\sqrt{1 - xx}}.$$

Solved, $x = {\cdot}47477$, &c.

Whence $\left\{\begin{array}{l} DB = 28^\circ\ 21' \\ CD = 40\ \ 58 \\ AD = 29\ \ 29 \end{array}\right\}$ whose complements $\left\{\begin{array}{l} 61^\circ\ 39' \\ 49\ \ 2 \\ 60\ \ 31 \end{array}\right\}$ are the alt. of the stars required.

N. B. *This method is universal, when the given angle exceed a right angle.*

The Proposer Mr. Ri. Lovatt's *Answer.*

Put $b =$ sine of $AB \div$ by sine of the $\angle ADB$; $p =$ cosine AC; $d =$ sine $BC \div$ by s. $\angle CDB$; s. $ABC = n$; and $m =$ its cosine; $a =$ s. $\angle ABD$. Then $ab =$ s. alt. AD. And $m\sqrt{1 - aa} - am$ will be $=$ s. $\angle DBC$; and $nd\sqrt{1 - aa} - amd =$ s. CD. Hence this analogy, as radius 1 : cosine $AD = \sqrt{1 - aabb}$ :: cosine $CD =$ $\sqrt{2anmd^2\sqrt{1 - aa} - aamm + aann - nn + 1} : p$.

Therefore

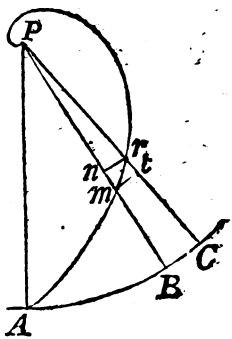
Therefore $\overline{1 - aabb} \times 2anmd^2 \sqrt{\overline{1 - aa - aamm + aann - nn + 1}} := pp$. Which gives $AD = 29° 29'$, $BD = 28°21$, $CD = 40°50$; whose complements are the altitudes.

* III. QUESTION 201 *answered by Mr.* J. May, *jun. of Amsterdam.*

The ship will arrive at the north pole, and the leagues run, will be the length of the loxodromic, beginning at the latitude of 51 north, and ending at the pole, after having performed infinite revolutions about the said pole; the length of the said loxodromic will be 1278·56 leagues, supposing the earth to be a true spherical figure; and the circumference according to Norwood's observations, likewise the ship infinite small.

The same answered by Mr. Walter Trott, *per Corollary to Prob.* XII. Stone'*s Appendix.*

Suppose a line drawn infinitely near to PB; then may the triangle made thereby be looked on a rectilinear; and making nm radius, mr will be the secant of the course, and $a = PA = PB$; $A\dot{m} = y$; and $mn = \dot{x} =$ fluxion of the diff. latitude; then per plain trigonometry, R : sec. course :: $\dot{x} : \dot{y}$. Hence, taking the fluents, $y =$ secant of the course multiplied by x, divided by the radius. The ship's distance sailed is 1103·08 leagues, and then coincides with the pole.



Mr.

* III. QUESTION 201.

A Complete Investigation of this question may be seen at Quest. 18 of our MATH. MISCELLANY.

Mr. Tho. Bird *anſwers thus:*

The ſmall decrements of the ſpiral, or the diſtance the ſhip runs while it paſſes through ſmall, but equal angles of longitude (as at every 0·01 deg.) are a ſeries of geometrical proportionals continued decreaſing and ending in 0, in the pole: whoſe ratio (1·0001745) together with the firſt or greateſt term (0·57736) and laſt or leaſt (0) being known; the ſum of all the ſeries or length of the ſpiral is eaſily found = 3309 minutes of a degree, or 1103 leagues; which may more eaſily be done by fluxions. However it may be ſolved to a geometrical exactneſs by this eaſy and known analogy.

As the coſine of the courſe	co. ar.	0·150515
To diff. lat. 780 leagues —		2·192095
So radius — — —		10·000000
To diſt. run 1103 leagues		3·042610

After the ſhip has paſſed through all the degrees of longitude, and is arrived to the ſame meridian itſelf, it will be but 4·632 minutes from the pole, according to artificial tangents for Wright's meridional parts being made from falſe ſecants, are not to be truſted) after which it will be no deviation from mathematical exactneſs to ſuppoſe the remainder to the pole a plane, in which caſe the proportion of the velocity of the ſhip's approach towards the pole, is nearly as 10000000 to 18151, ſhe in the next round will be but 0·00841 min. from the pole, and the next revolution falls into the pole itſelf, contrary to what the ingénious Oughtred ſuppoſed in his Cir. Prop. of Nav. p. 37.

Mr. Chr. Maſon *the Propoſer anſwers in this manner.*

Let P denote the north pole, A the place whence the ſhip ſet ſail, AmP its courſe, AP the meridian, and BP, CP two other meridians infinitely near. Nr a parallel of latitude. Let AP = co. lat. $= b$; $r =$ rad. $c =$ coſine of the courſe, $Bm = a$; $AmP = x =$ diſt. run. Then by plain trigonometry, $a : \frac{ra}{c} :: b : \frac{bra}{ca} = \frac{br}{c} = x = 1277{\cdot}73$ leagues.

The ſame anſwered by Mr. R. Dunthorne.

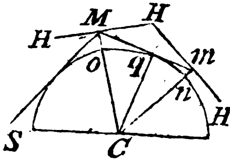
Since parts of rhumbs are every where to their correſponding parts of the meridians, as radius to the coſine of the courſe, it will be S. 45 : rad. :: 900·25 leagues (the diſt. of 51° from the pole) : 1273·14 leagues the anſwer.

This queſtion was alſo curiouſly ſolved by Mr. *George Brown*, jun.

IV. QUES-

* IV. QUESTION 202 *answered by Mr.* Rob. Heath.

Let r = earth's radius = $Co = Cq = Cn$ = 6980000 yards; d = diff. mountains heights = 119 yards, x = height of the lowest mountain + earth's radius = Cm? Then $\sqrt{xx - rr} = qm$, by 47 Eucl. 1.



Again $\sqrt{xx + 2dx + dd - rr} = Mq$; but $Mq + qm$ = 63 miles = 110880 yards = b. Consequently $\sqrt{xx - rr} - b = -\sqrt{xx + 2xd + dd - rr}$. Reduced, $xx + dx = \frac{bb - dd}{4} + \frac{bbrr}{bb - dd}$. In numbers $xx + 119x = 48723529707626{\cdot}7$, &c. Solved, $x = 6980164{\cdot}68$ yards; consequently the height of the lowest mountain = 164·68 yards; that of the highest = 283·68 yards.

The time when the sun first begins to shine on the highest mountain (M) is when he cuts the tangent to the earth's surface (SM) below the horizon (HMH) the angle HMS (found by trig.) = 31 min. also he first begins to shine on the top of the lowest mountain (m) when he cuts the tang. (Mm) below the horizon (HmH) the $\angle MmH$ = 23 min. Now if the sun's refraction be allowed for near the horizon = 33′, the angle HMS will be = 1° 4′; and angle HmM = 56 m. which the sun is below the horizon of each mountain when he first shines on their tops.

The hour or times will be found by spheric trigonometry; thus, at 10h. 8 m. 38 s. on the highest mountain, at 10h. 2 m. 4 s. on the lowest.

N. B. Sun rises in lat. 65, at 10h. 35 m. 16 s. in 64° 30′, at 10h. 22 m. 56 s. This shews how much those are mistaken, who suppose the sun would first shine on them, nearly at his rising.

Merones

---

* IV. QUESTION 202.

In this problem we have given the base Mm, the perpendicular Cq, and the difference of the sides $CM - Cm$; whence the $\triangle\, CMm$ may be easily constructed.

Merones gives the higheſt hill 283·69 yards, the lower 164·69; ſun riſes on the higher at 10 h. 21¼ min. on the lower at 10 h. 14⅗ min.

The propoſer, Mr. *Tho. Cooper*, ſays the ſun appears on the higheſt at 10 h. 21 m. 12 s. on the lower at 10 h. 13 m. 12 s.

V. QUESTION 203 *anſwered by Mr.* Ri. Dunthorne.

Let $a = 20$ feet, $b = 4$ minutes, $c =$ number of vibrations which the pendulum whoſe length is a makes in the time b, and $\dot{e} =$ a ſmall particle of time. Then $b : c :: e : \frac{ce}{b} =$ number of vibrations which the pendulum a makes in the time e; and $b : a :: e : \frac{ae}{b} =$ portion of the ſtring drawn up in the time e; then will $a - \frac{ae}{b} =$ length of the pendulum after the firſt time e; $a - \frac{2ae}{b} =$ that after the ſecond time e, &c. And $\frac{ccee}{b \times \overline{b-e}} = \square$ of the number of vibrations in the ſecond time e. Conſequently its ſquare root $=$ number of vibrations in ſecond, e. In like manner $\frac{ce}{\sqrt{b} \times \sqrt{b-2e}} =$ number of vibrations in the third time e. $\frac{ce}{\sqrt{b} \times \sqrt{b-3e}} =$ number in the fourth, &c. Whence 'tis manifeſt that the number of vibrations in the ſeveral times e, as above, are a ſeries of fractions, whoſe numerators are equal, and their denominators are ſquare roots, whoſe ſides are ſingle powers, decreaſe in arithmetical progreſſion from b, and $\frac{b}{e} =$ the number of terms. So that by the arithmetic of infinites, $2c$ will be the ſum of all the terms in the ſeries. But $c = 97$, whence $2c = 194$, the number of vibrations required.

The ſame anſwered by the Propoſer I. B. S. Tycho.

I obſerve, that the anſwer reſults to this, viz. To find the length of a pendulum, which remaining invariable, ſhall make the ſame number of vibrations in a given time, as one does, which is continually lengthening or shortening, in some given ratio.——This I find to be 56·25 inches nearly; wherefore the number of vibrations before the ball reach the pully will be about 200.

Note, The ball is supposed to be a point, and the string a mathematical line, and the oscillations performed in similar arcs.

Merónes says, the ball will make 194 oscillations, being twice the number which a pendulum of the whole length of 20 feet would have made in the same time.

Mr. *Tho. Bird* answers 194·49 vibrations.

VI. QUESTION 204 *answered by Mr.* Rob. Heath.

Let $b = 25\frac{214}{725} = 25{\cdot}2935528$ gallons of rum left in the cask.
$q = 41$ gallons, the liquor drawn off each time.
$n =$ the number of times of drawing. And
$x =$ the quantity of neat liquor the cask held.

Say $x : x - q :: x - q : \frac{\overline{x - x}|^2}{x} =$ rum left at 2d drawing.

$x : \frac{\overline{x - q}|^2}{x} :: x - q : \frac{\overline{x - q}|^3}{xx} =$ rum left at 3d drawing.

$x : \frac{\overline{x - q}|^3}{xx} :: x - q : \frac{\overline{x - q}|^4}{xxx} =$ rum left at 4th drawing.

Whence it is evident, that the quantity of neat liquor, left at any number of times drawing off, will be universally $\frac{\overline{x - q}|^n}{x^{n-1}}$; consequently $\frac{\overline{x - q}|^4}{xxx} = b$. Reduced $x - q = b^{\frac{1}{4}} x^{\frac{3}{4}}$. In numbers $x - 41 = 2{\cdot}1426 x^{\cdot 75}$; solved, [according to a new method of managing exponentials] $x = 124{\cdot}671$, &c.

Now, if each quantity left in the cask, at any time of drawing off, be subtracted from the quantity left the time preceding, the neat rum at each time drawn off will be found; and is as follows:

Rum drawn off	1.	41·000	gallons Total 124·67.
	2.	27·516	
	3.	18·467	
	4.	12·394	
add gallons left		25·293	

Mr. *T. Robinson*, Mr. *J. May*, *Merones*, Mr. *W. Rubins*, Mr. *El. Colbourn*, Mr. *J. Badder*, Mr. *Waller Trott*, Mr. *Tho.*

Tho. Cooper, Mr. *T. Woodward*, Mr. *J. Parminter*, Mr. *J. Prichard*, Mr. *Paul Sharp*, Mr. *E. Verrall*, Mr. *J. Bir* , Mr. *Powle*, Mr. *W. Spicer*, Mr. *Rob. Cooke*, Mr. *Forster*, A. B. Mr. *Hitton*, Mr. *Mitchel*, Mr. *Young*, Mr. *Williams*, Mr. *G. Brown*, Mr. *Ballard*, Mr. *Dunthorne*, Mr. *Wade*, Mr. *Lord*, and others have given true answers to this question.

The PRIZE QUESTION *answered by Mr.* Hen. Travis.

Let *FB* and *AB* (in the scheme to the quest.) be called b and c, the abscissa and ordinate x and y; then $bbcc + 2bccy + ccyy - bbyy - 2byyy - y^4 = yyxx$, will be the equation, expressing the nature of this curve, &c. The first in fluxions, $2bcc\dot{y} + 2ccy\dot{y} - 2by\dot{y} - 6byy\dot{y} - 4y^3\dot{y} = 2y\dot{y}xx + 2x\dot{x}yy$; again by an uniform velocity $ccy\ddot{y} - 6by\dot{y}\dot{y} - 6yy\dot{y}\dot{y} = \dot{y}\dot{y}xx + 2y\ddot{y}xx + 2y\dot{y}x\dot{x} + \dot{x}\dot{x}yy$; and from the 1, 2, and 3 steps we have $y^3 + 3byy = 2bcc$. Therefore $y = 12{\cdot}0885$ the answer.

Answered by Mr. J. May.

Put *AB* or *DF* $= 16 = a$, *BP* $= 24 = b$, *BE* or *DM* $= x$, and *EF* $= y$. Then by the similar triangles *DMF* and *PEF*, we have $DM = x : MF = \sqrt{aa-xx} :: PE = b+x : EF = y$. Multiplying extreams and means, and dividing by x, have $y = \frac{\overline{b+x}\times\sqrt{aa-xx}}{x}$; in fluxions $\dot{y} = \frac{\overline{x^3+aab}\times\dot{x}}{xx\sqrt{aa-xx}}$; again $\ddot{y} = \frac{\overline{2a^4b - aax^3 - 3aabxx}\times\dot{x}\dot{x}}{\overline{aax^3 - x^5}\times\sqrt{aa-xx}} = 0$. Then is $x^3 + 3bxx - 2aab = 0$; by means of this equation the value of x may be found by the section of a parabola and circle. But if brought into numbers, and reduced, will give $x = 12{\cdot}0885$ inches for *BE*; being the distance of the point of inflection of the curve from the asymptote.

This prize question was taken from *Infiniments Petits par Mar. de l'Hopital*, p. 65, Paris edit. 1696, or *Hay*'s Fluxions, p. 91, and is in Stone's Fluxions, p. 87. *Simpson*'s Fluxions, p. 87.

The prize of 10 Diaries fell to the lot of Mr. *John Withered.*

Of the Eclipses in 1739.

Within the ſphere of the earth's orbit will happen five eclipſes this year; three times will the moon in her wandering courſe interpoſe and hide the ſplendour of the ſun from falling on the earth or its atmoſphere; and twice will the earth in its courſe, ſo fall in a line between the ſun and moon, as to hinder her receiving the light ſhe borrows from the ſun, to enlighten the earth by reflection.

The firſt eclipſe is of the moon, Jan. 13, at night.

Calculated by		Beg. h. m.	Mid.	End	Dur.	Dig.
From Aſtronomia, Coventry		9 59	11 20	12 41	2 42	6 40
Mr. Chattock,	London, Coventry	9 24	11 0	12 35	3 11	8 9
Mr. Leadbetter for London		9 32	10 47	12 15	2 42	6 22
Mr. T. Robinſon,	London	9 57	11 26	12 54	2 57	7 1
	Newcaſt.	9 51	11 20	12 48		
Mr. Ra. Hulſe, Cheſhire		9 32	10 47	12 15	2 42	6 22
Mr. T. Cooper, Wellingbor.		9 40	11 5	12 30	2 50	6 52
Mr. J. Canton,	London	9 36	11 1	12 26	2 50	6 51
	Stroud	9 27	10 52	12 17		
Mr. W. Brown,	Cleobury	9 54	11 15	12 36	2 42	6 39
	London	10 4	11 25	12 46		
	Jeruſalem	12 26	1 47	3 8		
	Barbadoes	6 5	7 26	8 47		
	Iſpahan	1 37	2 58	4 19		
Mr. J. Wilſon, Morpeth		9 28	10 48	12 8	2 40	6 21
Mr. Sam. Bamfield, Devonſh.		9 39	11 5	12 31	2 50	6 52
Mr. T. Glaſpool,	Wincheſter	9 29	10 51	12 12	2 43	6 21
	Norwich	9 35	10 56	12 17	2 42	6 22
Mr. Rob. Cooke, Newcaſtle		9 58	11 22	12 46	2 46	7 0
Mr. N. Oats		9 18	10 45	12 12	2 54	6 33
Mr Geo. Forſter, Branſpath		10 10	11 26	12 43	2 32	6 41
Mr. Ed. Dale, Sunderland		10 10	11 26	12 43	2 32	6 41
Mr. Schoolcroft, Hovingham		9 55	11 22	12 49	2 53	7 7
Mr. E. Silborow, Buntingford		10 2	11 26	12 49	2 47	7 6
Mr. Jo. Benwell, Highworth		9 15	10 42	12 8	2 50	7 4
Mr. Tho. Williams		9 24	10 40	12 9	2 45	6 30
Mr. Cha. Facer, per Hamſtead		9 32	10 56	12 21		6 36
Mr. Sparrow, Edmundſbury		9 35	10 56	12 17	2 42	6 31
Mr. Tho. Wade, Leiceſter		10 55	11 44	12 53	2 18	7 7

The

The second eclipse is of the sun, Jan. 28, at 4 morn. invisible.

The third eclipse of the moon, July 9, 4 aftern. invisible.

The fourth eclipse is of the sun, July 24, at 4 afternoon.*

Calculated by	Beg.	Mid.	End	Dur.	Dig.
Astron. Caroline, Coventry	3 23	4 33	5 37	2 14	7 12
Mr. Chattock, London	3 4	4 19	5 27	2 23	7 26
Mr. Chattock, Coventry	2 54	4 10	5 19	2 2[illegible]	7 28
Mr. Leadbetter, London	3 10	4 22	5 29	2 18	7 6
Mr. Robinson, Newcastle	3 20	4 34	5 42	2 2[illegible]	8 35
Mr. Hulse, London	3 10	4 25	5 30	2 2[illegible]	7 6
Mr. Cooper, Wellingborrow	3 12	4 25	5 33	2 2[illegible]	7 43
Mr. W. Brown, Cleobury	3 18	4 28	5 32	2 1	7 10
Mr. W. Brown, Copenhagen	4 16	5 25	6 27	2 11	8 42
Mr. W. Brown, Paris	3 49	4 56	5 58	2 9	7 0
Mr. Wilson, Morpeth	3 15	4 27	5 35	2 20	7 30
Mr. Bamfield, Honiton	3 37	4 48	5 55	2 18	7 18
Mr. Glaspool, Winchester	3 8	4 20	5 26	2 18	7 5

The fifth eclipse is of the sun, on wednesday December 19, in the morning, beginning before sun-rising.†

Calcu-

---

* This Eclipse was observed at *Wittemberg* by JOH. FRIED. WEIDLER.

The Beginning — —	4h. 15m. 30s.	P. M.
The End — — —	6 27 20	

† The End of this Eclipse was observed in *Surry-street*, *London*, at 9 h. 1 m. 45 s. Ap. time, by Mr. SHORT.

A COMET

Was observed this year at *Bologna* by EUST. ZANOTTI, and from his observations he determines its Elements thus for a Parabolic Orbit.

Place of the Perihelion —	♋	5°	11′
——— Descending Node	♈	25	18
Inclin. Orbit to the Ecliptic		53	25
Was in the Perihelion June 9 d. 9 h. 59 m.			
——— Desc. Node July 18 4 57			

The Comet's motion in its proper orbit was Retrograde; and its Perihelion was between the Orbits of Mercury and Venus, its distance from the Sun being ·69614 parts of the Earth's Mean Distance.

Calculated by	Beg.	Mid.	End	Dur.	Dig.
Mr. Leadbetter, London	8 10	8 48	9 30	1 19	2 10
Mr. Robinson, Newcastle	7 42	8 9	9 50	1 7	2 7
Mr. Cooper, Wellingborrow	7 59	8 38	9 18	1 19	2 15
Mr. W. Brown, Cleobury	7 56	8 32	9 11	1 15	2 4
Mr. T. Glaspool, Winchester	8 7	8 46	9 27	1 20	2 9
Norwich	8 12	8 52	9 38	1 21	2 9
Mr. Rob. Cooke, Newcastle	8 18	8 48	9 38	1 19	2 18
Mr. T. G. Forster, Branspath	8 8	8 47	9 28	1 20	3 26
Mr. Ed. Dale, Sunderland	8 8	8 47	9 28	1 21	3 26
Mr. W. Schoolcroft, Vienna	8 41	9 24	10 11	1 29	2 30
Mr. J. Benwell, Highworth					
Mr. T. Williams	8 4	8 44	9 24	1 20	2 18
Mr. Cha. Facer	8 2	8 39	9 17	1 14	2 3
Mr. T. Sparrow, Edmondsbury	8 11	8 50	9 32	1 21	2 7
Mr. Tho. Wade, Leicester	7 36	8 20	8 35	1 4	1 50

New Questions.

I. QUESTION 205, *by Mr.* John Turner.

Where Derwent's streams with gentle murm'rings glide,
Kissing the flow'ry banks on either side,
A lovely meadow lies, whose fertile soil
Amply rewards the painful lab'rer's toil.
Fenc'd with tall shady trees, a cool retreat
From the meridian sun's intenser heat.
Here, to relax my weary mind, I stray'd,
And, as I walk'd, these observations made:
The form of the abovesaid piece of ground,
A triangle obtuse, I quickly found:
The lengths of whose two shorter sides I knew,
AC fourteen, *CB* chains twenty-two.
From *C*, a drain was carried on to *D*,
A circle's arch exact, (as in the figure see)
Its center *A*, the radius is *AC*.
Lastly, the analyst I must inform
That the third side *AB*, as yet unknown,
Was to the arch *CE* as ten to four;
From whence the unknown side he may explore.

See the Fig. to the Solution.

II. QUESTION 206, *by* Merones.

If a cannon ball be projected upwards in a direction perpendicular to the horizon, half a mile high, and in the latitude of 53 degrees; where will it fall?

III. QUESTION 207, *by Mr.* John May, *jun.*

There came three Dutchmen of my acquaintance to see me, being lately married; they brought their wives with them. The men's names were Hendrick, Claas, and Cornelius; the womens, Geertruii, Catriin, and Anna: but I forgot the name of each man's wife. They told me they had been at market to buy hogs; each person bought as many hogs as they gave shillings for each hog; Hendrick bought 23 hogs more than Catriin, and Claas bought 11 more than Geertruii; likewise, each man laid out 3 guineas more than his wife. I desire to know the name of each man's wife.

IV. QUESTION 208, *by Mr.* Hen. Travis.

Given this equation, viz. $Axy + Bu + z - C = 0$; expressing the relation of the sides of a trapezium inscribed in a circle whose diameter is known to be 75 feet (or d): Required the sides separately, and area, by a general method that will resolve all such problems?

N. B. $A = 100$; $B = 5$; $C = 432246$.

V. QUESTION 209, *by Mr.* Robert Heath.

Ingenious ladies of the British isle,
Whose minds are fraught with scientific arts;
Renown'd for virtue, wit, and excellence;
The admiration of the learned world:
Whose bright reflections, swift, like lightning, pierce
The secret pow'r, and hidden depth of things:
Disclose to light * this dark mysterious truth,
And distant nations shall your praise resound.

* $\overline{x^{\frac{1}{3}}}|^{x^{\frac{2}{3}}}$ *a minimum. Quere* x^2.

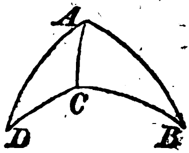
VI. QUESTION 210, *by Mr.* Rich. Dunthorne.

Suppose a cask in form of the middle frustum of an hyperbolic spindle, whose length is 24 inches, bung diameter 30, head diameter 20, and transverse axis of the generating hyperbola 100 inches. Required its content in ale gallons?

VII. QUES-

VII. QUESTION 211 *by Mr.* Ri. Lovatt.

Suppose that in the spherical triangle ABD, there is given $AB = 80° 3'$, $AD = 60° 10'$, $AC = 40° 21'$, the angle $BAD = 73° 0'$, and $DCA = BCA$: What are the sides DC and BC.



VIII. QUESTION 212, *by Mr.* Chr. Mason.

There is a triangular piece of ground, whose center of gravity measures from each angle 12, 16, and 20 chains: It is required to find the periphery of the greatest inscribed ellipsis; and also the content of each angular piece without the ellipsis?

The PRIZE QUESTION, *by Mr.* R. Heath.

Given the latitude of three places, Moscow 55° 30', Vienna 48° 12', Gibraltar 35° 30', all lying directly in the same arc of a great circle: The difference of longitude between Vienna (situated in the middle) and Moscow, easterly, is equal to that between Vienna and Gibraltar, westerly: It is required to find the true bearing and distance of each place from the other, and the difference of longitude, according to the convexity of the globe?

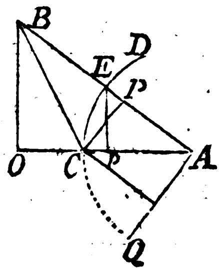
Questions

1740.

Questions answered.

I. QUESTION 205 *answered by the Proposer*, J. Turner.

LET $AB = x$, $AC = 14 = b$, $CB = 22 = c$, $cc - bb = 288 = m$. Then the cosine of the angle $BAC = \frac{xx - m}{2x}$: but the arch $CE = \cdot 4x = nx$; and the radius $= b$; consequently the cosine of the said arch $= b - \frac{nnxx}{2b} + \frac{n^4x^4}{24b^3} - \frac{n^6x^6}{720b^6}$ &c. Therefore $\frac{xx - m}{2x} = b - pxx + qx^4$, &c. From which equation by converging series, the value of x comes out $= 28\cdot 9443$: And the angle $BAC = 47$ degrees 17 minutes.



Merones *answers this Question thus*:

Let the radius $AC = r$, $CB = s$; take an arch p, as near CE as possible, let $a =$ its sine, $b =$ its cosine to the radius r; and let $p + z = CE$: and per quest. $\frac{5}{2}p + \frac{5}{2}z = AB$; also the cosine $AP = b - \frac{az}{r} - \frac{bz^2}{2rr} + \frac{az^3}{6r^3} +$ &c. And $BP - AP = \frac{5}{2}p + \frac{5}{2}z - 2b + \frac{2a}{r}z + \frac{b}{rr}z^2 - \frac{az^3}{3r^3} -$ &c. different segments of the base. But by Ax. 4th of plain trigonometry, $\frac{5}{2}p + \frac{5}{2}z : s + r :: s - r :$ different segments. Whence we have

$$\left.\begin{matrix} +\frac{5ap}{r} \\ +\frac{25p}{2} \\ -5b \end{matrix}\right\} z + \left.\begin{matrix} +\frac{5bp}{2rr} \\ +\frac{5a}{r} \\ +\frac{25}{4} \end{matrix}\right\} z^2 \left.\begin{matrix} -\frac{5ap}{6r^3} \\ +\frac{5b}{2rr} \end{matrix}\right\} z^3 = \left.\begin{matrix} +ss \\ -rr \\ +5bp \\ -\frac{25}{4}pp \end{matrix}\right\} = R.$$

Now

Now assume $p = 11\frac{1}{2}$: and then $\frac{180p}{r \times 3.141592}$ = the degrees of the arch P; from whence is had $a = 10.249675$; $b = 9.536464$; all which substituted in the foregoing series, and putting A, B, C, &c. for the known coefficients, and reversing the series, we shall have $z = \frac{R}{A} - \frac{BR^2}{A^3} + \frac{2BC - AC}{A^5}R^3$, &c. $= 0.704043$, and the arch $CE = 11.5704043$; and $AB = 28.926011$.

Mr. Robert Heath

Says, by a table of natural sines and a few trials, I find the angle $CAB = 47° 21'$; whence the required side $AB = 28.925$, &c. The method of solving this by infinite series, which converges so slow, renders it more tedious than useful.

Answered by Mr. Hen. Travis.

Let $CB = b$, $CA = a$, $AP = y$; and as in Simpson's Flux. p. 121, we have the arch $QC = y + \frac{y^3}{2.3a^2} + \frac{3y^5}{2.4.5a^4} + \frac{3.5y^7}{2.4.6.7a^6}$ &c. and $\frac{2a \times 3.1416}{4}$ = the side QE 21.991 $= q$; from which take QC, leaves $q - y - \frac{y^3}{2.3a^2} - \frac{3y^5}{2.4.5a^4} - \frac{3.5y^7}{2.4.6.7a^6}$ &c. $= CE$; which multiplied by $\frac{5}{2}$ gives $\frac{5q}{2} - \frac{5y}{2} - \frac{5y^3}{2.2.3a^2} - \frac{3y^5}{2.4a^4}$ &c. $= AB$, and per Eucl. 1. 47, $aa - yy = CP^2$; also $bb - aa + yy = BP^2$, or $288 + yy = BP^2$: Call $288 = rr$; then $PB = \sqrt{rr + yy}$, or $r + \frac{yy}{2r} - \frac{y^4}{8r^3} + \frac{y^6}{16r^5}$ &c. $= PB$; to which add y, gives $r + y + \frac{y^2}{2r} - \frac{y^4}{8r^3} + \frac{y^6}{16r^5}$ &c. $= AB$; hence $r + y + \frac{y^2}{2r} - \frac{y^4}{8r^3} + \frac{y^6}{16r^5}$ &c. $= \frac{5q}{2} - \frac{5y}{2} - \frac{5y^3}{2.2.3a^2} - \frac{3y^5}{2.4a^4}$; therefore $y + \frac{5y}{2} + \frac{y^2}{2r} + \frac{5y^3}{2.2.3a^2} - \frac{y^4}{8r^3} + \frac{3y^5}{2.4a^4}$ &c. $= \frac{5q}{2} - r$; multiplying by 2, and dividing by 7, gives $\frac{5q - 2r}{7} = z$;

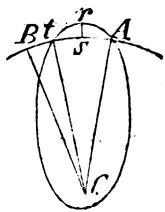
$= z$; $\because y = z - \frac{zz}{7r} + \frac{2z^3}{49r^2} - \frac{5z^3}{7\times 2a^3} + \frac{5\times 5z^4}{7\times 7\times 2ra^3} - \frac{5z^3}{7\times 7\times 7r^3} + \frac{z^4}{7\times 4r^3}$ &c. $\because \bar{y} = 9{\cdot}46$ and $AB = 28{\cdot}94$.

Mr. Nich. Farrer's *Answer.*

Let fall the perpendicular BO, on AC produced, alſo Ep from the point E; then let $r = AE = 14$, $m = BC = 22$; and put $x =$ the arch of a circle, whoſe radius is unity, ſimilar to the arch EC; then $rx =$ arch EC per ſimilar figures; and its ſine $= r\times x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$ &c. $= Ep$; and $\frac{5rx}{2} = AB$ per queſtion, and $\frac{25rxx}{8} - \frac{mm+rr}{2r} = OC$ per 12 Eucl. 2. ſubſtitute $a = \frac{25r}{8}$, and $b = \frac{mm+rr}{2r}$; then $axx - b = OC$; and per 47 Eucl. 1, $\sqrt{mm - aax^4 + 2baxx - bb} = BO$; and per 4 Eucl. 6, $AE : Ep :: AB : BO$, i. e. $r : r\times x - \frac{x^3}{6} + \frac{x^5}{120}$ &c. $:: \frac{5rx}{2} : \sqrt{mm - aax^4 + 2baxx - bb}$; therefore $\frac{2}{5rx}\sqrt{mm - aax^4 + 2baxx - bb} = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$ &c. and the value of $x = {\cdot}82646$; and $AB = 28{\cdot}9211$ chains required in anſwer.

II. QUESTION 206 *anſwered by* Merones.

Let the ball be projected from A, the time of its flight will be $25\frac{1}{4}$ ſeconds; in which time the point A will be carried to B, through a ſpace AB, of 23666 feet, by the earth's rotation. Now the ball (carried by a compound motion of its projection and the earth's rotation) will deſcribe an ellipſis whoſe focus is in the center of the earth; in which the elliptic area $ArtC =$ the circular ſector ABC: Or the area $ArtA =$ area BCt; but by reaſon of the ſmall ratio of rs to AC the portion Art may be taken for a parabola. Let $AC = b = 21000000$ feet, $AB = d$



$= d = 23666$ feet, $rs = b = 2640$, $a = Bt$; then will $\frac{ba}{2} = \frac{2}{3}b \times \overline{d - a}$; and $3ba + 4ha = 4hd$; whence $a = \frac{4hd}{3b + 4h} = \frac{4hd}{3b}$ nearly $= 3.967$ feet. Near 4 feet to the west.

Mr. Hen. Travis's *Answer.*

The time of the ball's ascending is equal to the time of its descending, according to the writers on projectiles; which time call (x) and the number of feet a heavy body will fall or descend freely, by the force of its own gravity, in one second of time $= n$. Then will $nxx = 2640 =$ the feet in half a mile; $\therefore x^2 = \frac{2640}{n} = \frac{2640}{16.1} = 163.98$ nearly, $\therefore x = 12' \; 48'' =$ the time the ball is ascending or descending; and consequently the ball will fall near 4 feet from the place it was projected.

Mr. *Bird* says the time of the projectile was 65.32 seconds.

III. QUESTION 207 *answered by Mr.* J. Hill.

Call the number of hogs any woman bought x; the number her husband bought $x + n$; money laid out by the woman is xx shillings; money laid out by the husband is $xx + 2nx + nn$ shillings. Equation $xx + 2nx + nn = xx + 63$. $\therefore x = \frac{63 - n}{2n}$. If $n = 1$, then $x = 31$, and $x + n = 32$; hence some woman bought 31 hogs, and her husband 32: If $n = 3$, then $x = 9$, and $x + n = 12$; therefore some other woman bought 9, and her husband 12: If $n = 7$, then $n + x = 8$; $\therefore$ some woman bought 1, and her husband 8. Consequently

Hendrick	bought	32,	and his wife	Anna	31
Claas	— —	12	— — —	Catriin	9
Cornelius	—	8	— — —	Geertruii	1

Answered by Merones.

	Men.			Women.		
For the persons put	A,	B,	C,	P,	Q,	R,
Hogs	a,	e,	y,	$e - c$,	$a - b$,	u,
Money	aa,	ee,	yy,	$\overline{e - c}^2$,	$\overline{a - b}^2$,	uu.

Let $b = 23$, $c = 11$. Compare B with Q, then per question $ee - \overline{a - b}^2 = 63$ shillings; that is, putting $e = a + z$; $2az + zz$

$+zz+46a=592$; therefore $a=\frac{23-z}{2}+\frac{63}{2z+46}$; now 'tis evident the last term cannot be a whole number; therefore z in the first term must be an even number, so the last term $\frac{63}{2z+46}$ must be the half of a whole number; let $\frac{63}{z+23}=v$. Whence $z=\frac{63}{v}-23$; hence v must be either 1, 3, 7, 9, 21, or 63: From each of which is had $\left.\begin{matrix} a\ 54, 32, 14, 22, 24 \\ e\ 32, 12, 12, 8, 8 \end{matrix}\right\}$. And again comparing C with P, then $yy-ee+22e=184$; and we find $\left\{\begin{matrix} y\ 12, 8, 8, 12, 52. \\ e\ 2, 10, 12, 20, 42. \end{matrix}\right.$

Whence e must be the same in both suppositions; therefore 'tis 12, if the question be possible in whole numbers. But since the other two persons A, R, must be compared, therefore $aa-uu=63$: From hence $a=32$, $u=31$, $e=12$, and $y=8$; but comparing the men and women in any other manner, it will appear there is no other answer in whole numbers. Therefore Hendrick and Anna, Claas and Catriin, and Cornelius and Geertruii, are man and wife.

The same answered by Mr. Rob. Heath.

Let $x=$ the hogs bought by either Hendrick, Claas, or Cornelius; then xx will be the shillings they cost, and $xx-63$ the shillings their wives hogs cost, which (as whole hogs) must always be a square number; because the square root of the shillings laid out for each parcel is equal to the number of hogs. Let $x-y=$ the side of that square, then $xx-63=xx-2xy+yy$. Consequently, by reduction, $x=\frac{63+yy}{2y}$; whence we find y may be

$$\left.\begin{matrix}1\\3\\7\end{matrix}\right\}\ \text{Conseq. } x=\left\{\begin{matrix}32\\12\\8\end{matrix}\right\}\ \begin{matrix}\text{Hogs. bought by the}\\ \text{men, coupled}\\ \text{with}\end{matrix}\ \left\{\begin{matrix}31\\9\\1\end{matrix}\right\}\ \text{bought by their}$$

Wives. Whence are joined Hendrick and Anna, Claas and Catriin, Cornelius and Geertruii.

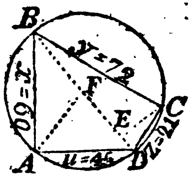
Mr. N. Farrer

Observes, that the number of hogs the three men and their respective wives bought will be expressed by three pair of numbers, the difference of whose squares must be 63. Now

 all

all the whole numbers whose squares will produce this difference are 1 and 8, 9 and 12, 31 and 32; therefore 8, 12, 32, the men bought; 1, 9, 31 the women.

IV. QUESTION 208 answered by the Proposer.

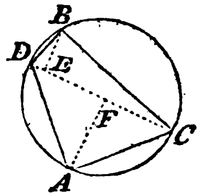


$d : y :: z : \frac{zy}{d} = CE$. Hence

$\frac{yy - zzyy}{dd}$, or $\frac{\sqrt{ddyy - zzyy}}{dd} = BE$,

$\frac{zz - zzyy}{dd}$, or $\sqrt{ddzz - zzyy} = ED$,

$\frac{\sqrt{ddzz - zzyy} + \sqrt{ddyy - zzyy}}{d} = BD$,

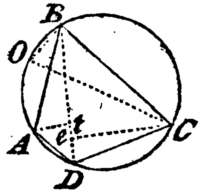


$\sqrt{ddxx - uuxx} + \sqrt{dduu - xxuu} = BD$.

$Axy + Bu + z = C$, per quest.

$z = C - Axy - Bu$; which substit. for z.

$\sqrt{ddxx - uuxx} + \sqrt{dduu - xxuu} = \overline{C - Axy - Bu} \times \sqrt{dd - yy} + \sqrt{ddyy - yy \times \overline{C - Axy - Bu}^2}$.



Here we have one equation including three unknown quantities, and yet the question is truly limited; and to be resolved as the following question is.

Given $2yx + \frac{2592}{uy} + 4ux = 48x - x^2$.

Quere x, y, and u? In fluxions,

$2y\dot{x} + 4u\dot{x} = 48\dot{x} - 2x\dot{x}$,

$2\dot{y}x - \frac{2592u\dot{y}}{uuyy} = 0$, and $-\frac{2592\dot{u}y}{uuyy} + 4\dot{u}x = 0$.

From the first, second, and third steps, by common algebra we get $x = 12$, $y = 6$, and $u = 3$; and by the very same method of reasoning, the sides of the trapezium are found to be $x = 60$, $y = 72$, $u = 45$, and $z = 21$; and the area $= 2106$ square feet. This trapezium may be placed 4 or 5 different ways in a circle, which I have proved by a large geometrical projection, and every way justly contain the remaining chords of the circle, and measured, amount to each the same area 2106 square feet. (1) In the first fig. the several lines are obvious; and the diagonal $BD = 75$, multiplied by the half of the two perpendiculars ($CE = 19.65$, $AF = 36.52$) will give

give the area as above. (2) In the second fig. $BC = y = 72$, $BD = z = 21$, $DA = x = 60$, and $AC = u = 45$. And the diagonal DC ($= 74 \cdot 96$) multiplied by half the perpendiculars ($AF = 35 \cdot 75$, and $BE = 20 \cdot 2$) gives nearly the same area. (3) In the third fig. y is the line CB, x is BA, z is AD, and u is x. The diagonal $BD = 70 \cdot 01$, the perpendicular $At = 16 \cdot 6$, $Ce = 43 \cdot 1$ nearly; and (4) in the same fig. z is represented by BO, u by OA, and x by AC; the diagonal OC is nearly $74 \cdot 95$, which by the perpendiculars give the area as before.

Mr. *Paul Sharp* has found the sides 72, 60, 45, and 21, in answer.

Mr. *Tho. Robinson* gives the sides 72, 2, 59, 9, 45, 1, 20, 7, nearly true. In this question it does seem to appear, that the number of quantities sought exceed the number of given equations, and (as my ingenious correspondents have observed) is unlimited. But I presume since the numbers given in the question, viz. $A = 100$, $B = 5$, and $C = 432246$; and the four numbers sought are together obliged to extend the chords of 360 degrees, and the diameter of the circle is given; it may be said to be limited; but I shall rather leave it to the speculation of those ingenious persons who are pleased to appear in the emendata next year.

V. QUESTION 209 *answered by* Merones.

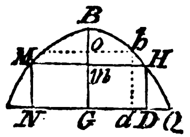
$\overline{x^{\frac{1}{3}}}|^{x^{\frac{2}{3}}}$ = a minimum. Therefore $x^{\frac{2}{3}} \times \log. x^{\frac{1}{3}}$ minimum; whence $\frac{2}{3}x^{-\frac{1}{3}}\dot{x} \times \log. x^{\frac{1}{3}} + \frac{1}{3}x^{-\frac{1}{3}}\dot{x} = 0$; or $2 \log. x^{\frac{1}{3}} + 1 = 0$. And therefore hyp. log. $x^{\frac{1}{3}} = -\frac{1}{2}$. And tab. log. of $x^{\frac{1}{3}} = -\frac{1}{2} \times \cdot 434294 = -\bar{1} \cdot 782853$. Therefore $x^{\frac{1}{3}} = \cdot 60653$, and $x = \cdot 22313$.

The Proposer, Mr. Heath, *answers thus.*

Its log. is a minimum. Let $y^3 = x$, the expression will become y^{y^2}; its log. $= \text{l}.y \times yy$, fluxed, $\dot{y}y + \text{l}.y \times 2y\dot{y} = 0$. Reduced, $\text{l}.y = -\frac{1}{2}$, consequently $\text{l}.y^3 = \text{l}.x = -\frac{3}{2}$; Whence the natural number corresponding thereto is $= \cdot 223130$, &c. and accurately, which is the value of x required.

N. B. This problem shews the difuse of Mr. Simpson's series, (at p. 165 of his Fluxions) for finding the number answering to any hyperbolical logarithm: For instead of converging, it diverges in many cases; as if $\overline{x^{\frac{1}{6}}}\big|^{x^{\frac{1}{6}}}$ were a minimum, $l.x = -6$; and $x = \cdot 00247875$, &c. Here it diverges very swift; but converges very slow in the former case as to be useless.

VI. QUESTION 210 *answered by Mr.* Rob. Heath.

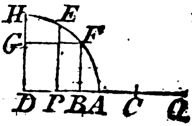


In the hyperbola, there is given the transverse axis in the figure $= 100$ inches $= p$, $BG = 15 = a$, $HD = mG = 10$, $Bm = 5 = b$, $GD = mH = 12 = c$; let $x = Bo$, and $y = oh$ required. By the property of the curve, $\overline{p+b} \times b : cc :: \overline{p+x} \times x : yy$; $\because$ $xx + px = \frac{p+b}{cc} \times byy$, and $x = \sqrt{\frac{1}{4}pp + \frac{p+b}{cc} \times byy} - \frac{1}{2}p$; $a - x = a + \frac{1}{2}p - \sqrt{\frac{1}{4}pp + \frac{p+b}{cc} \times byy} = oG = hd$. (put $f = a + \frac{1}{2}p$, $g = \frac{p+b}{cc} \times b$); it is plain the fluxion of $Gd \times$ into the area of the circle whose radius is hd, is equal the fluxion of the indefinite solid generated by the rotation of the curve (Bh) about Gd; $ff + \frac{1}{4}pp + gyy - 2f\sqrt{\frac{1}{4}pp + gyy} : \times 3\cdot 1416\dot{y}$; whose fluent is $3\cdot 1416y \times \overline{ff + \frac{1}{4}pp} + 3\cdot 1416y^3 \times \frac{g}{3}$ (let $n = 3\cdot 1416$) $- fpny - \frac{2fngy^3}{3p} + \frac{2fng^2y^5}{5p^3} - \frac{4fng^3y^7}{7p^5} + \frac{10fng^4y^9}{9p^7}$ — &c. and if c be put instead of y, in the expression, we have the solidity of half the cask $= 6754\cdot 8$ inches, true to a decimal; consequently $13509\cdot 6$ inches the whole content, or $47\cdot 893$ ale gallons.

Merones's

Merones's Answer.

Let the semi-transverse $CA = r = 50$, $CD = a = 65$, the rectangle $QPA = pp = 525$, $PE = q = 12$; $dd = aa + rr$, $c = 3{\cdot}14159$; $CB = x$, GF or $DB = a - x$, BF or $GD = y$. Then per conics, $qq : pp :: yy : xx - rr$; whence $x = \sqrt{rr + \frac{pp}{qq}yy}$; and $BD^2 = \overline{a-x}|^2$



$= dd + \frac{pp}{qq}yy - 2a\sqrt{rr + \frac{ppyy}{qq}}$: Therefore $DB^2 \times c\dot{y}$

$cdd\dot{y} + \frac{cpp}{qq}yy\dot{y} - 2ca\dot{y}\sqrt{rr + \frac{pp}{qq}yy}$ = flux. solid $AFGD$ revolving round GD; put $L = 2{\cdot}302585 \times \log. \frac{p + \sqrt{rr + pp}}{r}$; and write q for y in the fluent, and we shall have $cddq + \frac{1}{3}cppq - caq\sqrt{rr + pp} - \frac{caqrr}{p}L$ = half the cask = 6754·88 inches, and the whole = 13509¾ = 47·907 ale gallons.

Mr. *Farrer*, Mr. *Turner*, and Mr. *Travis*, have also curiously wrought this answer by a different process.

VII. QUESTION 211 *answered by Mr.* R. Heath.

Given	°	′	Sines	Cosines	
AD =	60	16	·86747 = b	·49747 = c	$\angle DCA$ =
AC =	40	21	·64745 = d	·76210 = f	ACB
AB =	80	3	·98495 = g	·17278 = h	Required DC
$\angle DAB$ =	73	0	·95630 = s	·29237 = t	and CB?

It must be noted that DC and CD are not two continued arches, for so the question would be over-limited and absurd, but sides of different triangles. Let x = cos. $\angle DAC$, $\sqrt{1 - xx}$ its sine, radius = 1. Then (per Anderson's Theor.) $bdx + cf$ = cos. DC; its sine $\sqrt{1 - \overline{bdx + cf}|^2}$. By trigonometry, s. DC : s. $\angle DAC$ :: s. DA : s. DCA = $b\sqrt{\frac{1 - xx}{1 - \overline{bdx + cf}|^2}}$: Now cos. $\angle CAB = xt + s\sqrt{1 - xx}$, its sine $xs - t\sqrt{1 - xx}$, and by the aforesaid theorem

 dgtx

$dgtx + dgs\sqrt{1-xx} + hf =$ cosine CB. Whence $\sqrt{1 - \overline{dgtx + dgs\sqrt{1-xx} + hf}^2}$ = its sine: Per trigonometry, s. CB : s. $\angle CAB$ :: s. AB : s. $\angle ACB$ =

$$\frac{xs - t\sqrt{1-xx}}{\sqrt{1 - \overline{dgtx + dgs\sqrt{1-xx} + hf}^2}} \times g = h\sqrt{\frac{1-xx}{1 - \overline{bdx + cf}^2}};$$

By reduction, $1 - \overline{bdx - cf}^2 \times \overline{xs - t\sqrt{1-xx}}^2 \times g^2 = hh \times \overline{1-xx} \times 1 - \overline{dgtx + dgs\sqrt{1-xx} + hf}^2$. In numbers, $1 - \overline{\cdot561643x + \cdot379121}^2 \times \overline{\cdot941907x - \cdot287969\sqrt{1-xx}}^2 = \cdot7525 \times \overline{1-xx} \times 1 - \overline{\cdot18647x + \cdot60993\sqrt{1-xx} : + \cdot131675}^2$. Here x [by a new method of solving equations] is found $= \cdot8770$, &c. Whence $DC = \cdot4900 = 29^\circ\ 21'$, and $BC = \cdot8086 = 53^\circ\ 58'$. *Q.E.I.*

Mr. *J. Turner* observes the scheme is false drawn, and so makes no question at all; but correcting it, and putting b = rectangle s. AD, AC; c = their cosines; d = rectangle AC, AB; f = cos. g = s. AD; h = s. AB; m = s. $\angle BAD$, n = its cos. x = cos. $\angle DAC$, $\sqrt{1-xx}$ = its sine. Then the sine of $BAC = mx - n\sqrt{1-xx}$, and its cos. $= nx + m\sqrt{1-xx}$; cos. $DC = bx + c$; cos. $BC = dnx + dm\sqrt{1-xx} + f$. Sine $DC = \sqrt{1 - cc - 2bx - bbxx}$; s. $BC = \sqrt{1 - ddnnxx - 2dnfx - ff - ddmm + ddmmxx - 2ddnmx - 2dmf\sqrt{1-xx}}$.

As $1 - cc - 2cbx - bbxx : 1 - xx :: gg : \dfrac{gg - ggxx}{1 - cc - 2bcx - bbxx}$ = square s. $\angle DCA$. Again $BC^2 : \angle BAC^2 :: BA^2 : \angle DCA^2$. Consequently $\dfrac{gg - ggxx}{1 - cc - 2bcx - bbxx} =$

$$\frac{hhmmxx - 2hhmnx\sqrt{1-xx} + hhnn - hnx}{1 - ddnnxx - 2dnfx - ff - ddmm + ddmmxx - 2dnmx\sqrt{1-xx} - 2dmf\sqrt{1-xx}};$$

which when all the terms affected with $\sqrt{1-xx}$ are brought to one side of the equation, and involved, will produce an equation of the 8th power; in which $x = \cdot87719$. Consequently the $\angle DAC = 28^\circ\ 42'$; $BAC = 44^\circ\ 18'$; side $DC = 29^\circ\ 19'$; and $BC = 53^\circ\ 59'$.

* VIII. QUESTION 212 *anſwered.*

There is no ſolution printed to this queſtion this year; but in the Emendations in the next year the Diary Author ſays the printer omitted it for want of room, and that of ſeveral who anſwered it, Mr. *Hill*'s numbers for the ſides of the triangles are 34·151, 28·521, and 19·549.

Again in the Emendations in the year 1744 he mentions it, ſaying that Mr. *Heath* had fully anſwered it at firſt; and that Mr. *James Terey* now puts the ſide ſought $= x$, from one angle to the interſection $= d$, another $= c$, the 3d $= b$; then $x = \sqrt{2dd + 2cc - bb}$: Whence he gives the ſides 34·176, 28·844, and 20; the area $= 288$. Then 1 : ·6046 :: area of the equilateral △ : area inſcr. circle. Hence area of inſcr. ellipſis $= 174·124$, and each angular piece $= 37·95$; diam. inſcr. circle 10·88 = conjugate of the ellipſe; longeſt 20·36.

The

---

* VIII. QUESTION 212.

The method of finding the ſides of the triangle may be thus: Put x, y, and z for the three ſides, and a, b, c, ($= 12$, 16, and 20) for the diſtances between the angular points and the center of gravity; then, becauſe the lines a, b, c, if produced would biſect the oppoſite ſides, and are each $\frac{2}{3}$ of the whole biſecting line, by a known theorm in geometry, we have

$$\left\{\begin{array}{l} x^2 + y^2 - \frac{1}{2}z^2 = \frac{9}{2}c^2, \\ x^2 + z^2 - \frac{1}{2}y^2 = \frac{9}{2}b^2, \\ y^2 + z^2 - \frac{1}{2}x^2 = \frac{9}{2}a^2 \end{array}\right\};$$

the ſum of theſe being taken, and cleared of fractions, we have $xx + yy + zz = 3aa + 3bb + 3cc$ [which is a very curious Theorem]; from this laſt each of the three former equations being ſubtracted, &c. we have

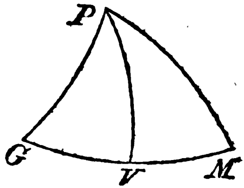
$$x = \sqrt{2bb + 2cc - aa} = 4\sqrt{73} = 34·176,$$
$$y = \sqrt{2aa + 2cc - bb} = 4\sqrt{52} = 28·8444,$$
$$z = \sqrt{2aa + 2bb - cc} = 4\sqrt{25} = 20.$$

OTHERWISE. Or the triangle might be eaſily conſtructed. For, ſince the three lines drawn to the center of gravity divide the whole triangle into three equal parts or triangles, and each triangle being equal to half the ſine of its angle at the center of gravity drawn into the product of the two ſides or lines about it, therefore the

ſines

The PRIZE QUESTION *anſwered by Mr.* J. Turner.

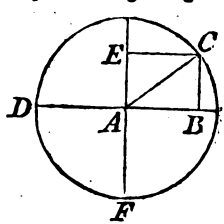
Let P repreſent the pole of the world; M, Moſcow; V, Vienna; G, Gibraltar. Put $x =$ coſ. $\angle GPV = \angle VPM$; $\sqrt{1-xx}$ = its ſine; ſine $GP = b$; of $PM = c$; s. $\angle GVP = \frac{\sqrt{1-xx}}{z}$; the rectangle of the ſines of GP, $PV = d$; rect. of coſ. $= f$; rectangle of the ſines of VP, $PM = g$; rect. coſines $= h$. By Anderſon's Theorem, $dx + f =$ coſ.



GV, and $gx + h =$ coſ. VM; as $\frac{\sqrt{1-xx}}{z} : b :: \sqrt{1-xx} : bz =$ ſine GV, and its coſ. $= \sqrt{1-bbzz}$; as $\frac{\sqrt{1-xx}}{z} : c :: \sqrt{1-xx} : cz =$ ſine VM, and its coſ. $= \sqrt{1-cczz}$.

Conſe-

ſines of the three angles formed by thoſe three given lines are reciprocally as the products of each two about them; but thoſe ſides being 12, 16, and 20, are as 3, 4, and 5; therefore the ſines of the angles are as $\frac{1}{3\times 4}$, $\frac{1}{3\times 5}$, and $\frac{1}{4\times 5}$; or as 5, 4, and 3; and ſo univerſally the ſines of the three angles are always as the three given lines. But the three angles about one point are equal to four right angles or 360°; whence the problem is to divide 360° or a circle into three ſuch parts, that their ſines ſhall obtain the given ratio. In the preſent caſe the ratios 5, 4, and 3 form a right-angled triangle; let it be ABC; then the ſupplements of the $\angle$s A, B, and C, of this triangle are the angles required to be formed by the given lines at the center of gravity. For, with the center A and radius AC deſcribe the circle; then it is evident that CB is the ſine of the arc DC or $\angle DAC$, that AB or CE is the ſine of CF or $\angle CAF$, and that AC is the ſine of the remaining quadrant FD or $\angle FAD$; and theſe are reſpectively the ſupplements of the three angles of the triangle ABC.



The other parts of this queſtion will be done as queſt. 430 propoſed in the year 1757, where the ſubject is reſumed, and to which therefore we refer.

Consequently,

$$\left.\begin{array}{l} ddxx + dfx + ff = 1 - bbzz, \\ ggxx + 2gbx + hh = 1 - cczz; \end{array}\right\} \begin{array}{l} zz = \dfrac{1 - ddxx - 2dfx - ff}{bb}, \\ zz = \dfrac{1 - ggxx - 2gbx - hh}{cc}. \end{array}$$

And therefore these two are equal to one another. But $\frac{dd}{bb} = \frac{gg}{cc}$; so the two terms wherein xx is found destroy each other. We have therefore $x = \frac{cc + bbhh - bb - ccff}{2ccdf - 2bbgh}$ = ·969343 the cosine of 14° 13′ 27″, and the difference of longitude of Gibraltar from Moscow 28° 26′ 54″.

Vienna and Gibraltar bears from Moscow S. 56° 4′ westerly.
Moscow from Vienna, north 44° 50′ easterly.
Gibraltar from Vienna, south 44° 50′ westerly.
Viennna and Moscow from Gibraltar, north 35° 16′ easterly.
Vienna is distant from Gibraltar 16° 29′ = 1146 English geom. miles; Vienna from Moscow 11° 23′ = 791 miles; Gibraltar from Moscow 27° 52′ = 1937 miles. This answer is performed by a simple equation.——The same was answered by Mr. *Rob. Heath* the proposer, and by *Merones*.

Mr. *N. Farrer*, Mr. *Rob. Robinson*, Mr. *H. Travis*, Mr. *J. Powle*, Mr. *Jos. Young*, and some others, have also curiously investigated the answer to the prize question.

Of the Eclipses in 1740.

Mr. *W. Schoolcroft* gives the Transit of Mercury over the Sun, April 21, 1740, invisible at London, but may be seen in the western parts of America. The beginning at 10 h. 22′ at night; middle 11 h. 41′; end 1 h. 1′ in the morning; duration 2 h. 39′; apparent time at London.

Venus over the Sun, May 26, 1761; apparent time at York, beginning at 2 h. 26′ morn. middle 5 h. 30′; end 8 h. 34′.*

Within

---

* A TRANSIT *of* MERCURY *over the Sun*

Was observed this year at *Cambridge*, in *New-England* by Mr. J. WINTHORP; who found the Internal Contact to be at 5 h. 1 m. *P. M.* April 21, app. time.

Within the sphere of the earth's orbit will happen six eclipses this year; three times will the moon in her wandering course interpose and hide the splendour of the sun's rays from falling on the earth or its atmosphere; and thrice will the earth in its course, so fall in a line between the sun and moon, as to hinder her receiving the light she borrows from the sun, to enlighten the earth by reflection.

1. Moon eclipsed the 2d of January, at 10 at night.

Calculated by		Beg. h. m.	Mid.	End	Dur.	Dig.
From Astron. Car. Coventry		8 34	10 28	11 18	3 48	20 51
Mr. Chattock, London		8 22	10 27	12 35	4 11	21 41
Mr. Leadbetter for London		8 30	10 25	12 20	3 50	20 29
Mr. Peachey, Mildenhall		8 9		12 5	3 56	21 0
Mr. Bamfield, Honiton		6 18	7 44	9 20	3 2	16 54
Mr. May, jun.	London	8 31	10 29	12 27	3 55	
	Paris	8 41	10 39	12 37		
	Amsterdam	8 50	10 48	12 46		
	Coventry	8 27	10 25	12 23		
Annus Amanuensis, London		8 32	10 26	11 18	3 50	20 30
Mr. Robinson,	Guisbrough	8 7	10 6	12 5	3 58	21 12
	Whitby	8 9	10 8	12 7		
Mr. W. Brown,	Cleobury	8 30	10 24	12 18		20 52
	London	8 40	10 34	12 28		
	Paris	8 49	10 43	12 37		
	Madrid	8 26	10 20	12 14		
Mr. Ab. Donn,	London	8 42	10 40	12 38	3 58	22 30
	Bideford	8 24	10 22	12 20		
	Virginia	3 42	5 40	7 38		
Friend Montague, London		8 37	10 32	12 27	3 50	21 13

Mr

The above Eclipse of the 2d of January was observed in *Fleet-street, London*, by Mr. SHORT.

Beginning about — —	8 h.	25 m.	0 s.	App. time.
Beg. Total Darkness —	9	31	10	
End of Total Darkness	11	15	20	
End of the Eclipse about	12	22	0	

			h. m.	h. m.	h. m.	h. m.	
Mr. Nic. Farrer.	Leadb. Tables	London	8 10	10 9	12 8		
		Sunderland	8 5	10 4	12 3	3 58	21 12
		Morpeth	8 4	10 3	12 2		
	Scien. Steta.	London	8 31	10 25	12 20		
		Sunderland	8 26	10 20	12 15	3 49	20 29
		Morpeth	8 24	10 19	12 13		
Mr. J. Bulman,		London	9 1	10 30	12 25		
		Edinburgh	8 50	10 19	12 14		
		Dublin	8 33	10 2	11 57	3 24	20 28
		Carlisle	8 50	10 19	12 14		
		Deptford	9 2	10 31	12 26		
Mr. T. Glaspool,		Winchest.	8 27	10 22	12 17		
		Aylsham	8 31	10 26	12 21	3 50	20 28
Mr. Jo. Taylor by Astr. Angl.		London	8 32	10 35	12 38		
		Spaith	8 28	10 31	12 34		
		Coventry	8 26	10 29	12 32	4 6	21 30
		Jerusalem	11 34	13 37	15 40		
		Liverpool	8 22	10 25	12 28		
Mr. Cockson,		Lambton, Dur	8 29	10 22	12 16	3 46	20 31
Mr. R. Hughs,		Pentrefioden	8 34	10 27	12 21	3 47	20 39
Mr. J. Wilson,		Morpeth	8 25	10 15	12 5	3 40	21 14
Mr. J. Hilton,		———	8 1[illegible]	10 9	12 2	3 47	20 30
Mr. P. Pilbrow,		———	9 [illegible]	11 5	12 54	3 54	20 30
Mr. J. Canton,		———	8 22	10 20	12 18	3 56	20 56
Mr. Sparrow,		Edmundsbury	8 32	10 27	12 22	3 50	20 27
Mr. W. Leighton,		London	8 29	10 24	12 18	3 48	20 23
		Arlington	8 39	10 33	12 27	3 48	
Mr. Ch. Facer,		Watlington	8 34	10 30	12 26	3 52	20 50
Mr. W. Schoolcroft at Hovington, Yorkshire, by		Astr. Insta.	8 17	10 23	12 28	4 10	21 37
		Astr. Carol	8 36	10 30	12 24	3 47	20 51
		Sci. Stella.	8 27	10 22	12 16	3 49	20 28
		Flamst. T.	8 28	10 25	12 23	3 55	21 5
		Leadbett.	8 22	10 22	12 21	3 58	21 10
Mr. J. B. Smith,		Oxford	8 26	10 24	12 22	3 56	
M. Jo. Benwell,		Highworth	8 21	10 17	12 13	3 51	20 50
Mr. Cooper,		Wellingborrow	8 24	10 21	12 18	3 54	20 37

The second eclipse is of the sun, Jan. 17, at 8 at night, invisible to us.

The third eclipse is of the sun, June 13, at 2 in the morning, invisible.

The fourth eclipse is of the moon, June 28, at 9 in the morning, invisible.

The fifth eclipse is of the sun, Decemb. 7, at 11 at night, and invisible.

The

The ſixth eclipſe is of the moon, at 11 at night.

Calculated by		Beg. h. m.	Mid.	End	Dur.	Dig.
Aſtron. Carolina,	Coventry	10 22	11 36	12 50	2 29	5 4[illegible]
Mr. Chattock,	London	10 21	11 49	1 17	2 56	7 17
Mr. Leadbetter,	London	10 32	11 48	1 4	2 32	5 49
Mr. J. Peachey,	Mildenhall	10 16	11 36	12 56	2 40	6 0
Mr. S. Bamfield,	Honiton	9 43	11 5	12 27	2 44	6 32
Mr. J. May,	London	10 25	11 53	1 21	2 56	6 33
	Amſterdam	10 44	12 12	1 40		
	Peterſburg.	12 26	1 54	3 22		
Annus Amanuen.	London	10 34	11 48	1 7	2 36	5 46
Mr. Robinſon,	Guiſbrough	9 59	11 20	12 41	2 42	6 27
Mr. W. Brown,	Cleobury	10 17	11 32	12 46		
	Dublin	9 59	11 14	12 28		5 43
Mr. Ab. Donn,	Biddeford	10 25	11 44	1 3	2 38	6 15
	Virginia	5 43	7 2	8 21		
Friend Montague,	London	10 36	11 59	1 23	2 47	7 5
Mr. N. Farrer,	London	10 33	11 49	1 5	2 32	5 49
	Sunderland	10 28	11 44	1 0		
	Morpeth	10 27	11 43	12 59		
Mr. J. Bulman,	London	10 30	11 51	1 8	2 38	5 51
	Carliſle	10 19	11 40	12 57		
	Deptford	10 31	11 52	1 9		
Mr. Glaſpool,	Wincheſter	10 31	11 18	1 5	2 34	5 18
	Nottingham	10 36	11 52	1 7	2 31	
	Bungay	10 34	11 51	1 7	2 33	
Mr. Jo. Taylor,	London	10 18	11 43	1 9	2 51	7 9
	Snaith	10 14	11 39	1 5		
	Liverpool	10 8	11 33	12 59		
Mr. C. Cockſon,	Lambton	10 26	11 42	12 59	2 32	5 48
Mr R. Hughs,	Pentrefioden	10 30	11 44	1 5	2 25	
Mr. J. Wilſon,	———	10 27	11 42	12 57	2 33	5 54
Mr. J. Hilton,	———	10 16	11 30	12 45	2 29	5 43
Mr. J. Canton,	London	10 32	11 5[illegible]	1 12	2 40	6 19
	Stroud	10 23	11 43	1 3		
Mr. Sparrow,	Edmundſbury	10 34	11 50	1 6	2 32	5 48
Mr. Cha. Facer,	Watlington	10 21	11 36	12 51	2 30	5 43
Mr. Schoolcroft,	York	10 23	11 44	1 5	2 41	6 24
	London	10 27	11 48	1 9		
	Jeruſalem	12 49	2 10	3 31		
Mr. J. B. Smith,	Oxon	10 27	11 46	1 5	2 38	6 2
Mr. J. Benwell,	Highworth	10 26	11 42	12 58	2 32	5 44
Mr. T. Cooper,	Wellingbor.	10 14	11 34	12 54	2 39	6 13

New

New Questions.

I. QUESTION 213, *by Mr.* Rob. Heath.

A miser thus, fair ladies, makes request;
What pounds are those, at compound interest,
He must, for time, on these conditions lend,
To gain an equal value in the end?
Square root of years, square root of pounds per cent,
Must equal square root of the money lent:
To make it clear, the square root of each three,
Compar'd with each, must equally agree;
Time, rate per cent, and principal unfold,
And wed him, fair one, for his bags of gold.

II. QUESTION 214, *by Mr.* Nich. Farrer.

Sometime in the spring quarter, in 1739, in the forenoon, an observation being made of the sun, his altitude was found 33° 41′ 40″; and azimuth from the north 102° 40′ 52″; and sometime after, on the same forenoon, his altitude was found 48° 46′ 53″, and azimuth 134° 39′ 56″. From whence the latitude of the place of observation, month, day, and hours of observation may be found, and are here required? With a general theorem for all questions of this nature.

III. QUESTION 215, *by Mr.* J. May.

Going to pass a leisure hour at billiards, I wondered to find the tables an irregular hexagon, when seeing the balls fly very strangely in striking the several gins, made me think, If two balls, *A* and *B*, lay on the said table, and the ball *A* was struck against the gin *RS*, from thence reversing to *ST*, from thence to *TV*, then to *VW*, then to *WX*, thence to *XR*, thence reversed, and struck the ball *B*; to find geometrically the points in the several gins, where the ball *A* will strike; and that by a general construction for all polygons, supposing the balls to be geometrical points?

IV. QUESTION 216, *by Mr.* Henry Travis.

At Matlock, near the Peak in Derbyshire, where are many surprizing curiosities in nature, is a rock by the side of the river Derwent, rising perpendicularly to a wonderful height;

which, being inacceſſible, I endeavoured to meaſure in a mathematical method. From a ſtation at ſome diſtance, (nearly level with the bottom of the rock) I took an angle of altitude to its top 47° 30′; and having deſigned a ſecond ſtation, I took an horizontal angle 87° 5′, between the foot of the rock and that ſtation; the meaſured diſtance between the ſtations was 4 chains and 29 links, (per Gunter) or 283·274 feet. At that place I had an angle of altitude 40° 12′, but forgot from hence to take an angle between my firſt ſtation and the foot of the rock; yet am in hopes ſome curious artiſt will, from this data, determine the perpendicular height of this ſtupendous rock.

V. QUESTION 217, *by Mr.* Ant. Thacker.

Given the * equation of the exponential curve, *MDSEB*, together with the axis $AB = b = 1000$; to find the greateſt ordinate (*SR*) and inſcribed parallelogram *DEQP*, and to give the analytical inveſtigation of the ſame?

* $PB^{AP} = PD^{PD}$; i. e. $\overline{b-x}|^{x} = y^{y}$.

VI. QUESTION 218, *by Mr.* Rich. Gibbons.

I will undertake, with 12 fair dice, to throw 42 once in 15 times; and between 37 and 47, at every throw. Quere, Whether I ſhall be a gainer or loſer by theſe chances, and the exact odds?

The PRIZE QUESTION, *by Mr.* Rob. Heath.

Near Twickenham's banks, the muſes ſeat, where Thames
Rolls thro' the valley his ſmooth clearer ſtreams,
A fabric does in peaceful order riſe,
Whoſe owner's virtues reach the lofty ſkies!
Secure of fame, he ſlights all c—rt renown
For Maro's glory, an immortal crown.
His generous fancy, free and unconfin'd,
Well ſuits the buſineſs of a noble mind;
Beholding flatt'ry with a pitying eye,
And, than be guilty, ſooner chuſe to die!
Wrapp'd in himſelf, he can his thoughts approve,
Of truth, of juſtice, poetry, or love;
Can, meditating on life's various ſcene,
See folly's rocks, and ſeas ingulph'd between;
And ſmoothly gliding down amuſements ſtream,
Make gardens, ſhady bowers, or grots, his theme.

Or,

Or, from aloft, tall ſpires, domes, waving woods,
Re-echoing hills, fair fields, and chryſtal floods;
Hear the wing'd choir in warbling conſort ſing,
The ſweet-tun'd praiſes of their heavenly king;
See ſwans below, boats, beauteous nymphs, and men,
All moving on ſerenely, and agen.
Who'd not refuſe the gaudy pomp of ſtate,
To live ſo bleſs'd, ſo nobly, good, and great.
T' enrich the proſpect, let it be ſuppos'd,
A park is purchas'd, thus to be inclos'd;
Two ſpreading trees, on Thames ſtreight other ſide,
(Three furlongs diſtance) ſhade the ſilver tide;
And from the muſe's ſeat do equally divide;
From whence a fence of pailing muſt ſurround,
(In length a mile) the yet unfaſhion'd ground;
On this condition carried from each tree,
To make the park the biggeſt that can be.
Again, ſuppoſe a line drawn from each tree,
To the contrary fartheſt boundary;
Theſe, and the fence, to touch two * circling ſhades
On right and left, each ſhelt'ring as it ſpreads:
Hemm'd with a range of trees, to ſcreen the deer,
The middle ſpace wide op'ning to the year.
Ingenious ladies, you are deſired to ſhew,
The park's true form, content, ſhades, area too.
Apollo thus — ſweet ladies, when you've done,
Bring all your harps, and taſte the veniſon.

* *Circular encloſures touched by oppoſite ſides of the park, and interſections of the longeſt lines drawn from each tree a-croſs the park.*

1741.

Queſtions anſwered.

I. QUESTION 213, *by Mr.* J. Turner.

LET $x =$ principal, rate, and time; as $100 : x :: 1 : {\cdot}01x$. And putting $b = {\cdot}01$; what Ward in his Comp. Intereſt calls R, will be $= 1 + bx$; conſequently by his 1ſt prop, and per queſt. $x \times \overline{1+bx}|^{x} = 2x$; or $\overline{1+bx}|^{x} = 2$. Hence log. $\overline{1+bx} \times x =$ log. 2, $= {\cdot}693147$ (or c). But the log. of $\overline{1+bx}$ is $= bx - \frac{b^2x^2}{2} + \frac{b^3x^3}{3} - \frac{b^4x^4}{4} + \frac{b^5x^5}{5}$ &c.

 which

which multiplied by x is $= c$. Reverted, it gives $x = 8{\cdot}4983$ the time and rate, with which any sum will gain the principal.

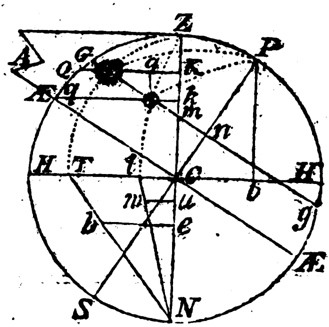
Answered by Mr. Peter Kay.

Let x be the number sought; therefore $\overline{1+\frac{x}{100}}\Big|^{x} = 2$, per quest. this in log. gives $x \times \frac{x}{100} - \frac{x^2}{20000} + \frac{x^3}{3000000}$ &c. $= {\cdot}693147$, whence $x = 8{\cdot}4983$ &c.

After the same manner Mr *Dunthorne*, *Merones*, Mr *Powle* have curiously answered this quest. Also Mr *May*, Mr *Brown*, Mr *Sharp*, Mr *Clifford*, Mr *Story*, Mr *Gibbons*, &c.

II. QUESTION 214 *answered by Mr.* Ant. Thacker.

CONSTRUCTION. With radius = sine of 90° describe the primitive circle on the plane of the meridian *HZHN*; draw *HH* thro' the center for the horizon, and *ZN* the east azimuth at right angles thereto; with the versed sines of 102° 40′ 52″ and 134° 39′ 56″, set off *Ht* and *HT*: And with the sines of 33° 41′ 40″ and 48° 46′ 53″ lay down *Ck* and *CK*; then draw *kq* and *KQ* parallel to *HH*: make *Nu* and *Ne* = *kq* and *KQ*; join *TN* and *tN*; and draw *wu* and *be* parallel to *HH*; then make *k*☉ and *K*☼ = *wu* and *be*; through ☼☉ draw *G*☼☉*mng*, and draw the equator *ÆCÆ* parallel thereto; and *Cnp* the elevation of the pole at right angles to it. From whence *Cn* will be the sine of the sun's declination, and *Pb* the sine of the latitude, &c.



1. From which projection, the investigation of a general theorem will easily arise. For per similar triangles, say as NC (radius = 1) : Ct (z) :: Nu (q) : $qz = uw = k$☉. And NC (1) : CT (m) :: Ne (s) : $ms = be = K$☼; ∴ $ms - qz =$ ☼a. Also as ☉a $(s-p)$: ☼a $(ms - qz)$ :: CZ (1) : $AZ = \frac{ms - qz}{s - p}$ = the tang. of the required latitude.

2. For

2. *For the Sun's Declination.* As Pb (x) : Cb (y) :: ☉k (qz) : $km = \frac{qzy}{x}$; and as mC $(p - \frac{qzy}{x})$: Cn (d) :: CP (1) : Pb (x); therefore $px - qyz = d$, the sun's declination.

3. *Lastly, for the hour of the day.* Put $c =$ cosine of sun's declination; and s and e for the sine and cosine of the hour from noon; then will $px - qyz = d$; and $dx + cye = p$;* and by substitution $xxp - xqzy + cye = p$; but $1 - yy = xx$; ∵ $ce = py + xqz$; and as $s : q :: n$ (sine of ☉ azimuth) : c; ∵ $\frac{qn}{s} = c$; which substituted for c, gives $\frac{qne}{s} = py + xqz$; therefore $\frac{e}{s} = \frac{py + xqz}{qn} =$ cotangent of the hour from noon, at the first observation.

* N. B. If the sun's azimuth is less than 90° (from the north) then tC must be taken on the contrary side of C, and therefore negative with respect to what it is (in this quest.)

Hence $\frac{qz - ms}{s - p}$; $px + qyz = d$; and $\frac{s}{e} = \frac{py - xqz}{qn}$ are general expressions for the quest. as above. Only whenever any angle or side exceeds 90°, its cosine must be expressed by a negative sine.

This theorem $px + qyz = d$, will be found to answer all the ends proposed by that of Mr. Anderson, in the Diary, 1732. And is indeed more simple, and is better adapted to the uses than his; tho' I freely own, that the above method of deriving it, was first hinted to me some years ago by my ingenious friend Mr. *T. Simpson.*

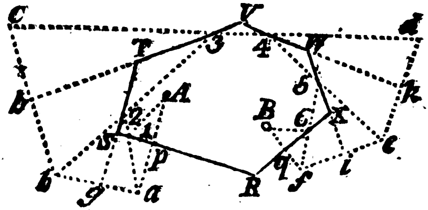
This theorem, viz. $\frac{e}{s} = \frac{py - xqz}{qn}$ is found to be more useful than the other, e. g. In the solution of the prize question 1739, solved by it, comes out $\frac{st}{2bc} = x$; where the sine $= s = GP + MP =$ sine 89°; and $t =$ tangent PV; and all the others as by Mr. Turner, Diary 1740. Also quest. 211 may readily be answered by this theorem, and will come out $xx + 2px = n$.

These theorems above being brought out in numbers, by help of the artificial and natural sines, to facilitate the labour; gave, 1. The latitude $= 54° 51'$. 2. The sun's declination $= 20° 24'$ answering to the 10th day of May. And 3d, the hour $60° = 4$ h. or 8 in the morning, and $30° = 2$ h. or 10 o'clock, agreeing precisely with that of the ingenious proposer's true answer.

This question was very methodically and truly solved by Mr. *J. Turner*, Mr. *Dunthorne*, Mr. *R. Robinson*, Mr. *Heath*, Mr. *Farrer*, Mr. *Rob. Price* in trigonometrical method; Mr. *Dan. Boote*, Mr. *H. Travis*, Mr. *May*, Mr. *Powle*, Mr. *Badder*, Mr. *Story*, Mr. *Ardon*; and the solution was given by Mr. *Brown*, Mr. *Webber*, and others.

III. QUESTION 215 *answered by* Merones.

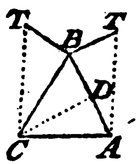
Produce all the sides of the figure as in the scheme: on *RS* let fall the perpend. *Apa*, and make $ap = Ap$; draw $ab \perp ST$, and make $bg = ga$; draw $bc \perp TV$, and make $ch = bh$. Also $Bf \perp RX$, and make $qf = Bq$; draw $fe \perp WX$, and make $ei = fi$; draw $ed \perp VW$, and make $dk = ke$. Then draw the lines *cd*, 3*b*, 2*a*, 4*e*, 5*f*, and 6*B*, cutting the sides of the figure; then *A*123456*B* will be the path of the moving body *A*.



After the same manner Mr. *Brown* and Mr. *Webber* have solved it, and Mr. *J. B. Smith* and Mr. *Betts*, by protracting the lines and angles, in a progressive form, have made the demonstration not only curious but very easy and general, which is omitted here for another place, as will be shown farther on.

IV. QUESTION 216 *answered by Mr.* N. Farrer.

In the annexed figure, if *BT*, *BT*, represent the height of the rock perpendicular to the horizontal plane *ABC*; the points *T* and *T* (on the turning up the two triangles to which and the plane they are perpendicular) are supposed coincident; *A* and *C* the two stations, and *BAC* the horizontal angle given: by letting fall a perp. *CD* by plain trigonom. I find $CD = m$, and $AD = p$. Then put $q =$ sine $\angle BAT$, (the angle at the first observation) and $s =$ its cosine; $h =$ sine $\angle BCT$ ($\angle$ altit. at the second observation) and $d =$ its cosine. Let $x = DB$, then (per 47 Eucl. 1.) $\sqrt{xx + mm} = CB$, and

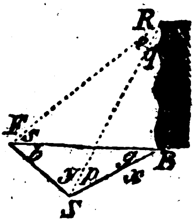


and $x+p=AB$; then as cosine $\angle BAT$: its sine :: AB : BT; cos. BCT : its sine :: $BC : BT$; i. e. $s : q :: x+p$: $\frac{qx+qp}{s}=BT$; $d : b :: \sqrt{xx+mm}$, $\frac{b\sqrt{xx+mm}}{d}=$ BT. Then $\frac{b\sqrt{xx+mm}}{d}=\frac{qx+qp}{s}$; hence $sb\sqrt{xx+mm}$ $=dqx+dqp$. This equation squared and brought into numbers, the square compleated and root extracted, &c. I find $x=321{\cdot}24$ feet; and BT the rock's height $=118{\cdot}47$ yards, the answer to the question.

N. B. By a mistake in printing, the angle was 40° 12′ instead of 41° 12′; so the true height was more, as in the following answer.

The same answered by H. Travis.

Let FSB represent the ground lines, as supposed truly horizontal; in which is given FS the stationary line 429 links $=a$; and the horizontal angle $SFB=87°\ 5'=b$; and in the vertical $\triangle\ BFR$ the angle of altitude at $F=47°\ 30'=s$; in the other $\triangle\ BSR$, right-angled at B, the second angle of altitude $BSR=41°\ 12'$ $=p$. Then say, as $q : x :: p : BR$ $=\frac{px}{q}$; as $s : \frac{px}{q} :: e : BF=\frac{pex}{sq}$; and as $b : x :: y : BF=\frac{xpe}{sq}$; ∵ yx $=\frac{pebx}{sq}$, and $y=\frac{peb}{sq}=\angle S$.



The Operation by Logarithms.

$p=9{\cdot}8186807$	
$e=9{\cdot}8296833$	$s=9{\cdot}8676309$
$b=9{\cdot}9994370$	$q=9{\cdot}8764574$

$peb=29{\cdot}6478010-qs=19{\cdot}7440883$ gives $9{\cdot}9037927=53°$ $15'$, which added to 87° 5′, then subtracted from 180, leaves 39° 40′ $=\angle B$, whose natural sine call g; then as $g : a :: b : x=\frac{ba}{g}$. Which substitute for x in the value of BR, gives $\frac{bap}{gq}=$ the height of the rock. The operation:

$b=$

$b = 9.9994370$

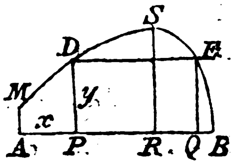
$a = 2.6324573 \quad g = 9.8050385$

$p = 9.8186807 \quad q = 9.8764574$

$bap = 22.4505750 - gq = 19.6815959 = 587$ links or 129 yards.

This question was truly solved by *Marcellus Beighton*, Mr. *Will. Daniel*; and the answer to it as printed, by Mr. *R. Dunthorne*, Mr. *D. Webber*, Mr. *Heath*, Mr. *Robinson*, *Merones*, *J. B. Smith*, Mr. *J. May*, Mr. *Arden*, &c.

V. QUESTION 217 *answered by Mr.* Rob. Heath.



When an ordinate (or any quantity) is a maximum, the logarithm of it, or any power of it, is a maximum; consequently the log. of RS^{RS} = log. of RB^{AR} $= x \times$ log. of $b - x$ is a maximum.

In fluxions $-\frac{x\dot{x}}{b-x} + \dot{x} \times \log. \overline{b-x} = 0$; where x is easily determined $= 836.0532$ &c. by a new method of solving exponential equations; $\because \overline{163.9468}^{863.052} = y^y$, and $y = 657.1442$ &c. very true to a decimal, the length of the greatest ordinate RS required. By drawing the true figure, or trying the increase of the parallelogram, it appears that Ap is greater than 400, when the $\square\, QEDP = DP \times PQ$ is a maximum; and by the table of logarithms, it also appears that $DP = EQ$ is greater than 420, and AQ less than 987. This must be observed in order to have the following logarithmetic series converge. Put $400 + x = AP$, $420 + y = DP = EQ$, and $987 - z = AQ$; to find DP and PQ. By the equation of the curve, $PB^{AP} = PD^{PD}$ $= QE^{QE} = QB^{AQ}$, or $\overline{600-x}|^{400+x} = \overline{420+y}|^{420+y}$ $= \overline{13+z}|^{987-z}$, or $\overline{m-x}|^{n+x} = \overline{p+y}|^{p+y} =$ $\overline{q+z}|^{r-z}$ by substitution; in logar.

$$\text{l.}\,m\times n - \left.\begin{matrix}\frac{n}{m} \\ + \text{l.}\,m\end{matrix}\right\} x \left.\begin{matrix}-\frac{n}{2mm} \\ -\frac{1}{m}\end{matrix}\right\} xx \left.\begin{matrix}-\frac{n}{3m^3} \\ -\frac{1}{2mm}\end{matrix}\right\} x^3 \text{ \&c.} = \left.\begin{matrix}\text{l.}\,p\times p+1 \\ +\text{l.}\,p\end{matrix}\right\} y$$

$$+\frac{3}{2p}y^2 + \frac{5}{6pp}y^3 \text{ \&c.} = \text{l.}\,q\times r + \left.\begin{matrix}\frac{r}{q} \\ -\text{l.}\,q\end{matrix}\right\} z \left.\begin{matrix}+\frac{r}{2qq} \\ -\frac{1}{n}\end{matrix}\right\} zz \left.\begin{matrix}+\frac{3}{3q^3} \\ +\frac{1}{2q^2}\end{matrix}\right\} z^3 \text{ \&c.}$$

&c. Now the value of any two single quantities may be found in terms of the other. The above in numbers (g) 21·8648 + (a) 5·730263 x — (b) ·0032221 x² — (k) ·000002 x³ &c. = (c) 7·0402547 y + (d) ·00357147 y² + (e) ·0000047 y³ &c. and = (c) 73·358128 z + (d) 2·843195 z² + (e) ·146791 z³: I find the values of y and z in terms affected with x, by assuming an equation of this form, viz. y or $z = A + Bx + Cx^2 + Dx^3$ &c. Where by substitution for their assumed values, and making the coefficients of like terms on different sides of each equation equal, we shall have the real respective values of the coefficients, viz. $A^3 + dA^2 + cA = g$; $A =$ 3·1008 and ·36508; $B = \frac{3eA^2 + 2dA + C}{a} =$ ·811364 and ·075904; $C = \frac{-b - 2dB^2 - 3eAB^2}{3eA^2 + 2dA + C} = -$·0006478 and −·0002586; $D = \frac{-k - 2dBC - 6eABC - eB^3}{3eA^2 + 2dA + C} =$ ·0000001005 and + ·000000686. Consequently $y = 3·1008 + ·811364x - ·0006478xx$ &c. $z = ·36508 + ·075904x$, $-$ ·0002586xx &c, wherefore 586·63492 — 1·075904 x + ·0002586 xx &c. $= PQ$, and 423·1008 + ·811364 x — ·0002586 x x &c. $= DP$: the product of these two is a maximum, which fluxed and reduced, there results 20·759 — 2·28625 x + ·003212 xx &c. = 0; here $x = 9·2$ nearly; whence $PQ = 576·758$, $DP = 430·5105$, and the area of the greatest ▭ 248300¼. Q.E.F.

Answered by Mr. J. Turner.

The equation of the curve is $\overline{b-x}|^x = y^y$; $AB = 1000 = b$; hence $x \times \text{l}.\overline{b-x} = y \times \text{l}.y$. In fluxions, $\dot{x} \times \text{l}.\overline{b-x} - \frac{x\dot{x}}{b-x} = 0$; $\overline{b-x} \times \text{l}.\overline{b-x} = x$. Or finding the logarithm thereof, $\overline{b-x} \times \text{l}.b + \frac{x}{b} - \frac{x^2}{2b^2} + \frac{x^3}{3b^3} - \frac{x^4}{4b^4}$ &c. $= x$. This reduced gives $x = 836$ nearly; whence RS the greatest ordinate is = 657, &c.

For the greatest Parallelogram:

Put $z = PQ$, $DP = EQ = y$; the area $= A$.

By the nature of the curve, $\overline{b-x}|^x = y^y$,

and $\overline{b-x-z}|^{x+z} = y^y$,

and $zy = A$.

Which thrown into fluxions and reduced (by expelling z) gives $x = 407·6$, $y = 429·2$, $z = 578·4$, $AQ = 986$, and the required area 248250.

The

The same answered by Merones.

1. For the greatest ordinate: $\overline{b-x}|^{x} = y^{y}$, and $x \times$ log. $\overline{b-x} = y \times$ log. $y =$ max. whence $\dot{x} \times$ l. $\overline{b-x} - \frac{x\dot{x}}{b-x} = 0$; and $\overline{b-x} \times$ log. $\overline{b-x} = x$. Put $a+v=x$; $b-a=g$; and then $\overline{g-v} \times$ l. $\overline{g-v} = a+v$. Let $p=$ log. g: take $a=836$; then $gp-a-pv-2v+\frac{v^2}{2g}+\frac{v^3}{6g^2}+$ &c. $=0$. And by reversion of series $v = \text{·}0531493$, $x = 836\text{·}0531493$, and $y = 657\text{·}02682$ the greatest ordinate.

2. Then for the greatest inscribed parallelogram; let $AP = x$, $AQ = s$; then $y \times \overline{s-x} =$ max. put $d+u=s$, $f=b-d$, $r=$ log. f; then $\overline{d+u} \times$ l. $\overline{f+u} = \overline{d+u} \times$ l. $\overline{g-v}$ ($= y$ log. y). These turned into series, and reverted, will give the value of u: whence is had s. Also let $c+z=y$, then will $\overline{a+v} \times$ log. $\overline{g-v} = \overline{c+z} \times$ log. $\overline{c+z}$. This turned into series, and the series reverted, finds z, and consequently y; then these values of s and y being substituted in the maximum, and its flux made $=0$, and the series reverted, gives v, x, and y, and thence the inscribed parallelogram $= 247675$ nearly.

The same answered by the Proposer Mr. An. Thacker.

Assume $AP = 407+x$; $PD = EQ = 429+y$; and $QB = 13+v$: then will PQ or $DE = 580-x-v$; and per nature of the curve we have $\overline{593-x}|^{407+x} = \overline{429+y}|^{429+y}$ and $\overline{13+v}|^{977-v} = \overline{429+y}|^{429+y}$. These in log. are $\overline{407+x} \times$: l. $593 - \frac{x}{593}$ &c. $= \overline{429+y}$: $\times$ l. $429 + \frac{y}{429}$ &c.

And $\overline{987-v} \times$: l. $13 + \frac{v}{13}$ &c. $= \overline{429+y} \times$: l. $429 + \frac{y}{429}$ &c. And these equations ordered make $a+bx+cxx$ &c. $= A+By+Cyy$ &c. And $f+gv+hvv$ &c. $= A+By+Cyy$ &c. Consequently $x = F+Gy+Hyy$ &c. and $v = K+Ly+myy$ &c. Hence the area of $PDEQ$ will be expressed by $\overline{O-Py-Qyy}$ &c. $\times \overline{429+y}$, which per quest. must be a maximum; then put into fluxions and reduced, gives $y = 1\text{·}5104$ &c. ∴ the greatest area $= 248300\text{·}3749$ &c.

Again

Again, for the greatest ordinate, put $835 + x = AR$, then is $\overline{165 - x}|^{835+x}$ to be a maximum; therefore put into fluxions and reduced, gives $x = 1\cdot0531$ &c. and $SR = 657\cdot0268$ the true solution. *Q. E I.*

VI. QUESTION 218 *answered by Mr.* Peter Kay.

It is found from theorem 2, page 53, of Simpson's Laws of Chance, that the odds between 37 and 47 coming up, at any assigned throw, with 12 dice, is as $1\cdot162$ &c. to 1, or as 7 to 6 very nearly; which is an answer to one part of the question. And by theorem 1 in the same page, the probability of throwing just 42 comes out $= \frac{1}{15\cdot22}$, &c. Therefore

$$\overline{1 - \frac{1}{15\cdot22}\ \&c.}|^{15} = \cdot3604$$ is the probability of losing in the other part of the question; and consequently the required odds for winning as 6396 to 3604, or as 16 to 9 nearly.

The same solved by Mr. J. May, *jun.*

Put the number of dice $= n$; the number of sides on each dice $= m$; $42 = p$; (but if p was greater than $\frac{1}{2}n + \frac{1}{2}mn$, subtract it from $n + nm$, and put the remainder $= p$). Then the chance the proposer has to win in the first throw, or to throw just 42, will be $\frac{n \times n+1 \times n+2 \times n+3 \times n+4}{1 \times 2 \times 3\ 4 \times 5}$ &c.

The numb. of terms must be $p - n - \frac{n}{1} \times \frac{n+1 \times n+2}{2 \times 3}$ &c.

(in terms less than the foregoing) $+ \frac{n \times n-1}{1 \times 2} \times$

$\frac{n \times n+1 \times n+2 \times n+3 \times n+4}{1 \times 2 \times 3 \times 4 \times 5}$ &c. (In terms less)

$- \frac{n \times n-2 \times n-3}{1 \times 2 \times 3} \times \frac{n \times n+1 \times n+2 \times n+3}{1 \times 2 \times 3 \times 4}$ &c.

(in terms less) which put in numbers is 144840476; now all the chances in the said dice will be m^n or 2176782336; so that there is 2031941860 chances to lose, that is very near 3365 to win, against 47207 to lose the first throw: Put 3365 + 47207 $= r$; 47207 $= s$; and 15 $= t$; the proportion which the proposer has to throw 42 once in 15 times, is to the

the chances he has not, as $1 - \frac{s^{-t}}{r^t}$ to $\frac{s^{-t}}{r^t}$ (see Struyck p. 52) which is about 71199 to 128801. The second query is easily deduced from the above series, and the proposer has 1400482914 chances to win, against 776299422 to lose; i. e. if 37 and 47 are included; but if excluded, then 1194427194 to win, against 982355142 to lose.

Mr. Rob. Heath *answers this Question.*

The number of chances for 42 happening in one throw with 12 dice (by his theorem) will be 144840476; which taken from all the chances on all the dice, leaves 2031941860 chances for failing in one throw.——The advantage in wagering to throw 42, once in 15 times with 12 fair dice, will be 6440045 to 3559955; or nearly as 9 to 5. To find the number of throws to make an equal wager; make $\frac{bx}{\overline{a+b}|^x} = \frac{1}{2}$, which comes to $x = \frac{\log. 2}{\text{l.}\, a+b - \text{l.}\, b}$ when reduced; and solved $x = 10{\cdot}0666$ throws by common logarithms: Mr. De Moivre says $\frac{b}{a} \times {\cdot}7$ shews the trials requisite to that effect, when b is pretty large in respect of a; but in this case it shews it to happen in 9·82 throws; very near the truth: but his Table of Limits at p. 42 Doct. Chances, 2d edit. is not very exact, as being not deduced from exponential equations truly solved.

The sum of the chances for each party, N. in one throw (by the series) are found = 1194307074; which taken from all the chances on all the dice, leaves 982475262 chances for missing; therefore the odds are nearly 6 to 5. *Q.E.F.*

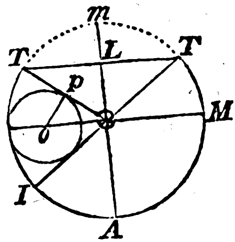
Mr J. Turner (in ans. to 218 quest.) by prob. 22. Mr Simpson's Laws of Chance, the odds of throwing (in this quest.) as 6439971 to 3560042; nearly as 8½ to 4½; and the latter part of the question as 6 to 5 nearly.

Mr J. Hill says the odds are as 113156 l. 12 s. 5½d. to 105830 l. 6 s. 1¼d. And when he undertakes to throw between 37 and 47 (exclusive) he ought to lay 622097 l. 9s. 11¼d. to 511643 l. 6 s. 0¾ d. The proposer Mr Gibbons has given tables for these questions, but as too long to insert here, if he pleases to revise them, may be put in another place.

The

The PRIZE QUESTION *answered by Mr.* H. Travis.

Put 1 = radius of the circle; *TMAIT* the park; $q =$ 1·570796 &c. ($\frac{1}{4}$ periph. when rad. is 1) = quadrantal arch *AM*; *TM* $= x$; $TL = y$; then, as $8 : 3 :: q + x : y$; ∵ $8y = 3q + 3x$; and by Sir Isaac Newton's series, $y = 1 - \frac{x^2}{1.2} + \frac{x^4}{1.2.3.4} - \frac{x^6}{1.2.3.4.5.6}$ &c. ∵ $8 - 4xx + \frac{x^4}{3} - \frac{x^6}{90}$ &c. $= 3q + 3x$. Solved, $x =$ ·0611928 &c. ∵ $y =$ ·81849 &c. and the area of the park = 89 a. 0 r. 14 p. For the area of the shades: Put $s =$ sine arc *TM*; $z = po =$ radius of the circular shades; then, as $s : z :: 1 : 1 - z$; ∵ $z = \frac{s}{1+s}$; hence the area of each is 14 acres and 7 perches. *Q.E.I.*



The same answered by Mr. Heath *the Proposer.*

Since the river *TT* (part of the boundary) is given straight, therefore the nearest the park can approach the form of a circle, is the segment *TMAIT* the true form; which comprehends for its area a maximum. Put $b = 320$ poles $=$ *TMAIT*; $d = 60$ poles $= TLT$; $p = 6{\cdot}28318$ &c. $x =$ arch mT; then $\frac{b + 2x}{p} =$ rad. $= \odot T$. And, by a series for finding the sine from the radius, and arch given, $d = x - \frac{x^3}{6 \times \left(\frac{b+2x}{p}\right)^2} + \frac{x^5}{120 \times \left(\frac{b+2x}{p}\right)^4} - \frac{x^7}{540 \times \left(\frac{b+2x}{p}\right)^6}$ &c. Where x is found $= 70{\cdot}287$ &c. whence rad. $= 73{\cdot}302$ &c. poles $= b$; and the area of the park 14255·15 poles, or 89 a. 0 r. 15 p.

Put $f =$ ·574459 = sine $\angle o \odot p = \angle \odot TT$; $y =$ radius of each shade $= op$. Then $b - y = o\odot$; say rad. $1 : b - y :: f : y$; whence $y = \frac{fb}{1+f} = 26{\cdot}745$ &c. poles; consequently the area of each shade = 14 a. 0 r. 7 p.

 This

This question may be more easily solved, by a table of nat. sines (with rad. unity). Seek a chord and arc in the ratio of the given chord and arc, as 3 to 8, which are found by 2 or 3 trials = 1·6370516 and 250° 7′ 30″ similar to the chord *TT*, and arc *TMAIT*; (for ·8185258 is sine of 54° 56′ 15″ similar to the sine *LT*, and arc *Tm*, half the supplement of the given arc to 360°) say 1·6370516 : rad. 1 :: 120 (*TT*) : 73·302 &c. poles = *T*⊙, radius as found before.

Merones, by a curious and short process, has brought out the true answer to the question. Mr. *Robinson*, Mr. *Brown*, Mr. *Webber*, Mr. *Trott*, Mr. *Turner*, and others, have also given true solutions.

The same solved by Mr. J. Powle.

Put $b = 5400$ the min. in a quad. $c = \cdot 0000000846161$; $d = 4 = \frac{3}{4}$ arc; $q = 3\cdot 14159$; $p = 1\frac{1}{2}$ = half the given chord; $x = \angle \odot TL$ in minutes; $y = T \odot L$; radius = 1. It will hold, $b + x : d :: 2b : \frac{2bd}{b+x}$ length of the semicircle; $q : 1 :: \frac{2bd}{b+x} : \frac{2bd}{qb+qx} = \odot T$; then, per sim. △s, $1 : y :: \frac{2bd}{qb+qx} : p$; i. e. $\odot a : ab :: \odot L : LT$; ∵ $\frac{2bdy}{qb+qx} = p$. Here $x = \frac{2bdy - qpb}{qp}$: $xx = \frac{4bbddyy - 4dbbpqy + qqbbpp}{qqbb}$.

Then by Mr. Ward's equation for finding the sine corresponding to a given arc (Math. Introd. p. 358) we have $y^4 + 28y^3 + 195y^2 + 36cxxyy + 108cxxy - 28y = 196 - 81cxx$; and exterminating xx, by means of its value before found, there will be produced this equation:

$$\frac{qqpp}{144cbbdd}y^4 - \begin{array}{l}28qqpp\\144dbbpqc\\432cbbdd\end{array} y^3 - \begin{array}{l}195qqpp\\36qqbbppc\\432cdbbpq\\324cbbdd\end{array} y^2 \begin{array}{l}108qqbbppc\\-28ppqq\\-324cbbdqp\end{array} y$$

$= 196qqpp - 81qqbbppc.$

In numbers, $-5707\cdot 1022y^4 - 10979\cdot 01y^3 + 998\cdot 257y^2 + 9733\cdot 248y = 85\cdot 7091935$; solved, $y = \cdot 8185287$ the sine of 54° 56′ 16″; by trigonom. s. 54° 56′ : (*LT*) 1·5 :: rad. 90° : *CL* = 1·83255; then 1·8325 × 4 = 7·3302 = 73 a. 1 r. 8 p. the sector; and $\sqrt{3\cdot 3325 \times \cdot 3325 \times 1\cdot 5 \times 1\cdot 5}$ = 15 a. 3 r. 6 p. = area △. Their sum 89 a. 0 r. 14 p. the true area of the park. Shade 14 a. 0 r. 7 p.

Of

Of the Eclipses in 1741.

To the inhabitants of Great Britain or the adjacent islands, there will happen no visible eclipse this year, but at Fort St. George, the sun will appear eclipsed the 27th of November, beginning at 8h. 40′ morning. Middle at 10h. End at 11h. 14′. Duration 2h. 30′. Digits eclipsed 2h. 48′ [*John Skay*]. On the 2d of June the moon would be eclipsed, but by her depression of latitude is rendered invisible to us; yet will appear in the southern parts of the world as follows, [*R. Hale.*]

	Beg.	Mid.	End	Dur.	Dig.
At Gibraltar in Spain	8 20	9 6	9 52	1 32	4 5
Alexandria in Egypt	11 18	12 9	12 59	1 41	5 24
Caluent in the Indies	2 5	3 5	4 5	2 0	9 0
Jerusalem	0 16	1 9	2 2	1 46	5 31

Observations of Eclipses at the Observatory.

At Trinity College, Cambridge, was took the beginning of the moon's eclipse Jan. 2, 1740, at 8h. 27′ apparent time, the total shadow entered the moon's N. E. limb about the 118th degree in Hevelius's chart, near Mount Audus, then clouds interposed. The emersion was about 11 h. 15′ over-against M. Pherme.

At 3 miles from Beverley, Abr. Buncholot observed the moon's eclipse Jan. 2, 1740, beginning 8 h. 20′. Begin. total darkness 9 h. 30′. End total darkness 11 h. 10′. End 12h. 20′.

New Questions.

I. QUESTION 219, *by Mr.* J. May *of Amsterdam.*

Here in Holland, the land lying so very low, they are obliged to raise banks or dikes to keep the sea from overflowing it; yet some time ago a great storm, with a high tide, broke through one of the banks, and laid the country for some miles under water. Going with some friends to the place where this inundation happened, and walking upon one of these narrow banks, we saw at a distance three trees (*A*, *B*, *C*) standing in the water, which we were told, stood at an equal distance from each other at the corners of a triangular field; and a pole equidistant from each tree, that

 formerly

formerly was used as a mark to shoot at with bows and arrows. Now one of our party proposed to find the content of the inundated field by the help of a staff, or measuring rod only, which we had with us. As we walked along this bank in a straight line (*DP*) at *D* we came in a straight line with the trees *C* and *A*; thence measuring 304 feet, at *E* we came in a right line with the trees *C* and *B*; thence continuing straight 121·6 feet, at *R* we made a right line with the pole in the middle of the field, and the tree at the farther corner *B*; lastly from *R* measuring right forward 159·6 feet, at *P* we came in the line of *BA*. From this, which was all we were then capable of doing, is required the content of the field *ABC*.

II. QUESTION 220, *by* Merones.

Being at sea on the first of May, and a clear forenoon, I made two observations of the sun, and found the difference of altitudes 16° 30′, the difference of azimuths 34 deg. and the difference of the times 2 h. 15 m. Required the latitude of the place and hours of the day.

III. QUESTION 221, *by Mr.* Peter Kay.

One with six dice undertakes to bring up four faces of a sort at a throw; that is, either four aces, four duces, &c. in seven trials? What is the odds against him?

IV. QUESTION 222, *by Mr.* Daniel Boote.

Some time in the spring quarter 1740, an observation was made of the sun, at 24 minutes past eight o'clock in the morning, and his altitude found 39° 7′. Also at 15 minutes past ten o'clock (on the same forenoon) the altitude was 55° 57′. From whence the latitude of the place of observation, month, and day may be found; and are required, with a general theorem for all questions of this nature.

V. QUESTION 223, *by Mr.* Peter Kay.

Supposing a body in 51° 32′ north latitude, to be projected with a velocity of six miles per second, in a south-east direction, and an angle of 30° with the horizon: required the trajectory, the place of the earth where the body will fall, and the time it is in motion; allowing the earth to be spherical, its circumference = 25020 miles, that a heavy body descends 16·1 feet in a second of time, and that the projectile moves in a non-resisting medium.

VI. QUES-

VI. Question 224, *by Mr.* Nich. Farrer.

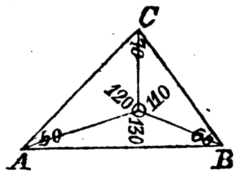
Two ports bear north and south on British shore,
Their distance 50 leagues, nor less nor more;
A ship from the south port sets out to gain
The northern port, and plows the wat'ry main,
On larboard tacks a certain distance, then
Alters her course, so gains her port; but when
The distances compar'd, the first was more
By true sea leagues exactly half a score;
The distance of the ports and distance run,
Include a space * as here below is shewn:
Now, artists, by these data it's requir'd,
To find the distance run, † and courses steer'd:
To make it plainer, and require less art,
Give the solution by a plain sea chart.

* *796 square leagues.* † *Variations and lee-way allowed for.*

Now let us suppose the ship to have sailed uniformly at the rate of four miles an hour, and at the same time that she sailed from the south port, another ship sails from the north port directly south, at the rate of $5\frac{1}{2}$ miles an hour, till she arrive at a port under the same meridian with the former ports, and is then known to be at the nearest distance from the first ship, that she could possibly have been during her whole passage, if she had continued her course to the south port: Required the bearing aud distance of the middlemost port from the first ship, when she brings her starboard tacks on board.

VII. Question 225, *by Mr.* J. Corbett.

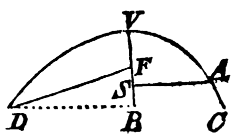
In order to survey and divide the triangular field *ABC*, I bid my men measure the three sides round, whilst I took the angles. The owner would have it in three equal parts nearly divided, but as there was no water but one spring near the middle, in order to have the three fences run straight from it to the three corners, I took the angles there, as well as those at the spring to each corner, as in the scheme are set down; for he would have the content of each part separate. Having thus much, as angles, sides, &c. I thought surely I might from hence find the lines to the corners (and areas) as well as Mr. Beighton in his survey of Warwickshire could find the situation of Newbold, and distance to High Cross, Tripontium, Eathorpe, &c. as he sets forth in the compartment of his map; for that besides angles, I should have three sides measured, whereas he had but one and a piece. As soon as we had done, night approaching, I repaired home; where I found they, blundering, had taken the three lines all in one sum just 100 chains. And I, being ambitious to match this mapmaker, have been puzzling to find a theorem less complex than his: Now if any of you could assist me to do this by a simple equation, I doubt not but his *F. R. S.* might be torn away from his name, and more deservedly put to mine. The angle at $A = 50$ deg. at $B = 60°$; and at $C = 70°$. Angles from the spring to *A* and *C*, 120°; *A*, *B*, 130°; *C*, *B*, 110°.



The PRIZE QUESTION, *by Mr.* J. Turner.

A certain 'squire, whose fertile grounds
The silver-streaming Mesbrook bounds,
Met me the other day, and said I shou'd
Survey for him a small adjacent wood.
With his request I straight comply'd,
And when got there, my skill I try'd;
But all in vain; so therefore crave
The ladies' help: * Below they have
Such data, as my friend and I
Cou'd then procure, most accurately.
And for her pains, she who unties
The knot † (with B—gh—n's leave) shall have the prize.

* *DVF* and *CVF* are two different Apollonian parabolas; *V* the vertex, and *VB* the transverse diameter of both; *DB* and *AS*, ordinates rightly applied: There is given the area *DFV* equal to $1473\frac{1}{2}$ poles, and $VF = FB$; *F* being the focus of the parabola *DVB*.



Again, *VAC* is the other Apollonian parabola, whose arch *AV*, is to the arch *DV*, as 1 to 2. And the ordinate *AS* (rightly applied) being let fall on the common axis *VB*, makes the area *ASV* a maximum. Required its parameter, and the area of the wood in statute measure.

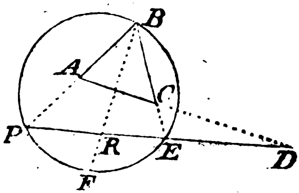
† By a simple equation.

1742.

Questions answered.

I. QUESTION 219 answered by Mr. R. Fall of Dunbar.

As $PR+RE$ (281·2) is to $PR-RE$ (38), so is the tang. of half the sum of the (opposite) angles P and E; to the tangent of half their difference (13° 10′); and half the sum more ½ the difference is equal to the greater angle ($PEB=$ 73° 10′); the half sum (60) less ½ difference is the lesser angle ($EPB=$ 46° 50′.)



Then say, as sine $\angle B$ (60°) : PE (281·2) : : s. P (46° 50′) : BE (236·9). Again, s. $\angle ECD$ (60) : ED (304) : : s. $\angle D$ (13·10) : EC (79·959); ∵ $BE-EC=$ 156·94 $=$ BC the side of the triangle required. Which squared and multiplied by $\sqrt{\frac{3}{4}}$ gives 10664·857 square feet, the area of triangle ABC required.

There is a great many ways this problem may be solved, which I shall reserve for another place, where the solutions to the questions in the Diaries shall be further discussed and illustrated. Only here observe, that by 3 Eucl. 6, the line BLR bisecting the $\angle B$, cuts the opposite side into two segments PR and RE, in proportion to the other two sides (BP and PE), and consequently the sines of the angles they subtend in like ratio.

This question was truly solved by Mr. *Da. Hastings*, Mr. *F. Parrot*, *J. Corbet*, *G. Neal*, *R. Beighton*, *D. Boote*, *W. Kingston*, *J. Jackson*, *S. Bamfield*, *E. Cross*, *J. Milbourn*, *T. Ramsay*, *Philoginus*, *J. Powle*, *R.* and *J. Robinson*, *W. Spicer*, and Mr. *Tompson*.

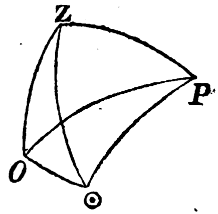
II. QUES-

II. QUESTION 220 *anſwered by Mr.* Ant. Thacker.

As ſine 90° : coſine ſun's declin. (71° 47′) : : s. diff. of time (2 h. 15′) : 32° a fourth number = ⊙*O* the diſtance in the parallel of declin. Then to the nat. coſ. of the diff. of azimuths (34° = ·829037) add radius, the ſum is 1·829037, which multiplied by the nat. coſ. 16° 31′ = ·958819) the difference of altitudes gives 1·753717282920, which made leſs by twice the coſine (32° = 2 × ·848048) = 1·696096, leaves ·057621082920; which divided by the verſed ſine of the difference of azimuths (34° = ·170963) quotes ·337453; which anſwers to the nat. ſine of 70° 18′; one half of which 35° 9′ made leſs by half the given difference of altit. (8° 15′) leaves 26° 54′ the altitude of the ſun at the firſt obſervation, and more by 8° 15′ is = 43° 24′ the altit. at ſecond obſervation. Whence there is now ſufficient given to find, by the common canons in trigonometry, the latitude = 57° 5′, and that the time of the firſt obſervation was at 7 h. 26′, and the ſecond at 9 h. 41′.

The analytical Inveſtigation of the aboveſaid Theorem.

Given ⊙*P* = 71° 47′ comp. declination, the ∠ ⊙*PO* = 33° 45′ diff. time, whence ⊙*O* is found = 32°, whoſe coſine call m, put z = coſine ⊙*ZO* the diff. azim. a and b for ſine and coſine of $\frac{1}{2}$ diff. alt. then will $bb - aa$ or $1 - 2aa$, or $2bb - 1$ $(= n)$ be the coſine of the diff. of the altitudes. Alſo let x and y expreſs the ſine and coſine of half the ſum of the altitudes; then $yy - xx$ $(= v)$ will ſtand for the ſum, $xb + ya$ and $yb - ax$ the ſine and coſine of the greater altitude, $xb - ya$ and $yb + ax$ the ſine and coſine of the leſſer. Then by a theorem in page 176 of Simpſon's fluxions, $xxbb - aayy + yybbz - aaxxz = m$; or writing $1 - xx$ for yy, and reducing the equation we have $xx = \frac{m + aa - bbz}{1 - z}$



$$\therefore 1 - xx = \frac{1 - z - m - aa + bbz}{1 - z} = yy, \text{ and } \frac{1 - 2aa - z + 2bbz - 2m}{1 - z}$$

$$= yy - xx = \frac{1 - 2aa + \overline{2bb - 1} \times z - 2m}{1 - z} = \frac{n + nz - 2m}{1 - z}$$

=

$= \frac{n \times \overline{1+z} - 2m}{1-z} = v$ the cosine of the sum of the altitudes. In numbers $\frac{\cdot 9588 \times \overline{1 + \cdot 8290} - 1\cdot 6960}{1 - \cdot 8290} = \cdot 3375$, which answers in the tables to 70° 16′ for the sum of the sun's altitudes, whose half (35° 8′) added to the half diff. (8° 15′) gives 43° 23′) the greater altitude; whence the lesser is = 26° 53′ and the latitude = 57° 7′. And the times of observation 7h. 27′ and 9h. 42′. *Q.E I.*

III. QUESTION 221 *answered by Mr.* Farrer.

First (per p. 7 Simpson's Laws of Chance) $\frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times 6 = \frac{3125}{7776}$ = the probability that one face of a sort comes up at one throw with 6 dice. Then $\frac{3125^4}{7776^4}$ is the probability of four faces of a sort coming up at the first throw with six dice; and that of the contrary $\frac{7776^4 - 3125^4}{7776^4}$, and the odds $7776^4 - 3125^4$ to 3125^4, or as B to A. Then by page 13, $\frac{A^p \times B^{n-p}}{\overline{A+B}|^n} \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3}$ &c. to p, factors: here $n = 7$, and $p = 1$; which substitute in the above theorem, and it becomes $\frac{3125^4 \times \overline{7776^4 - 3125^4}|^6}{7776^7} \times 7$ = the required probability $= \cdot 8134$, and that on the contrary $\cdot 1866$, and the odds 8134 to 1866, or as 4067 to 933, i.e. 2·1954 to 1.

Answered by Mr. Rob. Heath.

Raise the binomial a (1) + b (5) to the 6th power, the three first terms of which ($a^6 + 6a^5b + 15a^4b^2$) will be the chances for 6, 5, and 4 aces to come up, which (in this case) being multiplied by 6, = 2436 chances for 6, 5, and 4 aces, duces, trays, &c. to come up at one throw; wherefore $\frac{44220^7}{46656^7}$ are the chances for failing to throw 6, 5, and 4 like faces in 7 trials; whence the wager's disadvantage is 687033 to 312967, or 11 to 5 nearly, viz. 2·197 to 1.

The

The same answered by MERONES.

The number of combinations of 4 aces out of 6 is 15; and in any one case of these 15, any two left out are capable of 25 variations where no ace is found; therefore 375 gives all the cases where only 4 aces can be cast. But since 5 or 6 aces may be cast, the number of chances for these, which is 31, must be added; ∵ therefore all the cases wherein 4 aces can be cast with 6 dice, is 406. Now since there is the same variety for duces, trays, &c. therefore 2436 is the whole number of cases wherein 4 points of any one sort can be cast: Let the whole number of chances $46656 = s$; $46656 - 2436 = q$; the odds against the thrower for 7 casts will be as $\frac{q^7}{s^7}$ to $1 - \frac{q^7}{s^7}$, or as 2·1954 to 1.

Mr. *John May* has, from Mr. N. Struyck's method, given a very good solution to this problem; which are all the true solutions I have received.

IV. QUESTION 222 *answered by Mr.* J. MAY.

Let a be $=$ sine of sun's alt. at the first observation; $b =$ that of the second; $c =$ sine of the hour angle from 6 o'clock at the first observation; $d =$ that at the second; and let $r =$ radius; $b - a = p$; $d - c = q$; $r - c = h$; and $r + d = k$; then the sine of the sun's southern altitude will be $a + \frac{hp}{q}$, the degrees of which put $= m$: Likewise the sine of the sun's depression under the horizon in the north will be $\frac{kp}{q} - b$, whose degrees let be $= n$; then the sun's declination will be $\frac{m - n}{2} = 20° \; 25'$ N. answering to the 11th day of May; and the latitude is $90° - \frac{1}{2}m - \frac{1}{2}n = 46° \; 58'$ north.

This question was answered by Mr. *Boote* the proposer, Mr. *Robinson*, Mr. *N. Farrer*, &c. by a quadratic equation; and by *Merones*, Mr. *R. Robinson*, Mr. *Ramsay*, Mr. *Heath*, Mr. *Turner*, Mr. *Powle*, &c. by an equation of the fourth power. Also solutions were given by *Ens Rationalis*, Mr. *Bird*, Mr. *Pilgrim*, Mr. *Bamfield*, Mr. *Ant. Topham*, Mr. *Clarke*, Mr. *Howard*, and others.

But

But that we might ſhew what may be expected from the intended treatiſe of the diary queſtions, &c. where this and other queſtions will be ſolved by ſimple equations, which have their origin from the application of algebra to ſpherics, geometry, and all other branches of the mathematics; I will here give you the analytical inveſtigation of a theorem or two for ſolving this 222d queſtion.

Given $Z\odot = 50° 53'$, its coſ. $= b$; $ZO = 34° 5'$, its coſ. $= a$; the $\angle \odot PZ = 54° 0'$, its coſ. $= c$; $OPZ = 26° 15'$, its coſ. $= d$; (ſee the laſt figure). Put x and y for the ſine and coſine of half the ſum of the arcs $\odot P$ and ZP, z and v for ſine and coſine of half their difference; then will $xv + yz$ and $yv - xz$ be the ſine and coſine of the greater arc $\odot P$; $xv - yz$ and $yv + xz$ thoſe of the leſſer ZP. And (by p. 176, Simpſon's Fluxions) $yyvv - xxzz + dxxvv - dzzyy = d$. And $yyvv - xxzz + cxxvv - czzyy = b$; and ſubſtituting $1 - zz = vv$, and redu. equ. we have $yy - zz + dxx - dzz = a$; and $yy - zz + cxx - czz = b$; and ſubſtituting $1 - xx$ for yy, we get $vv + zz = \dfrac{a - ca - b + db}{d - c}$ $= \cdot 89449$ the coſine of the difference of the arcs ZP and $\odot P$ $(= 26° 34')$ or the ſine of the ſun's meridian altitude, which anſwers to $63° 26'$. Again, per equation above we have $\dfrac{b + db - a - ca}{d - c} = -\cdot 38410, = yy - xx$, the coſine of the ſum of thoſe arcs $(112° 36')$ or the ſine of the ſun's depreſſion at midnight, $= 22° 36'$. Whence $90° - \dfrac{63° 26' + 22° 36'}{2}$ $=$ latitude $46° 59'$; and $\dfrac{63° 26' - 22° 36'}{2} = 20° 25'$, the ſun's declination on the 11th day of May. *Q. E. I.*

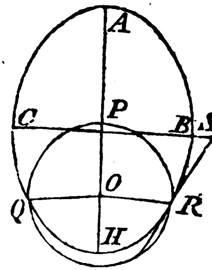
THEOREM.

$$\frac{s.\,90° - \text{coſ.}\,54° \times s.\,55° 57' - s.\,39° 7'}{\text{coſ.}\,26° 15' - \text{coſ.}\,54°} + s.\,39° 7' = s.$$ ſun's alt. at noon. $\dfrac{s.\,90° + \text{coſ.}\,54° \times s.\,55° 57' - s.\,39° 7'}{\text{coſ.}\,26° 15' - \text{coſ.}\,54°}$ $- s.\,39° 7' = s.$ ſun's depreſſion at midnight. Thence the meridian altitude is had $63° 34'$; and the midnight depreſſion $22° 34'$, half the difference of which is the ſun's declination; half of each ſubtracted from $90°$ is the latitude required.

V. QUES-

V. QUESTION 223 *answered by the Proposer Mr.* P. Kay, *and by* Merones.

Since the velocity with which the ball is projected is sufficient to carry it over a space of 6 miles per second; and the velocity of the place of projection thro' the earth's rotation round its axis is 0·1802 miles per second; the absolute velocity compounded of these two will be 6·1115 miles per second; and the angle which the true direction of the ball makes with the horizon 29° 24′, and the azimuth or bearing from the south 46° 22′. Let *PQRO* represent the earth, *R* the place of projection, *ACHB* the required trajectory, *AOH* its transverse axis, *CB* its conjugate, and *RS* a tangent to it at the point *R*. Putting $OR = d$; $6·1115 = v$; nat. sine of *ORS* (119° 24′) $= s$; radius $= 1$; and the distance descended in one second, in parts of a mile, $= r$. Then, by p. 23 of Mr. Simpson's Mathematical Essays, the transverse *AH* will be $= \dfrac{d}{1 - \dfrac{vv}{4rd}} = 17165$ miles, the conjugate $CB = \dfrac{vs\sqrt{AH}}{\sqrt{r}} = 12633$ miles; and the time of one entire revolution $=$ 4 h. 27 m. 13 s. from whence the time that the ball is in motion will be found 4 h. 6 m. 24 s. and the arch *RPQ* described, in the plane of a great circle round the center of the earth, in that time will be 209° 14′: but the angle, which that circle makes with the meridian, is found above to be 46° 22′; from which angle, and the two given sides including it, the third side of the triangle, or the latitude of the place where the ball descends, may be found, and comes out 28° 16′ south: Also the angle at the pole, from the solution of the same spherical triangle, will be 156° 30′, which being added to 61° 42′ the arch described by the earth about its axis during the time the ball is in motion, gives 218° 12′ westerly, for the difference of longitude required.



Merones's

Merones's *Anſwer.*

To anſwer every part of this problem at large, would require too much room: I ſhall therefore only explain the method of calculation:

1. Compounding the earth's motion, with the projectile's motion, I find its true velocity to be 6·11228, at an angle of elevation (above the horizon ſuppoſed at reſt) 29° $23\frac{1}{2}'$, making an angle with the meridian 46° $22\frac{1}{2}'$.

2. By propoſition 15 and 17 Princip. I. the latus of the orbit is = 9301·53; the tranſverſe axis of the ellipſis = 17252·42; and the periodic time = 4 h. 29 m.

3. By meaſuring the area of the ellipſis, and part cut off by the earth's radius; the time of the flight above the earth will be found 4 h. 8 m. 32·4 s. in which time it comprehends an arch of the earth (between its riſing and falling) = 208° 37'.

4. Therefore, by ſpherical trigonometry, the body falls in ſouth latitude 28° 48'; and the difference of longitude eaſt is 226° 29', from the point of projection ſuppoſed at reſt.

5. But ſince the earth's motion in the time of the flight, transfers the place of projection 62° 23' eaſtwards; therefore the place the body falls in will be in 164° 6' eaſt longitude from the place of projection.

Mr. *Dunthorne*, Mr. *Ramſhay*, and others have attempted the ſolution, but not with the ſame ſucceſs. In a queſtion ſo curious and intricate, it is very ſurprizing, that two perſons, at ſo great a diſtance as above 200 miles, ſhould ſo preciſely agree in the numbers: This ſhews, that, in theſe things, Mr. *Simpſon*'s Eſſays, &c. come up to the ſame perfection of the wonderful Sir *Iſaac Newton*'s Principia.

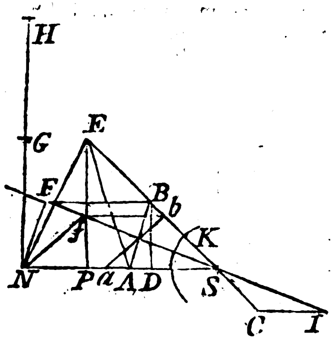
VI. QUES-

I. QUESTION 219 *Conſtructed.* [Which by miſtake was omitted in page 135]

CONSTRUCTION. Upon *PE* deſcribe the ſegment of a circle capable of containing an ∠ of 60°; and having completed the circle, from *F*, the middle of the lower ſegment, through *R* draw a line to cut the circumference in *B*; join the points *P*, *B* and *E*, *B*, and ⊥ *FB* draw *DL*, cutting *EB* and *PB* in *C* and *A*, and *ABC* will be the triangle whoſe area is required.——For the ∠ *ABC* being, by conſtruction, = 60°, the ∠ *FBE* = the half of it or 30°, and the ∠ *DLB* a right one, the ∠ *BCL* will alſo be = 60°, and of courſe the △ *ABC* equilateral.

* VI. QUESTION 224 answered by Mr. W. Daniel.

From an analytic process (which shall be published elsewhere) is deduced this theorem. The sq. root of $\frac{796^2}{25^2 - 5^2} + 25^2$; that is, $\sqrt{1681 \cdot 026} = 41$ nearly, made more by (5) half the compared distances, make 46 leagues, the distance between the south and middle port; and less by (5) half the compared distance $= 36$, between the middle and north port: therefore 82 leag. is the distance run. The area $796 \div$ by 25 gives $31 \cdot 84 = PE$ the perpendicular. Which squared and taken from



* VI. QUESTION 224.

CONSTRUCTION. Since the perpendicular of the $\triangle NES$ is given, being a 3d proportional to $\frac{1}{2} NS$ and the side of the square expressing the given area, the triangle itself may be constructed by prob. 76 of SIMPSON's *Algebra*, 2d ed. Or, perhaps, more elegantly thus: At $N \perp$ to NS erect $NG =$ the given perpendicular, and continue it to H so that $GH = NG$; with center S and radius $SK = SE - NE$ describe a circle; then by prob. XII. of Mr. LAWSON's *Tangencies*, find the center of a circle which shall touch that already described, and likewise pass through the points N and H, and that center will be the vertex of the triangle —— By either of these methods the $\triangle$ being described; on ES produced and parallel to NS take SC and CI proportional to the uniform velocities with which the ships move from S and N; then on an indefinite line drawn through the points D, S, let fall the $\perp NF$, and complete the parallelogram $NFBA$, so will BA be the *minimum* fought.——For constituting any other parallelogram $Nfba$; then since, by sim. $\triangle$s, $Sb : bf$ or (*Euc.* I. 34) $Na :: SC : CI$, and likewise $SB : BF = NA :: SC : CI$, the points b and a, B and A will be contemporaneous positions of the two ships; and by *Euc.* I. 18, Nf or ba is greater than NF or BA.

Had BA been required of a given length, NF need only have been taken equal to that length, and, it is manifest, the remainder of the construction would have been performed as above.

from the square of SE, leaves 1102·2144, whose root is 33·2 $= SP$. Let B and A represent two cotemporary positions of the ships; and their motion at B to that at P; as 4 to 5½, or as 1 to 1·375, and we have this theorem: $SB = \frac{SN \times 1{\cdot}375\, SE + SE}{1{\cdot}375^2\, SE + 2 \times 1{\cdot}375\, SA + SE} = 21{\cdot}95$. And as 1 : 1·375 :: 21·95 : 30·18 $= SP$, which taken from SN, leaves 19·82 $= PN$. Also $SP - SA$ leaves 13·38 $= PA$. And by 47 Euc. 1, $AE = 34{\cdot}21$ leagues the distance of the first ship from the middle port, when she brings her starboard tacks on board. Then by trigonometry, $\angle NSE = 43° 47'$. Therefore the course of the ship S to E was N.E. 1° 13′ northerly; and the $\angle PEN = 27° 51'$ ∴ the ship bears from E to N, W.N.W. 5° 21′ northerly. Lastly, the $\angle AEP = 21° 53'$. Hence the port A bears from E, W.S.W. 0° 37′ westerly.

The Proposer, Mr. Farrer's *Answer.*

Let $2m = SN = 50$ leagues; $2n = SE - EN = 10$; $2y = SE + EN$ sought. Then $m + y = \frac{1}{2}$ sum of the 3 sides, $n + y = SE$; $y - n = EN$; thence having the 3 sides the square area will be $m + y \times y - m \times m - n \times m + n$; that is, $y^2 - m^2 = \frac{A^2}{m^2 - n^2}$; ergo, $y = \sqrt{\frac{A^2}{m^2 - n^2}} = 41$ leagues; and $SE = 46$ leagues, and $EN = 36$. Hence the course on the larboard tacks is N. 43° 48′ E. and the distance 46 leag. and her course on the starboard N. 62° 11′ W. and the distance 36 leagues.

Now, let A and B represent the two ships at their nearest approach, and let fall the perpendicular BD; and put $q =$ sine $\angle S$; $p =$ its cosine; $s = 5{\cdot}5$; $r = 4$ miles; $d = SN$; and $z = AN$ sought. Then $d - z = AS$; and (per quest.) $s : r :: z : \frac{rz}{s} = SB$; and (by trig.) $1 : \frac{rz}{s} :: p : \frac{prz}{s} = SD$; and $1 : \frac{rz}{s} :: q : \frac{qrz}{s} = DB$; then $SA - SD = AD = \frac{sd - sz - prz}{s}$; substitute $h = 1 + \frac{pr}{s}$; then $AD = d - hz$: And (by 47 Euc. 1.) $AD^2 + DB^2 = AB^2 = \frac{qqrrzz}{ss} + dd - 2dhz + hhzz$, which must be a minimum, and its fluxion $\frac{2qqrrz\dot{z}}{ss} - 2dh\dot{z} + 2hhz\dot{z} = 0$.

 This

This reduced gives $z = \frac{dhzz}{qqrrhhss} = 29.5698$ leagues $= AN$; and $SA = 20.4320$ leagues; $SB = 21.5053$ leagues. And the middle port bears from the first ship, when she brings her starboard tacks on board, south 66° 52′ westerly, and distant 34.62 leagues. *Q. E. I.*

This question was answered by Mr. *R. Heath*, Mr. *Robert Robinson*, Mr. *John Jackson*, Mr. *J. Terey*, Mr. *Edw. Cross*, Mr. *T. Ramshay*, Mr. *J. Powle*, Mr. *J. Turner*, Mr. *W. Spicer*, Mr. *T. Bird*, Mr. *Walter Trott*, Mr. *Ri. Gibbons*, *Ens Rationalis*, Mr. *J. May*, Mr. *J. Hemmingway*, Mr. *Rich. Piercy*, Mr. *Proctor*, and others.

* VII. QUESTION 225 *answered by Mr.* John Watts.

The sum of the three sides (per quest.) = 100,

The angles $\begin{cases} CAB = 50° \\ ABC = 60 \\ ACB = 70 \end{cases}$ and from the point ⊙ within $\begin{cases} C\odot B = 110° \\ C\odot A = 120 \\ A\odot B = 130 \end{cases}$

Call CB unity (1); then per trig. as s. $\angle A : 1 :: $ s. $\angle C : AB = 1.2266$, and s. $\angle A : 1 ::$ s. $\angle B : AC = 1.1305$. And the sum of these three (proportional) sides is $= 3.3671$. Whence by ratio, or proportion,

As $3.3671 : 1 :: 100 : 29.786 = BC$
$3.3671 : 1.2266 :: 100 : 36.538 = BA$
$3.3671 : 1.1305 :: 100 : 33.674 = AC$

Let x and m express the number of degrees in the angles $AC\odot$ and $BC\odot$ respectively; n and y, those of $BA\odot$ and $CA\odot$; v and s, the angles $AB\odot$ and $CB\odot$. Then $x + m = 70°$, per quest. And $m + 110 + s = 180$ (by 17 Euc. 1.) $\therefore m + s = 70°$; consequently x is $= s$. Again, $y + n = 50$; and $n + 130 + v = 180$; $\therefore n + v = 50$; consequently $v = y$. Whence if x and y denote the sine and cosine of the $\angle CB\odot$; c and m that of ABC; then will $cy - xm$ express the sine of the angle $AB\odot$ or $CA\odot$. And

---

* VII. QUESTION 225.

The $\triangle ABC$ itself may be determined by the construction given to prob. 44 in the *Mathematician*, or by first describing a triangle similar to it; after which, if on AB and AC two segments of circles capable of containing $\angle$s = to 130° and 120° be described, their point of intersection will, it is plain, give the place of the spring.

And (per trig.) s. $\angle C\odot B$ (n) : CB (k) :: s. $\angle CB\odot$ (x) : $\frac{xk}{n} = \odot C$; s. $\angle A\odot C$ (c) : CA (p) :: s. $CA\odot$ ($cy - xm$) : $\frac{pcy - pxm}{c} = \odot C$. Consequently $\frac{pcy - pxm}{c} = \frac{xk}{n}$; which, out of fractions, is $pcny - pxmn = xkc$; $\therefore pcny = xkc + pxmn$; and $\therefore \frac{y}{x} = \frac{kc + pmn}{pcn} = \frac{k}{pn} + \frac{m}{c}$, the cotangent of $\angle CB\odot$. And $\frac{y}{x} = 1{\cdot}51866$ &c. answering in the tables to 33° 21′. Therefore the angle $AB\odot = 26°\ 41'$; whence (per trig.) is found,

Chains	Poles			a.	r.	p.
$\odot A = 21{\cdot}383$	$= 85{\cdot}5283$	The area of	$A\odot B =$	15	1	38·96
$\odot B = 18{\cdot}917$	$= 75{\cdot}6709$		$B\odot C =$	15	1	39·19
$\odot C = 17{\cdot}431$	$= 69{\cdot}7242$		$C\odot A =$	16	0	21·31

And the content of the whole field ABC = 47 0 20·46

Mr. John Jackson's *Answer.*

Calling AC (a); AB (b); BC (c). And finding the sides as above. Putting $m =$ s. $A\odot C = CBA$, and $d =$ cosine; $n =$ s. $\angle C\odot B$; and $s =$ s. $A\odot B$. Then $y =$ s. $\angle B\odot A = CA\odot$; $\sqrt{1 - y^2} = z =$ cosine; and tang. $= \sqrt{\frac{1-y^2}{y^2}} = q$; $x =$ s. $CB\odot = AC\odot$. Then $n : c :: x : \frac{cx}{n} = C\odot$; and $m : a :: y : \frac{ay}{m} = C\odot$; consequently $\frac{cx}{n} = \frac{ay}{m}$, $\therefore x = \frac{any}{cm}$, and $mz - dy = \frac{any}{cm}$; transposed, $mz = \frac{any}{cm} + dy = \frac{any + cmdy}{cm}$, and $z = \frac{any + dcmy}{cm^2}$, $\frac{z}{y} = \frac{an + dcm}{cm^2} = \sqrt{\frac{1-y^2}{y^2}}$; consequently $q = \frac{an + dcm}{cm^2} = 1{\cdot}999 = 63°\ 25'$, $\therefore B\odot A = 26°\ 35'$. Whence the lines and areas as above.

This question is very elegantly solved by Mr. *Turner* and Mr. *Farrer*, who say there's one of the same sort in *Ronayne's* Algebra, solved there two ways, but in a tedious and clumsey manner. Mr. *R. Robinson*, Mr. *T. Robinson*, Mr. *Bamfield*, Mr. *Williams*, Mr. *Chapple*, Mr. *Ramsay*, Mr. *Powle*, Mr. *Heath*, Mr. *May*, Mr. *Bird*, Mr. *Hastings*, Mr. *Kingston*, Mr. *Trott*, Mr. *Pilgrim*, Mr. *Gibbons*, and some others, have in various methods given solutions to this question.

The PRIZE QUESTION *solved by Mr.* Ant. Thacker.

Putting $x = FV$ the focal distance, and proceeding with the nature of the parabola, we get $x = 25 = FV$, $\therefore VB = 50$, and $DB = 70\cdot71$.

For any variable abscissa put x; and y for its corresponding semi-ordinate; p for the latus rectum; then will $px = yy$ express the nature of the curve DV; $\therefore \sqrt{\dot{x}^2 + \dot{y}^2} = \frac{\dot{y}}{p}\sqrt{pp + 4yy} =$ the fluxion of the arc DV; whose fluent is $\frac{y}{2p}\sqrt{pp + 4yy} + \frac{p}{4} \times \text{L.}\ \frac{2y + \sqrt{pp + 4yy}}{p} =$ the length of the variable arc, which when $y = 70\cdot71$ is $89\cdot8914 = DV$, the half of which $44\cdot9457$ is $=$ the arc VA; which call z, and put $\frac{2y + \sqrt{pp + 4yy}}{p} = h$; then will $\frac{h^4 p - p}{hh} + 4p$ L. $h = 16z$; and $\frac{\overline{hh - 1}\,^3}{h^3} \times pp = 64yx$; or if for p be put a^3, then (1st) $\frac{h^4a^3 - a^3}{h^2} + 4a^3$ L. $h = 16z$; and (2d) $\frac{hhaa - aa}{h} = 4\sqrt[3]{yx}$; whence per (1) equation $\dot{a} = \frac{2\dot{h}a + 4a\dot{h}h^2 + 2\dot{h}h^4a}{3h - 3h^5 - 12h^3\ \text{L.}\ h}$; and (2d) $\dot{a} = \frac{\dot{h}a + \dot{h}h^2a}{2h - 2h^3}$; consequently $\frac{2 + 4h^2 + 2h^4}{3 - 3h^4 - 12h^2\ \text{L.}\ h} = \frac{1 + hh}{2 - 2hh}$; hence $h^6 + h^4 - 12h^4$ L. $h - 12h^2$ L. $h - 1 = 0$; which in fluxions, and each term divided by $\dot{h}$, gives $6h^5 - 8h^3 - 4h^3$ L. $h - 24h$ L. $h - 14h = A$. And proceeding according to case 1st, p. 81 of Simpson's Essays, we get (if no mistake be made) $h = 4\cdot133926$; and from the equation $\frac{h^4p - p}{hh} + 4p$ L. $h = 16z$, is had $p = \frac{16hhz}{h^4 - 1 + 4hh\ \text{L.}\ h} = 31\cdot6$ &c. And the area $VSA = 616$ nearly.

Merones

Merones *answers it thus:*

In the parabola DV we have $2VB^2 = DB^2$, and $\frac{2}{3}VB \times DB = 1473\frac{1}{3}$; whence $DB^2 = 4999{\cdot}92$; $VB^2 = 2499{\cdot}96$.

In any parabola, where $x =$ abscissa, $y =$ ordinate, $a =$ latus rectum; the flux. curve $= \dot{y}\sqrt{1 + \frac{4yy}{aa}}$; and the curve $= \sqrt{\frac{1}{4}yy + \frac{y^4}{aa}} + \frac{1}{4}a \times \text{H. L.}\ \frac{2y}{a} + \sqrt{1 + \frac{4yy}{aa}}$; therefore $VD = 89{\cdot}8918$, and $VA = 44{\cdot}9459$. Let a parabola be such that $xy = 1$, and the curve a minimum; expunge a out of the foregoing expression of the curve, and put $z =$ H. L. of $\frac{2y}{a} + \sqrt{1 + \frac{4yy}{aa}}$; and then $\sqrt{\frac{1}{4}yy + \frac{1}{yy}} + \frac{y^3z}{4}$ is a minimum; the fluxion of this made $= 0$, there comes out $z = \frac{2}{3yy}\sqrt{1 + \frac{4}{yy}}$; or $\frac{2xx}{3}\sqrt{1 + 4x^4} =$ H. L. $2xx + \sqrt{1 + 4x^4}$; whence by reversion of series, $x = {\cdot}986408$; $y = 1{\cdot}01378$; the curve $= 1{\cdot}4787$; and in the parabola AV, where the curve is $44{\cdot}9459$; we shall have $VS = 29{\cdot}9823$; $SA = 30{\cdot}8142$; and the area $VAS = 615{\cdot}921$.

This question has exercised the faculties of a great number of persons versed in the most abstruse and higher parts of the mathematics, has occasioned a good deal of speculation and controversy; and the best of artists have been doubtful, whether it is possible to be solved by any scientific method; nor can I apprehend I have received any such solutions, unless these two above.

The latter by *Merones*, a person so profound in these sciences, that he is equal to the most arduous task; by his difference between the abscis and ordinate seems to be right, yet do not readily enough comprehend his process: And as I have long wish'd I could discover who he is, or how to direct to him; I would now heartily beg that favour, and that he would please a little further to exemplify how the expression above, $\sqrt{\frac{1}{4}yy + \frac{1}{yy}} + \frac{y^3z}{4}$ put into fluxions makes $z = \frac{2xx}{3}\sqrt{1 + 4x^4}$.

The

The proposer Mr. *Turner*, Mr. *Heath*, Mr. *Farrer*, Mr. *Dunthorne*, and several others (in their solutions) make the abscissa and ordinate equal, which though it does not much differ from the truth, yet is not really so.*

Mr.

PRIZE QUESTION

The Diary Author having as above expressed a desire of having some parts of the solution by *Merones* (Mr. *Emerson*) explained to him, and it no where appearing, that I know of, that he received any farther account of it; I think it my *duty*, as an *Editor*, to supply a particular explanation of the said solution, notwithstanding the Editor of the Repository has thought fit to pass this Question by in silence; though his declared intention at the beginning was to 'supply defects by rendering the Questions perfectly clear, and their subsequent answers as easy to be understood as the nature of the subject will admit.'

The first principle of the solution is to find the curve a minimum under a given area, instead of the area a maximum and the curve given, as in the question, which is the same thing. Mr. EMERSON assumes another curve similar to the curve required, whose area is $\frac{2}{3}$, or $xy = 1$; and if $a =$ the parameter, then $ax = y^2$; from these two equations we have $x = \frac{1}{y}$, and $a = \frac{y^2}{x} = y^3$; also $\frac{y}{a} = \frac{y}{y^3} = \frac{1}{y^2} = xx$: Now the general expression for the length of the curve being $\sqrt{\frac{1}{4}y^2 + \frac{y^4}{a^2}} + \frac{1}{4}a \times$ H. L. $\frac{2y}{a} + \sqrt{1 + \frac{4yy}{aa}}$, by writing y^3 for a in this expression, it becomes $\sqrt{\frac{1}{4}y^2 + \frac{1}{y^2}} + \frac{y^3}{4} \times$ H. L. $\frac{2}{y^2} + \sqrt{1 + \frac{4}{y^4}} =$ a minimum; the fluxion of which being found in the common way, and made $= 0$, the equation gives z or H. L. $\frac{2}{y^2} + \sqrt{1 + \frac{4}{y^4}} = \frac{2}{yy}\sqrt{1 + \frac{4}{y^4}}$; that is (putting x instead of $\frac{1}{y}$) $\frac{2xx}{3}\sqrt{1 + 4x^4} =$ H. L. $2x^2 + \sqrt{1 + 4x^4}$, which thrown into a series, and reverted, x is found; thence y, and the curve. Then, by sim. figures, he proportions thus, as the length of the curve thus found is to the length given in the question, so is x and y here found, to the abscissa and ordinate required.

	DV	VA	SA SV	SVA
Mr. *Turner*	89·89	44·945	30·39	615·7
Mr. *Heath*	89·8914	44·9457	30·3905	615·721
Mr. *Dunthorne*	———	44·94	30·39	615·7
Merones	———	44·9459	{ *SA* 30·814 *VS* 29·982 }	615·92

If any ratio could be found between the ordinates and abſciſſas, then this problem might be ſolved by a ſimple equation; but as the nature of ſuch curves do not admit of that, it ſeems impracticable.

Of the Eclipſes in 1742.

Within the ſphere of the earth's orbit will happen four eclipſes; two of the ſun, and two of the moon; but all inviſible in our iſland. The firſt of the moon, May the 8th, near noon; the ſecond of the ſun, May the 22d, at 12 h. 30′ p.m. the third of the moon, November the 1ſt, 27′ p.m. and the fourth of the ſun, November the 15th, at 18h. 16′ p.m.

New Queſtions.

I. Question 226, *by Mr.* Tho. Ramſay.

I'm in love with a damſel who has beauty in ſtore,
Beſides a large portion:—Her charms I adore;
Her eſteem and affections at length I've obtain'd,
But her father's conſent, as yet, is not gain'd,
Unleſs the conditions below I fulfil,
Which is a taſk far ſurmounting my ſkill.
He inſiſts that a method I to him make known,
How to find the content of a field of his own,
Without any more data than what he deſcribes,
I muſt find the ſaid * area, as alſo the ſides.
A triangular form is the ſhape of the ground,
The hedges as ſtraight as moſt are to be found:
Without the ſaid field grows an oak and a pine,
Betwixt which ſaid trees, if one ſuppoſe a ſtreight line,
To one of the ſides it would parallel lie,
As he ſays himſelf did accurately try.
If the field's longeſt ſide extended ſhould be,
Juſt eighteen chains farther, it would touch the oak tree:

* *The area to be a maximum.*

The

The pine in a streight line lies with the third side,
Distant exactly twelve chains, when 'twas try'd:
Moreover the distance betwixt the said trees
Is just forty chains.——And now, ladies, please
Some assistance to lend, my content it will prove:
Delays being dangerous in matters of love.

II. QUESTION 227; *by Mr.* Peter Kay.

Supposing a pendulum, whose length is 29·2 inches, and its bob or weight 27 pounds; to find at what part of the rod of that pendulum, a weight of one pound must be fixed, so as to have the greatest effect in accelerating the pendulum; or, so that the time of the vibration may be the shortest possible? The rod itself being supposed void of gravity.

III. QUESTION 228; *by Mr.* J. May.

The great inundations we have had here lately in Holland, has laid above six hundred thousand acres of land under water; and hath ruined and washed away the boundaries, that it is almost impossible again to determine each man's possessions; but, to help a friend, and prevent disputes, your assistance is desired.

He had a piece of land, *AFPVD*, which was divided into two equal parts by the right line *VF*; of which the part *ADVF* was a geometrical square; and the other part *VPF* an apollonian parabola, *V* the vertex, and *F* its focus: but all was defaced except the side *AD*; which we with some difficulty measured fifty-two chains. So that to fix the boundaries again there is required the lengths of the sides *VP* and *FP*: It is also expected that the points *V*, *F*, *P*, &c. be determined by a geometrical construction.

IV. QUESTION 229, *by* Hurlothrumbo.

Supposing an homogenous fluid, equal in density and magnitude to the earth, to revolve uniformly about an axis; so that the greatest diameter thereof may be just double the axis from pole to pole; to find the time of one entire revolution; with a general theorem for the solution of other questions of this nature.

V. QUES-

V. QUESTION 230, *by Mr.* John Turner.

Suppose the bung diameter of a spheroidal cask were 40 inches, and its diagonal 48 inches; it is required to find the head diameter of the least spheroidal cask possible, having the abovesaid dimension; and its content in ale gallons?

VI. QUESTION 231, *by Mr.* N. Farrer.

On the 20th of December, 1740, at ten minutes past eight at night, I observed two noted fixed stars on the meridian; the difference of their altitudes thirty-seven degrees; and at ten o'clock I found their azimuths 31° 40′ and 61° 45′, both west, and their difference of altitudes 32°. Quere the latitude of the place of observation?

VII. QUESTION 232, *by Mr.* Ant. Thacker.

If $x^3 + y^3 - 945yx = 0$ express the nature of the curve AM, and 150400 be the area of the space AMP; it is required to find the area of the greatest parallelogram that can be inscribed in the said figure AMP?

VIII. QUESTION 233, *by Mr.* Rich. Piercy.

If $x^n \times \overline{n-x}|^x = y$; when n is $= 1000$, required the value of x, y being the greatest number that possibly can be.

IX. QUESTION 234.

A father at his death bequeathed to his daughters these portions, viz.

To the eldest he gave $\cdot 2^2$ of 1000 pounds.
To the second $\cdot 3^3$ of 1000 pounds.
To the youngest $\cdot 4^4$ of 1000 pounds.
How much was each daughters's portion?

The PRIZE QUESTION, *by Mr.* Heath.

Vincit amor patriæ.——

Where, with his fleet, our noble patriot sails,
Success o'er all his conduct still prevails;
Intent on public wrongs, he stems the tide
Of Spain's oppression, insolence, and pride:

Brave,

Brave, generous-minded, not to be withstood,
A hero conqu'ring for his country's good.
Nor could vain boasts their Porto Bella save,
He took it, as he was resolv'd to have:
His thund'ring cannon rend the liquid sky,
Hot iron balls the iron castle try,
And storms of ruin round the harbour fly.
Pour'd on the land, behold each honest tar,
With sword in hand, ascending to the war,
Their country's wrongs are boiling in each breast,
Bold in revenge, dauntless, all forward press'd.
The Spaniards, aw'd, and in a deadly fright,
Shrink back for refuge to the woods in flight,
And leave the English masters of the fight,
While loud huzzas from deck and shore resound,
And the glad victor with applause is crown'd!
Th' obscuring smoak uncurls itself in air,
And British heroes bright in arms appear!
The conqu'ror now, as wisdom does approve,
Divides his treasure, and withal his love:
With love and joy each British bosom burns,
And in new conquests all express returns.
Chagre is conquer'd——America does shake,
Spain for her wrongs must restitution make;
From shore to shore the victor bears command,
And Vernon! Vernon! rings through all the land!
Whose actions an immortal record claim,
Amongst his shining ancestors of fame.
Late * news I've heard, ladies, I wish it true,
And as your hearts are there, so must all you.

The Question.

* Admiral Vernon sailing on a south course, from Jamaica to Carthagena, sees *Don Blass* right before him, steering due west, along the shore. He now continually bears directly upon him, in a right line; when coming up with him, it appears that the *Don* had sailed 8 leagues during the chace, and that the said admiral was 7 leagues distant from him when the chace began: Now, supposing each ship's motion to be uniform during the whole chace, to find from thence the distance sailed by Admiral Vernon.

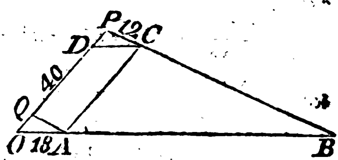
Questions

1743.

Questions answered.

I. QUESTION 226 *answered by* Hurlothrumbo.

LET AQ be parallel to CP; $OP = a = 40$; $OA = b = 18$; $AQ = (PC) = c = 12$, and $OQ = x$. Then the area of the $\Delta\ AOQ$ is $\sqrt{\frac{2b^2c^2 + 2b^2x^2 + 2c^2x^2 - b^4 - c^4 - x^4}{16}}$; therefore, as $xx\ (OQ^2) : \overline{a-x}|^2\ (AC^2) ::$ the said area : the area of the $\Delta\ ABC$, which, by the question, is a maximum; $\therefore \frac{\overline{a-x}|^4}{x^4} \times :$ $2b^2c^2 + 2b^2x^2 + 2c^2x^2 - b^4 - c^4 - x^4$ is also a maximum: Whence $a \times \overline{bb-cc}|^2 - \overline{bb+cc} \times ax^2 - \overline{bb+cc} \times x^3 + x^5 = 0$: from which the value of x is found $= 7{\cdot}6986$; $AC = 32{\cdot}3014$; $BC = 50{\cdot}3473$; $AB = 75{\cdot}52$; and the area 61 a. 2 r. 9 p.



Merones's *Answer.*

Draw DC parallel to AB; let $PC = b$, $OA = c$, $PO = a$, $PD = x$. Then $BC = b \times \frac{a-x}{x}$, $AB = c \times \frac{a-x}{x}$, $AC = x \times \frac{a-x}{x} = a - x$; and let $s = cc - bb$, $t = cc + bb$: Then, by the rule for finding the area from the three sides, $\frac{1}{4}\sqrt{\frac{-ss + 2txx - x^4}{x^4}} \times \overline{a-x}|^2 =$ area a max. this in fluxions, and reduced, gives $x^5 - tx^3 - atx^2 + ass = 0$, and $x = 7{\cdot}698$, and the area $= 615{\cdot}57$.

This question is truly solved by the proposer Mr. *T. Ramsay*, Mr. *Ant. Thacker*, Mr. *Hemmingway*, Mr. *Farrer*, and *Tho. Cowper*, by a process much alike. But as I was at some

some trouble in solving it in a method different from any of them, before the receipt of theirs, I will, for variety's sake, here give it, though I don't think it at all better.

$AQ = PC = 12$; $AO = 18$; $OQ = 2x$. The sum of the three sides is $30 + 2x$, half sum $= 15 + x$, the difference between half sum and three sides are $3 + x$, $x - 3$, and $15 - x$; then $\sqrt{\overline{15+x} \times \overline{15-x} \times \overline{x+3} \times \overline{x-3}}$, i. e. $\sqrt{234x^2 - x^4 - 2025} = z$, the area of OAQ. But as $xx : z :: \overline{20-x}|^2 : z \times \frac{\overline{20-x}|^2}{xx} =$ area of the $\triangle ABC =$ a maximum; which in fluxions is $\dot{z} \times \overline{20-x}|^2 - 2x\dot{x}z \times \overline{20-x} \times xx - 2x\dot{x}z \times \overline{20-x}|^2 = 0$; which divided by $x \times \overline{20-x}$, gives $\dot{z} \times \overline{20-x} - 2\dot{x}z \times x - 2\dot{x}z \times \overline{20-x} = 0$; or $20\dot{z}x - \dot{z}xx - 40\dot{x}z = 0$; $\therefore 20\dot{z}x - \dot{z}xx = 40\dot{x}z$; $\therefore \dot{z} = \frac{40\dot{x}z}{20x - xx}$; but $234x^2 - x^4 - 2025 = z^2$; in fluxions, &c. $\frac{234x\dot{x} - 2\dot{x}x^3}{z} = \dot{z}$; consequently $= \frac{40\dot{x}z}{20x - xx} = \frac{234x\dot{x} - 2\dot{x}x^3}{z}$; $\div$d by $2\dot{x}$ gives $\frac{20z}{20x - xx} = \frac{117x - x^3}{z}$; $\therefore 20zz = \overline{117x - x^3} \times \overline{20x - x^2}$, or $20 \times \overline{234x^2 - x^4 - 2025} = \overline{117x - x^3} \times \overline{20x - x^2}$; in numbers is $4680x^2 - 20x^4 - 40500 = 2340x^2 - 20x^4 - 117x^3 + x^5$; ordered, is $x^5 - 117x^3 - 2340x^2 + 40500 = 0$; from whence x is found $= 3{\cdot}8493$, and $2x = 7{\cdot}6986 = OQ$.

II. QUESTION 227 *answered by* Mr. Kay.

Put $a =$ the length of the pendulum $= 29{\cdot}2$ inches; $w =$ weight of the bob 27 pounds; $v =$ the weight to be fixed to the rod $= 1$ pound; and $x =$ the required distance thereof from the point of suspension; then the distance of the center of oscillation from that point will be $\frac{aaw + xxv}{aw + xv}$, which by the question ought to be a minimum; and therefore $2x\dot{x}v \times \overline{aw + xv} - v\dot{x} \times \overline{aaw + xxv} = 0$, whence x is $= a \times \frac{\sqrt{ww + wv} - w}{v} = 14{\cdot}4686$ inches.

Mr.

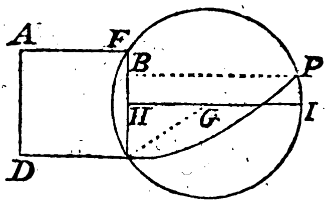
Mr. J. Watts.

The momenta of all moving bodies are as a rectangle of their celerity and mass, and the celerity of a pendulum as its length, &c. Then the center of oscillation of any compound pendulum (by Colin Mac Laurin's Fluxions, lib. 2, p. 453) is equal to the squares of the distances multiplied into their respective weights, and their aggregate divided by the sum of the momenta; therefore [the answer as above].

This question was answered by Mr. *Ramsay*, Mr. *Farren*, Mr. *May*, Mr. *Sharp*, and *Hurlothrumbo*.

III. QUESTION 228 *answered by* Hurlothrumbo

Upon the given line AD describe the square $ADVF$, and from the focus F, and vertex V, the parabola VP; bisect FV with the perpendicular HG, and take $HG = \frac{1}{4} AD$, and from the center G with the radius GV describe the circle VPF, and from the point P, where it cuts the parabola, draw FP, then will



$ADVPFA$ (by book 1 prop. 30 Newton's Princip.) be the figure required.

[*In this fig. a line drawn from* F *to* P *is omitted, which the reader may supply.*]

Answered by Mr. Farrer.

Let $m = VF = 52$ chains, $x = VB$; then $m - x = BF$, and, per conics, $PB = 2\sqrt{mx}$, and the area of $VPF = \frac{4x}{3}\sqrt{mx} + \overline{m-x}\sqrt{mx} = mm$; this equation reduced is $x^3 + 6mx^2 + 9m^2x - 9m^3 = 0$; hence $x = 34{\cdot}77163$, $BP = 85{\cdot}044$, $FP = 86{\cdot}77157$, and the arch $VP = 93{\cdot}7442$ chains. And the points V, P, and F, will be determined by the following construction.

Let $HG = \frac{1}{4}$ of VF bisect at right angles the side of the square, and on the center G, with GF describe the circle, which will cut the curve in P.

Mr. J. May *the Proposer,*

After an analytic answer, gives this geometrical construction: Having $xx = 4ay$ from the property of the parabola, and $yx = 6aa - 3ax$; then $yxx = 4ayy$, and $yxx = 6aax - 3axx$, whence $4ayy = 6aax - 3axx$, divided by $4a$ is $yy = \frac{3}{2}ax - \frac{3}{4}xx$, for $\frac{3}{4}xx$ put its value $3ay$, we have $yy \frac{3}{2}ax - 3ay$, to which add $xx = 4ay$, is $yy + xx = \frac{3}{2}ax + ay$ an equation to a circle, and $y = \frac{1}{2}a \pm \sqrt{\frac{1}{4}aa + \frac{3}{2}ax}$; put the surd $= 0$, then is $y = \frac{1}{2}a$; bisect FV in H, and draw HGI perpendicular to VF, in which the center of the circle will be; then $x = \frac{3}{4}a \mp \sqrt{\frac{13}{16}aa}$; make $HG = \frac{3}{4}a$; draw VG, this is $\sqrt{\frac{13}{16}aa}$ and the center is G; put $x = 0$, then $yy = ay + 0$, and $y = \frac{1}{2}a \mp \frac{1}{2}a$, or $y = 0$, or $= a$, then V, F are two points in the periphery. On the center G with GV, draw the circle which cuts the given parabola in the required points.

Mr. *Powle*, Mr. *Hemmingway*, Mr. *Ramsay*, Mr. *Watts*, *Teague of Exeter*, and others answered this, (which is in Sir Isaac Newton's Princ. prop. 30.)

IV. QUESTION 229 *answered by the Proposer.*

Let r be the time wherein a body would describe a circle about the earth, just above the surface, by means of its own gravity $=$ 1h. 24′ 45″, and let a be the arch of a circle whose radius is 1, and secant n, supposing the given ratio of the equatorial diameter to the axis be as n to 1. Then by Simpson's Mathematical Dissertations, we shall have $r \times \sqrt{\frac{2n^2 - 2 \times \sqrt{nn-1}}{nn+2 \times 3a - 9\sqrt{nn-1}}}$ for the exact time of one entire revolution; which therefore, when $n = 2$, or the equatorial diameter is just double the axis, will be 2h. 31′ 20″.

Merones's *Answer.*

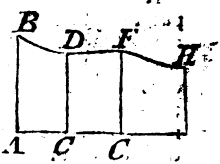
Let $r =$ earth's radius, $f = 16\frac{1}{12}$ feet. That the equinoctial diameter may be double the polar one, the centrifugal force must take away half the gravity; to do which, any point in the equinoctial must in 1″ describe the arch whose versed sine is $\frac{1}{2}f$; or, which is the same thing, the arch itself will be $\sqrt{rf}$; and the periodic time $3{\cdot}1416\sqrt{\frac{4r}{f}} =$

$= 7147'' = 2$ h. fere; and therefore the body will revolve 12 times as fast as our earth. But to give an accurate solution to this problem, the decrease of gravity arising from the earth's figure, ought to be taken into consideration; but this would render the calculus very intricate, and too long for this place.

V. QUESTION 230 *answered by* Hurlothrumbo.

Put the diagonal $= 48 = a$, half the bung diameter $= 20 = b$, and half the head diameter $= x$; then the content will be $\frac{2bb+xx}{1077} \times 8\sqrt{aa-\overline{b+x}^2}$ ale gallons, which by the question is to be a minimum. Now by making the fluxion of $\frac{2bb+xx}{1077} \times 8\sqrt{aa-\overline{b+x}^2}$, or that of $\overline{2bb+xx}^2 \times \overline{aa-\overline{b+x}^2} = 0$; we get $2a^2x - 2b^3 - 4b^2x - 5bx^2 - 3x^3 = 0$; that is, in numbers $-16000 + 3008x - 100x^2 - 3x^3 = 0$; whence $x = 7{\cdot}85$, or $x = 12{\cdot}63$, or $x = -53{\cdot}8$, but none of their roots is the required value of x.

For let $BDFH$ be a curve, whose abscissa AC is x, and ordinate $\overline{2b^2+x^2}^2 \times \overline{aa-\overline{b+x}^2}$, and it is evident that when the ordinate of this curve is a minimum, the cask will be so too; but it appears, from what has been found above, that the ordinate CD grows less and less, till x or AC becomes $7{\cdot}85$ (because till then its fluxion, or $-16000 - 3008x - 100x^2 - 3x^3$, is a negative quantity) after which it increases till x becomes $= 12{\cdot}63$ (the fluxion being affirmative) and then decreases again continually, till x arrives at 20, its greatest value; in which circumstance it will be less than in any former position, as will easily appear upon trial; therefore the head diameter is equal to the bung diameter indefinitely near, and the required content $236{\cdot}468$ ale gallons.



Mr. N. Farrer's *Answer.*

If $m = 40$, $n = 48$, $y =$ head diameter; then $\frac{m+y}{2}$ is the base of the right-angled triangle, and (per 47 Euc. 1) $\sqrt{\frac{4nn-mm-2my-yy}{4}} =$ semi-length. Let $4nn-mm$

$= d$; then its length is $\sqrt{d - 2my - yy}$, and its solidity $= \overline{2mm + yy} \times \sqrt{d - 2my - yy} \times .2618$; whose fluxion

$$2\dot{y}y \times \sqrt{d - 2my - yy} + \overline{2mm + yy} \times \frac{-m\dot{y} - y\dot{y}}{\sqrt{d - 2my - yy}}$$

$= 0$. Reduced, $3y^3 + 5my^2 + 2mmy - 2dy + 2m^3 = 0$. Hence $y = 15.701465$ = head diameter, 78.187894 = length; 250.174 ale gall. the content. And farther observes,

1. The cask will be greatest when it becomes a spheroid, (whose length is then 87.266 inches, and content 259.28 a. g.) and least when a cylinder (length 53.066, content 236.46 a. g.); therefore the question, properly speaking, does not admit of either maximum or minimum, for the least spheroidal cask will be infinitely near the cylinder, and greatest near the spheroid. 2. If the length be 76, the content is 252.18 a. g. If 84 inches long, the content is 252.85, between these two the least is that found above. 3. Between this least and the cylinder, there is another, whose capacity is a maximum.

Mr. Hemingway,

After his answer, remarks, That when any expression is put into fluxions, the roots of the equation determine so many limits of increase and decrease of the flowing quantity; whence if there's but one root which answers the conditions of the problem, we obtain a maximum or minimum; but if there be more than one, the limits those roots exhibit may not determine either. In the present case, if the semi-diff. of diam. $= x$ increases gradually from its first value 7.3749, till it arrives at its second value 12.149, the cask continually decreases from 250.87 to 250.174; but x still increasing, the cask will increase again, till it becomes a whole spheroid, whose content is 259.259 a. g. Again, let x gradually decrease from its first value, and the cask will decrease likewise, till it becomes a cylinder, whose length is 53.066 inches, and content 236.47, which is the minimum, and may be considered as the middle frustum of a spheroid of an infinite length.

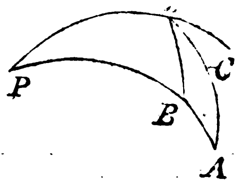
This question was answered by Mr. *Watts*, Mr. *Bamfield*, Mr. *Richardson*, Mr. *Ramsay*, Mr. *Wright*, Mr. *Robinson*, Mr. *Oats*, Mr. *Ant. Thacker*, and others.

VI. QUES-

VI. QUESTION 231 *solved.*

This question happened to be over-limited, the latter given time of observation being superfluous, for it will not agree with the rest of the data; (the hour angle may be only 26° 30′, and the times of observation 14 m. after 8 and 10 o'clock).

Given $\begin{cases} AB = 37° = \text{1st diff. alt.} \\ AZ - BZ = 32° = \text{2d diff. alt.} \\ AZB = 30°\ 5' = \text{diff. azim.} \\ BZP = 118°\ 15' = \text{zenith } \angle. \\ ZPB = 26°\ 30' = \text{hour } \angle. \end{cases}$



To find $AZ + BZ$ and PZ.

Put x and y for the sine and cosine of half the difference of AZ and BZ, s and q for those of half their sum. Then, by our new theorem, $sy + qx$ and $qy - sx$ is the sine and cosine of ZA, and $sy - qx$ and $qy + sx$ those of ZB; also let z represent the cosine of AZB, and d the cosine of AB. Then, by 1st theorem, $\overline{qy - sx} \times \overline{qy + sx} + \overline{sy + qx} \times \overline{sy - qx} \times z = d$; which when multiplied is $qqyy - ssxx + zssyy - zqqxx = d$, or $dqq + dss$; being transposed, $zssyy - ssxx - dss = dqq + zqqxx - qqyy$, $\therefore \frac{zyy - xx - d}{d + xxz - yy} = \frac{qq}{ss}$; to which add 1, its $\frac{zyy - xx - d}{d + zxx - yy} + 1 = \frac{qq}{ss} + 1 = \frac{qq + ss}{ss}$, or $\frac{z - 1}{d + zxx - yy} = \frac{1}{ss}$; $\therefore ss = \frac{d + zxx - yy}{z - 1}$, or $\frac{yy - d - xxz}{1 - z}$; $\therefore 2ss = \frac{2yy - 2d - 2xxz}{1 - z}$ = the versed sine of $AZ + BZ$.

Operation.

74°	16
74	16
148	32 Sum
Whose ver. s. is 1·8480481 = 2yy	Whose vers. s. is ·1519519 = 2xx
Sub. 2d + 2xxz = 1·7287543	multip. by z = ·8652972
2yy - 2d - 2xxz = ·1192934 rem.	gives 2xxz = ·13148337
this ÷ d by 1-z = ·1337028 gives	add 2d = 1·5972710
the quotient = ·8856064.	Sum 2d + 2xxz = 1·7287543

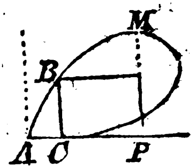
which gives = 83° 25′ 53″ = $AZ + BZ$, the half of which 41° 42′ 56″ added to half the diff. 16°, gives 57° 42′ 56″ = AZ the greater side, and subtracted, leaves 25° 42′ 56″ = BZ

BZ the lesser side; then as s. 37° : s. 30° 5' :: s. 57° 42' 56'' : s. 44° 44' $= PBZ$; again, as s. 26° 30' : s. 25° 42' 56' :: s. 44° 44' : s. 43° 13' $= PZ$ the complement of the latitude, ∴ 36° 47' is the latitude required.

This question has been truly answered by Mr. *W. Daniel*, Mr. *Jos. Spilsbury* of *Birmingham*, Mr. *John Worth*, and Mr. *Francis Parrot*.

VII. QUESTION 225 *answered by* Hurlothrumbo.

Put $a = 945$, $b = 150400$, $PM = y$, $AP = x = vy$; then, by writing vy instead of x in the equation of the curve, &c. we shall have $y = \frac{av}{1+v^3}$, $x = \frac{av^2}{1+v^3}$, and $y\dot{x} = \frac{2a^2v^2\dot{v} - a^2v^5\dot{v}}{\overline{1+v^3}|^3}$; whose fluent is $\frac{a^2}{6} - \frac{a^2}{2\times\overline{1+v^3}|^3} + \frac{a^3}{3\times\overline{1+v^3}}$, which,



by restoring x and y, will be $\frac{aa}{6} + \frac{xy}{2} - \frac{ayy}{6x}$ or $\frac{xy}{2} + \frac{axx}{6y}$.

Now by putting $z = 1 + v^3$, $\frac{aa}{6} - \frac{aa}{2\times\overline{1+v^3}|^2} + \frac{aa}{3\times\overline{1+v^3}} = b$, and $c = \frac{6b}{aa} - 1$, we shall get $z = \frac{1 - \sqrt{1 - 4c}}{c} = 1{,}512$; whence $v = 0{,}8$, $y = 500 = PM$, and $x = 400 = AP$. Hence the area of the greatest inscribed parallelogram *PB*, will be found $= 92133$.

Note, This curve, having its equation affected alike by x and y, returns into itself, and has its convex and concave parts exactly similar to each other.

VIII. QUESTION 233 *answered by Mr.* J. Watts.

Since $x^x \times \overline{n-x}|^x$ is to be a maximum, its h. l. $n \times$ l. $x +$ $x \times$ l. $\overline{n-x} =$ max. in fluxions $\frac{n\dot{x}}{x} + \dot{x} \times$ l. $\overline{n-x} - \frac{x\dot{x}}{n-x} = 0$; which divided by $\dot{x}$ is $\frac{n}{x} +$ l. $\overline{n-x} - \frac{x}{n-x} = 0$, i.e. $\frac{1000}{x} +$ l. $\overline{1000-x} - \frac{x}{1000-x} = 0$. By trials it is found x must

must be greater than 800, ∴ put $800+z=x$, which substituted above, gives $\frac{1000}{800+z}+1.\overline{1000-800-z}-\frac{800+z}{1000-800-z}=0$; contracted, it is $\frac{1000}{800+z}+1.\overline{200-z}-\frac{800+z}{200-z}=0$; out of fractions, is $200000-1000z+\overline{160000-600z-zz}\times 1.\overline{200-z}-\overline{640000-1600z-zz}=0$; or $-440000-2600z-zz+\overline{160000-600z-zz}\times 1.\overline{200-z}=0$; signs changed, $440000+2600z+zz-\overline{160000-600z-zz}\times 1.\overline{200-z}=0$; ordered by infinite series, is $440000+2600z+zz-\overline{160000+600z+zz}\times:1.200,-\frac{z}{200}-\frac{z^2}{80000}-\frac{z^3}{24000000}-\frac{z^4}{6400000000}$, &c. $=0$; or finding the hyp. log. of 200, viz. 5·2983, &c. its $440000+2600z+zz-\overline{160000-600z-zz}\times:5{\cdot}2983-\frac{z}{200}-\frac{z^2}{8000}-\frac{z^3}{24000000}$, &c. $=0$; that is, $-407728+6578{\cdot}98z+5{\cdot}2983z^2-{\cdot}005833z^3-{\cdot}0000125z^4=0$; or $6578{\cdot}98z+5{\cdot}2983z^2-{\cdot}005833z^3-{\cdot}0000125z^4=407728$; which divided by the coefficient of z, (per Simpson's Flux. p. 101) gives $z+{\cdot}0008054z^2-0000008z^3-{\cdot}000000001z^4=61{\cdot}9742$, compared with the series, in the same page, we have $b={\cdot}0008054$, $c={\cdot}0000008$, $d={\cdot}000000001$, &c. and $z=61{\cdot}9742$ which call y, then will $z=y-by^2+\overline{2bb-c}\times y^3$ &c. $=59{\cdot}37973$, which added to 800, gives $x=859{\cdot}37973$. *Q.E.I.*

IX. QUESTION 234 *answered by Mr.* J. Watts.

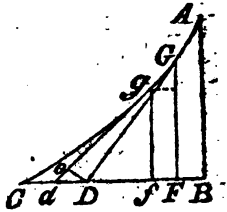
Subtract the log. of $2=0{\cdot}3010300$ from the log. of $10=1{\cdot}0000000$, the remainder is 0·69897000, which multiplied by 2, is ·1397940 = the log. of $\cdot 2^{\cdot 2}$, which subtract from unity, and the difference is ·8602060, which sought in the logarithm tables, gives the absolute number answering ·72478. And proceeding in like manner with $\cdot 3^{\cdot 3}$ and $\cdot 4^{\cdot 4}$, we shall have the eldest daughter's portion 724·78 = 724l. 16s 4d. the 2d = 696·84 = 696l. 16s. 11d. and the youngest = 693·14 = 693l. 2s. 8d.

This question was also answered by *Hurlothrumbo*, Mr. *Powle*, Mr. *Scyth*, Mr. *Ramsay*, Mr. *Farrer*, Mr. *Bamfield*, Mr. *May*, Mr. *E. Ruston*, Mr. *W. Miles*, and several others.

The

The PRIZE QUESTION *answered by* Merones.

Let $CB = b = 8$, $AB = c = 7$; let G, g be two points in the curve infinitely near each other, draw GF and gf parallel to AB, and to the points G, g draw the tangents GD, gd, and Do perpendicular to gd. Let $BD = x$, $BF = v$, $GD = y$, $AG = z$, and $z = ax$, and consequently $\dot{z} = a\dot{x}$, where a is a constant, but unknown, quantity.



By sim. triangles $\dot{z} : \dot{v} :: \dot{x} \left(= \frac{\dot{z}}{a}\right) : do = \frac{\dot{v}}{a}$; then y is increased by $d\dot{o}$, and decreased by $G g$, that is, $\dot{y} = \frac{\dot{v}}{a} - \dot{z}$; or $\dot{z} = -\dot{y} + \frac{\dot{v}}{a}$; and the fluent is $z = c - y + \frac{v}{a} = ax$; and when they arrive at C, $y = 0$, and $v = x = b$; whence $c + \frac{b}{a} = ab$; and $a = c + \frac{\sqrt{cc + 4bb}}{2b} = 1{\cdot}529$; and therefore the whole curve $z = c + \frac{b}{a} = 12{\cdot}2321$ leagues the distance sailed by Admiral Vernon.

Mr. Powle

Takes notice that this question is a particular case of prob. 8 p. 170 Simpson's Fluxions. And if $\frac{1}{x}$ be the ratio of the velocities of the two ships, and $AB = a$, and $BC = b$; then from the nature of the curve we have $\frac{ax}{1 - xx} = BC$, and $\frac{a}{1 - xx} = AC$ sought: Now the equation $\frac{ax}{1 - xx} = b$ will give $x = \frac{\sqrt{4bb + aa}}{2b} - \frac{a}{2b}$, $\therefore \frac{ax}{1 - xx} = \frac{2bb}{\sqrt{4bb + aa} - a} = 12{\cdot}2324 = AC$ required.

Mr. *Ant. Thacker*, Mr. *Scyth*, Mr. *Cross*, Mr. *Ramsay*, Mr. *Farrer*, Mr. *Moore*, Mr. *Watts*, Mr. *May*, Mr. *Robinson*, Mr. *W. Hanbury*, jun. Mr. *Jo. Spilsbury*, and several others have answered this question.

Of

Of the Eclipſes in 1743.

Within the ſphere of the earth's orbit will happen ſix eclipſes this year; three times will the moon, in her wandering courſe, interpoſe and hide the ſplendour of the ſun from falling upon the earth, or its atmoſphere; and three times will the earth in its annual elliptic motion, be ſo full in a line between the ſun and moon, as to hinder her receiving the light ſhe borrows from the ſun, to enlighten the earth by reflection.

1. Sun ecliped April 13, at 10 morning, inviſible.

2. Moon eclipſed April 27, inviſible.

3. Sun eclipſed May 12, at 5 morning, inviſible.

4. Moon eclipſed on thurſday the 22d day of October, at a quarter paſt 3 in the morning, total and viſible.

Calculated by	Beg. h. m.	Mid. h. m.	End h. m.	Dur. h. m.	Dig.
Aſtron. Carolina, Coventry	1 31	3 17	5 3	3 32	22 0
An. Amanuenſis, London	1 48	3 54	5 29	3 30	[illegible]
Jo. Taylor, Snaith, Yorkſhire	0 34	3 25	5 15	3 42	22 19
J. Williams, Whitford, Flint.	1 10	3 0	4 52	3 42	22 0
P. Timkin, London	1 29	3 17	5 6	3 36	
Ben. Claridge, Warwick	1 29	3 16	5 2	3 34	21 44
Will. Wollams, Oxford	1 24	3 10	4 57	3 33	20 14
John Thornet, Glouceſter	1 30	3 16	5 3	3 33	20 15
T. Cowper, Wellingborough	1 18	3 5	4 52	3 34	21 25
S. Bamfield, Honiton, Devon.	1 16	3 2	4 48	3 32	21 42
Ni. Farrer, Sunderland	1 18	3 6	4 53		
Ni. Farrer, Alexandria	3 23	5 11	6 5	3 35	21 51
Ni. Farrer, Cape Farewell	10 33	12 21	2 8		
W. Honor, Burcheſter	1 19	3 8	4 54	3 34	21 50
W. Honor, Ruſhden	1 23	3 10	4 58		
John Brown, Bp. Auckland	1 30	3 18	5 5	3 35	21 50
C. Facer, Benſon	1 26	3 13	4 59		22 [illegible]
Tho. Wade, London	1 23	3 11	4 58	3 35	21 50
Ens Rationalis, Newcaſtle	1 14	3 1	4 48	3 34	21 34
M. Rafton, Hartlepool	1 20	3 7	4 55	3 35	21 57
T. Robinſon, London	1 23	3 11	4 58		
T. Robinſon, Morton Hill	1 18	3 6	4 53	3 35	22 8
Nic. Oats, Topſham	1 14	3 4	4 52	3 38	21 54
Teague of Oxford, Greenwich	1 23		4 58		

The beginning of total darkneſs 2. 28. End 4. 6.

The other two eclipſes are very inconſiderable.

The

The passage of Mercury over the Sun, October 25.*

	Beg.	Mid.	End	Dur.
Joseph Taylor	9 25	11 37	1 43	4 18
Peter Timkin, for London	11 6	0 11	1 16	2 9
T. Cowper, Wellingborough	8 36	10 48	1 0¼	4 23
Annus Amanuensis, London	8 54	11 40	2 30	4 23
T. Wade, London	8 35	10 51	1 7	4 32
Ens Rationalis, Newcastle	8 23	10 40	0 56	4 33

New

* *This* TRANSIT *of* MERCURY

Was observed at *London* by Dr. BEVIS thus:

Temp. Ap.	h.	m.	s.	
Oct. 24.	20	28	57	The 1st exterior contact.
		29	48½	The ingress of the center.
		30	40	The 1st inter. contact.
25.	1	0	33	The last inter. contact.
		1	24½	The egress of the center.
		2	16	The last ext. contact.

Also at *London* by Mr. GEO. GRAHAM.

1	0	42	The last inter. contact.
	2	16	The last ext. contact.

And at *Giessen* by Mr. CHRIST. LEWES GERSTEN.

h.	m.	True time.
9	4	Central ingress.
1	37	Central egress.

A COMET

Was observed, of a remarkable size, in the month of December this year, and in the months of Jan. and Feb. the year following. From the observations Mr. JOS. BETTS determined that the place of the ascending node was in ♉ 15° 45′ 20″; the log. of the perihelion dist. 9·346472; the log. of the diurnal motion 0·940920; the place of the perihelion ♎ 17° 12′ 55″; the distance of the perihelion from the node 151° 27′ 35″; the log. sine and cosine of the inclination of the orbit to the ecliptic 9·865138 and 9·832616: and thence the time the comet was in the vertex of the parabola, or the time of the perihelion, Feb. 19 d. 8 h. 12 m. the motion of the comet in its orbit thus situated, was direct, or according to the order of the signs. This amazing comet was not less than the earth;

New Questions.

I. QUESTION 235, *by Mr.* Thomas Cowper.

When that immenſe amazing orb of day
Gilded the northern tropic with his ray,
From eaſtern ſkies emitting lucid ſtreams,
And ſpread his radiance in prolific beams;
In pleaſing ſolitude I did repair
To view the fields, and breathe in purer air;
Where joyous birds ſtretch'd forth their tuneful throats,
And pierc'd the yielding air with thrilling notes:
On verdant ſprays the thruſh and blackbird ſings,
While warbling larks diſplay'd aërial wings;

Artful

---

earth; and travelled with the wonderful velocity of 13 millions of miles in a day, or 9 thouſand miles in a minute; but when in its perihelion the rapidity of its motion exceeded that of lightning itſelf; and drew after it a tail of fire of above 16 degrees or 23 millions of miles in length.

Another COMET

Alſo appeared in the beginning of this year, but I do not meet with any determination of its elements.

A COMET

Was obſerved in the ſpring of the year 1742 by ſeveral perſons.

From the obſervations made at *Pekin* in *China*, Mr. JAMES HODGSON computed theſe following articles: It is manifeſt, he ſays, that the comet came to the equator March 3d about 6 h. *ant. merid.* and that it paſſed in r. aſc. 282° 30′, with inclination of its path to the equator 84° 30′ very nearly; and therefore that its longitude was 13° 35′ in ♑ with N. lat. 22° 54′. Hence we may collect, that the path of the comet, which did not ſeem to deviate from a great circle, met the ecliptic in ♑ and ♋ 9° 19′ with incl. of 80°: and the colure of the equinoxes in the diſtance of 50° 37½′ from the poles of the world toward the equinoctial points with the angle of incl. 77° 33½′: and the colure of the ſolſtices in the diſt. of 23° 57½′ from the poles of the world toward the ſolſticial points with ang. of incl. 13° 38′ equal to the greateſt elongation of its orbit from the ſame colure in the adverſe part, and to the diſt. of the poles of the orbit from the equinoctial points.

[*This Note was by miſtake omitted in its proper place laſt year.*]

Artful aſcending through the elaſtic way,
And hail'd triumphant the ſolſtitial day.
Each object round a grateful ſcene did yield,
While teeming plenty crown'd the blooming field,
And beauteous verdure deck'd the enamell'd plain,
And pearly dews hung on the rip'ning grain;
The fragrant flowers with diff'rent colours dy'd,
On ſmiling ground diſplay'd their gaudy pride:
The air with light effluvia did abound,
Which ſpread their aromatic odours round:
The winds were huſh'd, and zephyr's gentle breeze
Scarce heard to murmur through the ſhady trees.
To one of them a bough did appertain,
Whoſe ſhadow I obſerv'd upon the plain;
And found its diſtance from the bough to be,
One hundred inches wanting only three.
As towards the ſouth bright Phœbus journied on,
And on the earth with greater luſtre ſhone,
While up the ſkies progreſſive time he led,
Seventy-ſix minutes and one-fifth were fled.
It happen'd that another branch I found,
Whoſe altitude above the level ground
Was thirteen inches and juſt eight-tenths more
Than that which I obſerv'd the time before;
Yet, as the firſt, its diſtance from the ſhade,
No more than ninety-ſeven inches made;
And th' baſe was ſhorter at this ſecond view,
By one foot and ſix-tenths of one inch too.
Now, from this data, 'tis requir'd to ſhow
Their height*, the time, and latitude alſo?

* i. e. The perpendicular height of the end of each bough, from which the ſhadows were projected and meaſured.

N. B. That 17 minutes muſt be deducted from the ſun's altitude at the time of the firſt obſervation, and 16½ at the ſecond, as an allowance for his ſemidiameter and refraction.

II. QUESTION 236, *by* Hurlothrumbo.

If two bodies, *L* and *T*, whoſe maſſes are reſpectively equal to thoſe of the moon and earth, were projected at the ſame time, and in the ſame plane, from two places, *A* and *B*, at the diſtance of a hundred thouſand miles from each other; the former, *L*, with a velocity of 5 miles per ſecond, making an angle with *AB* of 100 degrees, and the latter with a velocity of 2 miles per ſecond, an angle (on the ſame ſide *AB*) of 60 degrees; it is required to find the diſtance and poſition of the two bodies with reſpect to each other,

alſo

also with respect to the points A and B, after they have been 48 hours in motion, supposing them, when in motion, to be only acted upon by each other.

III. QUESTION 237, *by Mr.* Nich. Oats.

—— *Hic validum Vernon cano magnâ sorte juvanti.*

Free from domestic broils and homeward jars,
The hero rushes in the din of wars;
On proud Iberia's coast, in dread alarms,
Appears the fury of Britannia's arms.
Each honest heart with indignation burns,
And long call'd vengeance on the insulter turns.
The elements are mix'd, with fire the main;
Destruction's hurl'd through the indignant plain:
The watchful squadrons triumph o'er the deep,
Whilst thunder-struck the dastard tribes can't peep,
Nor scarce a petty-auger from her moorings creep.
Him conquest still attend, and let your songs
Blazon his triumphs, num'rous as our wrongs:
So shall our ships in distant climates roam,
And bring the wealth of Peru's Indies home.
Then let each heart with gratitude be fraught,
Each sailor love, who for his country fought.
Ladies, ere long (believe the muses true)
Display your wit, the conq'rors you'll subdue.

The QUESTION.

A fleet of ships at Portsmouth is bound with military stores, &c. for our brave admiral in the West Indies; and being informed, by experienced navigators, that a ship, in sailing upon a wind, having her larboard tacks on board, which makes her way good six miles an hour, will, when got into a trade wind (which blows in the latitude of thirty) make her way nine miles an hour; now admit the fleet can sail at the rate above, I demand the course and distance, before and after their arrival in the trade wind, to be performed in the shortest time possible, from the Lizard to Jamaica, and the minimum, according to Wright's projection. Lizard in lat. 49° 56′, long. 5° 14′ west; Jamaica 18° and 76°.

IV. QUESTION 238, *by Mr.* Peter Kay.

To find in what arc of a circle a pendulum must vibrate, so that the time of one whole descent shall be equal to the time in which a heavy body would fall along the chord of the same arc.

V. QUESTION 239, *by Mr.* Robert Heath.

Four maids, wiſe and fair, and as Aſtrea rare,
(Impatient to tie Hymen's nooſe)
Burn with amorous flame for an artiſt of fame:
Then which for her age would you chooſe?

Suppoſing their ſeveral ages to be repreſented by the following equations.

$$a \times : ee + uu + yy = 20850.$$
$$e \times : aa + uu + yy = 23238.$$
$$u \times : aa + ee + yy = 24654.$$
$$y \times : aa + ee + uu = 24750.$$

What was each of their ages, and the analytical inveſtigation?

VI. QUESTION 240, *by* F. R. S.

Let there be the fruſtum of a cone, whoſe leſs diameter is 20 inches, its greater 40, and length 90; which being cut by a plane diagonally through the contrary extremities of its two diameters, will divide it into two parts (called hoofs) a greater and a leſs. There is required a ſcientific theorem for finding the ſolid content of each part; there being none yet given by any author (except perhaps thoſe got from a tedious ſeries) which will give the contents preciſely true, when at the ſame time we have found, by an analytical method, the true ſolidity of each. All which may be made fully appear, and ſhall be demonſtrated in the next year's Diary.

VII. QUESTION 241, *by Mr.* William Daniel.

It is univerſally agreed, that the heat at any moment of time, on any day, is proportionable to the rectangle made of the ſine of the ſun's altitude, and the arc of time expreſſing his continuance above the horizon: which being allowed, it is required to find what time of the day will be the hotteſt at Coventry (lat. 52° 30′) on Auguſt 14, 1743.

N. B. This is one of the two problems which Mr. Stone (in his tranſlation of L'Hoſpital's Fluxions) challenges the mathematicians of Europe to anſwer.

VIII. QUESTION 242, *by Mr.* J. May, *jun.*

Laſt ſpring, 1742, being at ſea in north latitude, we had great ſtorms for ſeveral days together, ſucceeded with cloudy weather, which hindered our making obſervations; at laſt, when it cleared up, with Mr. Hadley's octant, upon deck, I endeavoured to take the ſun's meridian altitude, but unfortunately thick clouds prevented it. Now being at a great

loſs

loss to know where we were, we endeavoured to contrive some other way; and accordingly waiting a few minutes, it cleared up again, and we took the sun's altitude (after allowance for refraction and dip of horizon, &c.) 57° 24′ 52″; tarrying 26 minutes by a good watch, we observed the sun's height 55° 35′ 19″; and 25 min. after this, the sun's altitude was 53° 16′ 15″. From whence is required the latitude we were in, the time of each observation, and the sun's declination, by a general theorem for all problems of this nature.

IX. Question 243, *by Mr.* John Powle.

Let there be three spherical and perfectly elastic bodies, *A*, *B*, *C*: the weight of $A = 3$ pounds, and of $C = 27$. Now it is required to find the weight of the intermediate body *B*, so that *A* striking *B* at rest, and *B*, with the motion acquired by the stroke, striking *C* at rest, the motion produced in *C*, shall be a maximum?

Prize Question, *by the late illustrious Sir* I. Newton.

Three staves being erected, or set up on end, in some certain place of the earth, perpendicular to the plane of the horizon, in the points *A*, *B*, and *C*; whereof that which is at *A*, is 6 feet long; that in *B*, 18; that in *C*, 8; the line *AB* being 33 feet long: It happens on a certain day in the year, that the end of the shadow of the staff *A* passes through the points *B* and *C*; and of the staff *B*, through *A* and *C*; and of the staff *C*, through the point *A*.

To find the sun's declination, and the elevation of the pole, or day and place where this shall happen.

Note, this is the 42d problem in Sir Isaac Newton's Universal Arithmetic; and it may seem a piece of vanity in attempting to give a solution after the greatest of men; but having in the winter 1740, taken a great deal of pains to bring out a solution, and never being able to get his numbers for the declination and latitude precisely the same, I was fond to think his were exact, and wrought it over and over again: at first it came out an adfected equation, then a quadratic, and at last happily by a simple equation; and having taken the pains to prove all the numbers (not depending on the logarithms) found them agree in every particular, and by construction to form a true conic section. We therefore humbly presume, that in a calculus so prolix and difficult (in Sir Isaac's method) there might happen a small error, or at least some press fault of the editions, or in the translation; which we hope to make more fully appear in the next year's Diary.

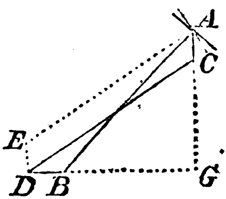
1744.

Questions answered.

* I. QUESTION 235 *answered by Mr.* J. Turner.

THERE is given $AB = CD = b = 97$ inches, $AC = 13{\cdot}8 = c$, and $DB = 12{\cdot}6 = d$; to find $CG = x$. Put $y = BG$.

By 47 E. 1. $\begin{cases} cc + 2cx + xx + yy = bb, \\ dd + 2dy + yy + xx = bb; \end{cases}$ consequently they are equal to one another, or $cc + 2cx = dd + 2dy$; and by the first equation, $y = \sqrt{bb - cc - 2cx - xx}$, which substituted for y, in $cc + 2cx - dd = 2dy$, is $cc + 2cx - dd = 2d\sqrt{bb - cc - 2cx - xx}$; and squaring both sides of the equation, makes $c^4 + 4c^3x - 2ccdd + 4ccxx - 4ddcx + d^4 = 4ddbb - 4ddcc - 8ddcx - 4ddxx$; transposed, $4ccxx + 4ddxx + 4c^3x + 4ddcx = 4ddbb - 2ddcc - c^4 - d^4$; and dividing all by the coefficient of the highest power, it will stand, $xx + cx = \dfrac{4ddbb - 2ddcc - c^4 - d^4}{4cc + 4dd}$.



The square compleated, and properly reduced, gives this theorem,

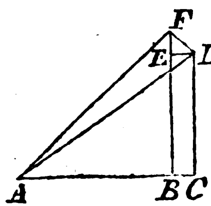
* I. QUESTION 235 *Constructed.*

Make DB = the given diff. of the shadows [Fig 1], and perpendicular thereto DE = the given diff. of the heights of the boughs; then from E and B as centers, with the same given distance from each bough as a radius, describe two circular arcs, and having drawn EA and BA to the point of intersection, A, draw DC ∥ and $= EA$, and AC drawn to meet DB, produced, in G, will determine CG and AG for the two required heights of the boughs.—— DC being, by construction, $= EA = BA$, AC (by *Euc.* I. 34) = and ∥ DE, and consequently the angle at G = the angle at D = a right one.

theorem, $x = \sqrt{\frac{ddbb}{ad+cc} - \frac{dd}{4}} - \frac{c}{2} = 58{\cdot}2 = GC$. Thence is found $y = 65$, $GA = 72$, and $DG = 77{\cdot}6$; whence, by common trigonometry, may be found the angles $GAB = 42°\ 4'\ 30''$, $ABG = 47°\ 55'\ 30''$, and $CDG = 36°\ 52'\ 12''$. Now having the two altitudes of the ſun found, and the differences of time given per queſtion $= 76\frac{1}{3}$ minutes, the azimuth is eaſily had; then by the theorem in the anſwer to queſtion 220, the latitude will come out $52°\ 19'\ 37''$, and the times of the day 8 h. 0 m. 11 s. and 9 h. 16 m. 23 s. the anſwer required.

The ſame anſwered by Mr. Betts.

Put x and y for the ſine and coſine of the angle EFD, radius $= 1$. Then by trigonometry, as s. $\angle EFD : ED :: $ s. $\angle EDF : EF$, i. e. $x : d :: y : c$; therefore $cx = dy$ and $\frac{x}{y} = \frac{d}{c} =$ tangent of $42°\ 23'\ 51''$ the angle EFD, or half the ſum of the angles BAD and $BAF =$ half the ſum of the two altitudes of the ſun. Again, ſine $EDF : EF ::$ rad. $: DF$, i. e. $y : c :: 1 : \frac{c}{y}$; and as AD : radius $:: \frac{1}{2}FD$: ſine of half the angle DAF, viz. $b : 1 :: \frac{c}{2y} : \frac{c}{2by}$; or as $2b : c :: \frac{1}{y} : \frac{c}{2by}$, viz. $2b : c ::$ ſecant $42°\ 23'\ 51''$: ſine $5°\ 31'\ 39''$ equal to half the angle DAF, which is half the difference of the required altitudes; whence the altitude $CAF = 47°\ 55'\ 30''$ and $CAD = 36°\ 52'\ 12''$, and the times of the day 8 h. and 9 h. 16 m. the latitude $52°\ 19'\ 37''$, the anſwer required.



This queſtion was anſwered by *Bironnos*, Mr. *S. Bamfield*, Mr. *J. Aſh*, Mr. *R. Sowerby*, Mr. *Ramſhay*, Mr. *Watts*, Mr. *Scyth*, Mr. *Tho. Cowper* the propoſer, Mr. *W. Kingſtone*, and Mr. *Daniel*, in the former method, and by Mr. *Dan. Howard* in the latter.

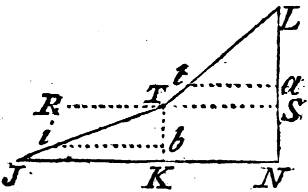
II. QUESTION 236 *anſwered.*

After the two bodies have been 48 hours in motion, *T* is diſtant from the point of projection (*B*) 393270 miles, and makes an angle with it of 59° 55′. Alſo *L* makes with *A* an angle of 98° 23′, and is diſtant from it 861570 miles. Through the induſtrious labours of ſome of our correſpondents we have

have got an anſwer to this abſtruſe and curious philoſophical problem; but as it is doubted whether ſome errors are not in it, and believing no one ſo equal to the taſk as the author, whoſe knowledge and penetrability in ſuch difficult and uncommon problems, is ſcarce to be rivalled in an age; we have given only the numbers above (beſides the ſcheme and anſwer we have inſerted in our 1ſt vol. of Diary Queſtions, p. 193) * till ſuch time as the propoſer favours us with his.

III. QUESTION 237 *anſwered by* Bironnos.

Let LN repreſent the enlarged diff. lat. between the Lizard and Jamaica, LS that between the Lizard and the trade wind. Then we have $LS = a = 1579{\cdot}9$; $TK = 790{\cdot}2 = b$; $Tb = 720 = q$; $JN = 4246 = n$. Let $y = TS$; then $n - y = JK$; and per ſim. triangles, $a : y :: m : \frac{my}{a} = ta$; $b : n - y :: q : q \times \frac{n-y}{b} = ib$; and per 47 Eu. 1, $\sqrt{m^2 + \frac{m^2 y^2}{a^2}} = Lt$; also $\sqrt{qq + qq \times \frac{\overline{n-y}|^2}{b^2}} = Ti$; hence the required minimum is $\frac{m}{6a}\sqrt{aa+yy} + \frac{q}{9b}\sqrt{bb + \overline{n-y}|^2}$; which being fluxed, there comes out $\overline{qn - qy} \times 2a\sqrt{aa+yy} = mq \times 3b\sqrt{bb + \overline{n-y}|^2}$; which ſolved, $y = 1857{\cdot}9215 = TS$. Hence the courſe from the Lizard to the trade wind is ſouth 49° 37′ 25″ weſterly, diſtance = 1846·23 miles; and thence to Jamaica S. 71° 41′ 27″ W. diſtance 2291·95 miles.



Anſwered by Mr. J. Powle.

RS is the parallel of 30°, L the Lizard, J Jamaica, T the point where the fleet muſt come to in their way, the $\angle TLS$ the firſt courſe ſteered, &c. (per Wright's projection)

LS

* The 1ſt vol. of queſtions here referred to, is *Thacker*'s Miſcel. where the calculation of this queſtion is given.

$LS = 15978979$ miles, which call b; $SN = TK = 790\cdot1607 = c$; $JN = 4246 = d$. Put $x = TS$; $m = 6$; $n = 9$: then (per 47 E. 1) $TL = \sqrt{bb + xx}$, $TJ = \sqrt{cc + dd - 2dx + xx}$; and, by a uniform velocity, the times of description will be $\frac{\sqrt{bb+xx}}{m}$, and $\frac{\sqrt{cc+dd+2dx+xx}}{n}$, the sum of which two must be a minimum; being fluxed, made $= 0$, and ordered, there will arise this equation:

$$\left.\begin{matrix}+n^2\\-m^2\end{matrix}\right\} x^4 \left.\begin{matrix}-2dn^2\\+2dm^2\end{matrix}\right\} x^3 \left.\begin{matrix}+ddnn\\+ccnn\\-bbm^2\\-ddm^2\end{matrix}\right\} xx + 2dbbm^2 x = bbddm^2.$$

In numbers, $52x^4 - 441584x^3 + 771997102\cdot9985x^2 + 763080806290\cdot03x = 1620020551753144$.

Solved, $x = 1458\cdot016$, or $2891\cdot56$, or $3488\cdot45$, or $1346\cdot03$; but it is the first of these values ($1458\cdot016$) which serves our present purpose. Now having found TS, by plain trigonometry $TL = 2149\cdot86$ is had) being the distance sailed before the fleet's arrival in the trade wind) and the course steered S. W. by S. 8° 27′ westerly: Thence to Jamaica TJ is $2897\cdot79$; course W. S. W. 6° 40′ westerly.

The result of some other answers are these following:

	1 course °	′	Dist. sailed. miles	2 course °	′	Dist. sailed. miles	x's val. miles	Time d.	h.
Mr. N. Oats	S. 48	44	1813·56	72	5	2343·41	1801	23	10
Mr. Bamfield	49	43	1849·6	71	39	2287·6	1864		
Mr. J. Ash	49	43	1849·7						
Mr. Ramsay	39	50	1555	73	33	2542		22	6
Mr. J. Watts	148	44	1813	172	5	2343		32	10½
Mr. Rr. Scyth	48	40	1853	72	8	2346	1796	23	17
Mr. Cowper	39	44	1555·4	73	32	2542		22	13

IV. QUESTION 238 *answered by* Merones.

Put c = chord of the arch required; x = any variable part of it; r = the length of the pendulum; t = time of the descent in the chord; and z = time of the descent in the arch. Then (by ex. 10 prop. 13 Mr. Emerson's Doctrine of Flux. p. 114) $\dot{z} = \frac{tr}{2} \times \frac{\dot{x}}{\sqrt{cx-xx}\sqrt{4rr-cx}} = \frac{t\dot{x}}{4\sqrt{cx-xx}}$ $\times : 1 + \frac{cx}{2.4r^2} + \frac{3.c^2x^2}{2.4.16r^4} + \frac{3.5.c^3x^3}{2.4.6.64r^6}$ &c. And by forms

10 and 17 (pages 62, 68, ibid) of the table, $z = \frac{tt}{2} \times$ arch whose sine is $\sqrt{\frac{x}{c}} \times : 1 + \frac{cc}{16r^2} + \frac{3^2c^4}{4.16^2r^4}$ &c. = (when $x = c$) $\frac{3.1416t}{4} \times : 1 + \frac{cc}{16r^2} + \frac{3^2c^4}{2^2 16^2 r^4} +$ &c. $= t$, per quest. ∴ (putting $r=1$) $cc + \frac{3^2c^4}{2^2 16} + \frac{3^2 5^2 c^6}{2^2 3^2 16^2} + \frac{3^2 5^2 7^2 c^8}{2^2 3^2 4^2 16^3}$ &c. = 4·3744: whence, by reversion, $cc = 2·5107$; and $c = 1·5846$, the chord of 104° 48′ the arch through which the pendulum must descend.

The same answered by the Proposer Mr. Peter Kay.

Let $P = 3·14159$; $a =$ length of the pendulum; $c =$ the versed sine of the arch described in the descent; then the time of descent, or half the time of vibration, will be $\frac{a^{\frac{1}{2}}P}{\sqrt{2}} \times : 1 + \frac{c}{2.2.2a} + \frac{c^2}{2.2.4.4.4a^2}$ &c. and the time along chord by $2\sqrt{2a}$; (as demon. p. 140, 141. of Simpson's Flux.) which two expressions must by the question be equal to each other; divide both by $\frac{a^{\frac{1}{2}}p}{\sqrt{2}}$, and put $x = \frac{c}{2a}$, and the equation will become $1 + \frac{x}{2.2} + \frac{3.3x^2}{2.2.4.4}$ &c. $= \frac{4}{p}$, and therefore $x + \frac{3.3x^2}{4.4} + \frac{3.3.5.5x^3}{4.4.6.6} + \frac{3.3.5.5.7.7x^4}{4.4.6.6.8.8}$ &c. $(\frac{16}{p} - 4) =$ 1·093; whence $x = \frac{c}{2a}$, will be found = ·642, and therefore $\frac{c}{a} = 1·284$; and consequently the required arch = 106° 32′.

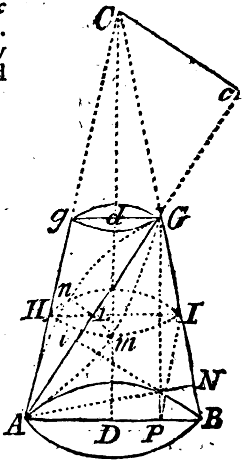
Mr. *Ramshay*, by an easy process, finds the arch 104° 5′, Mr. *Watts*, = 104° 48′. *Gamston Retford*, = 106° 32′.

QUES-

V. QUESTION 239 was anſwered by the propoſer, by Mr. *John Landen*, Mr. *Samuel Bamfield*, Mr. *Ramſbay*, Mr. *Watts*, and *Gamſton Retford*. The ages 15, 18, 21, 25.*

VI. QUESTION 240 *anſwered by* F. R. S.

In the given fruſtum of the cone let $AB = D = 40$; $gG = d = 20$; $GP = h = 90$. And put $AG = t$ the tranſverſe diameter; $nm = c$ the conjugate; $GB = z$; and $Cc = x$. Extend the tranſverſe to c, and draw the two perpendiculars to it cC and iB, alſo $f = GC$, and $x = Cc$.



Then by ſim. triang. $\begin{cases} t : h :: D : \dfrac{Dh}{t} = Bi, \\ z : \dfrac{Dh}{t} :: f : \dfrac{Dhf}{zt} = Cc, \\ \dfrac{D-d}{2} : z :: \dfrac{d}{2} : \dfrac{dz}{D-d} = f. \end{cases}$

Subſtitute $\dfrac{dz}{D-d}$ for f, then Cc (x) $= \dfrac{Ddh}{Dt - dt}$, and x is nearly 38; which multiplied by $\frac{1}{3}$ of the area of the ellipſis, $AnGm$, viz. $\dfrac{.7854tc}{3}$, or ntc, gives $\dfrac{Dhdnc}{D-d}$, the ſolidity of $ACGmA$; from which take the ſolid gCG, i. e. $\dfrac{ddnhd}{D-d}$, leaves $\dfrac{Dhdnc - d^3nh}{D-d}$ the content of $AgGA$. But nm $(=c)$ is

---

* V. QUESTION 239.

The ſolution of this queſtion, as given by Mr. *Aſh*, is omitted, as being ſo very erroneous. It is not perhaps worth one's while to reduce the equations by a regular algebraic proceſs, which would be a very tedious operation: but they might be much eaſier ſolved by the method of Trial-and-Error, or ſome other approximating method. Or by a few trials the numbers are found to be as above.

is known to be a mean proportional between AB and gG; i. e. $\sqrt{Dd}=c$; whence $\frac{Dhdn\sqrt{Dd}-d^3nh}{D-d}$ = lesser hoof $AgGA$. Also $\frac{D^3hn-Ddn\sqrt{Dd}}{D-d}$ = greater hoof $ABGA$.

But in this example, as $2d=D$, the theorems are reducible, $\frac{D\sqrt{Dd}-dd}{D-d}\times{}$·2618$dh$ is $\frac{2d\sqrt{dd}-dd}{2d-d}\times{}$·2618$dh$, i. e. $\overline{2dd\sqrt{2}-dd}\times{}$·2618$h$, or $\overline{2\sqrt{2}-1}\times{}$·2618$bdd$. In numbers, 1·8284 × 90 × 400 × ·2618 = 17232·304 the lesser hoof. And $\frac{D^2-d\sqrt{Dd}}{D-d}\times{}$·2618$Dh$ becomes $\frac{4dd-dd\sqrt{2}}{2d-d}\times{}$·2618$hhd$, that is, $\overline{4dd-dd\sqrt{2}}\times{}$·2618 × $2h$, or $\overline{4-\sqrt{2}}$ × ·2618 × $2hdd$: but $\sqrt{2}$ = 1·4142, and 4 − 1·4142 = 2·5858; therefore in numbers 2·5858 × 2618 × 180 × 400 = 48741·2956 the greater ungula. Which ungulas together make the whole frustum 65973·6.

The same answered by Merones.

Let L be the center of the ellipsis $AnGm$, mLn the common section of it and the circle $HnIm$, and draw IP parallel to HA, &c. Since GA is bisected in L, ∴ $LI = \frac{1}{2}AB$, and $LH=\frac{1}{2}gG$; whence $nL^2 = HLI = \frac{1}{4}gG\times AB$, and $nm=\sqrt{Gg\times AB}$; and the area of the ellipsis $c\times GA\sqrt{Gg\times AB}$ (c being = ·7854). By similar triangles, $AB-Gg:GP::AB:CD=\frac{AB\times GP}{AB-Gg}::Gg:Cd=\frac{Gg\times GP}{AB-Gg}$; and, by the same, $GA:Gg::CD:Cc=\frac{CD\times Gg}{GA}$; ∴ the area ellipsis $\times Cc=\frac{c\times CD}{3}\sqrt{Gg^3\times AB}$ = solidity of GCA; but $c\times Ggg\times Cd$ = solidity GCg; and $\frac{c\times AB^2\times CD}{3}$ = solid BCA; whence $\frac{c\times CD}{3}\times\overline{AB^2-Gg\sqrt{Gg\times AB}}$ = ungula GAB; and $\frac{c\times CD}{3}\times\sqrt{Gg^3AB}-\frac{c\times CD}{3}Ggg$ = ungula GgA. Hence, putting the height $GP=H$, $AB=B$, $Gg=b$, c = ·7854; then $\frac{cHB}{B-b}$

×

$\times \frac{B^2 - b\sqrt{Bb}}{3}$ = the greater ungula GAB = 48740·55; and $\frac{cHB}{B-b} \times \frac{b\sqrt{Bb} - bb}{3}$ = the leſſer ungula GgA = 17232·055. *Q.E.I.*

Bironnos *has anſwered this Queſtion in a Method ſomething different from the two laſt.*

Let fall the $\perp$ AN upon GB; put $2a$ = greater diameter AB, $2q$ = leſſer gG. Their half ſum $AP = m$; half diff. $PB = n$; the fruſtum's height $GP = d$; and ·785398, &c. $= s$; then $\sqrt{d^2 + m^2} = AG$ the tranſverſe, and $2\sqrt{aq}$ = the conjugate: Then, per ſim. Δs, $PB : GP :: gd : Cd = \frac{dq}{n}$, and $:: BD : DC = \frac{da}{n}$; and, per 47 Euc. 1, $CG = \frac{q\sqrt{nn+dd}}{n}$; again, $GB : GP :: AB : AN = \frac{2da}{\sqrt{dd+nn}}$, and $AG : AN :: CG : cC = \frac{2daq}{n\sqrt{d^2+m^2}}$; hence the ſolidities, of the cone $ACB = \frac{4a^3sd}{3n}$, of $CgG = \frac{4q^3sd}{3n}$, and the ſcalenous CAG, is $= \frac{4sdaq\sqrt{aq}}{3n}$; whence the ſolidity of the greater hoof $AGB = \frac{4asd}{3n} \times \overline{aa - q\sqrt{aq}}$, and leſſer $\frac{4sdq}{3n} \times \overline{a\sqrt{aq} - qq}$. Or putting the fruſtum's height $= A$, greater diameter $= D$, leſſer $= d$, diff. $= x$, mean proportional between B and ·78539 $= s$; then

Greater hoof $= \frac{DAs}{3x} \times \overline{DD - Bd}$ = 48741·05.

Leſſer hoof $= \frac{dAs}{3x} \times \overline{DB - dd}$ = 17232·55, &c.

Q.E.I.

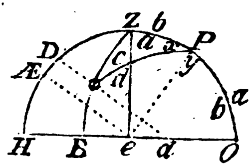
Mr. *Tho. Atkinſon*, and Mr. *Tho. Ramſhay*, have each brought out theorems and true ſolutions by the common method of algebra. Solutions were alſo given by Mr. *Turner* and Mr. *Arch. Scyth.*

VII. QUESTION 241 *answered by Mr.* J. Turner *of York*.

Let b = length of the semidiurnal arch = 1·8233; x = arch of the hour from noon, its cosine $= 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$ &c. d = rectangle of the sines of the lat. and declination = ·1493; c = ·5979 the rectangle of their cosines; then $c \times : 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$ &c. $+ d$ = sine sun's altitude; and $\overline{cb + cx} \times : 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$ &c. $+ db + dx$ by the question must be a maximum; which put into fluxions and reduced, gives $cbx + \frac{3cx^2}{2} - \frac{cbx^3}{6} - \frac{5cx^4}{24} + \frac{cbx^5}{120}$ &c. $= c + d$; or $1{\cdot}09015x + {\cdot}8968x^2 - {\cdot}1817x^3 - {\cdot}1246x^4 + {\cdot}0991x^5$ &c. $= {\cdot}7472$. Here $x = {\cdot}5049$ = an arch of 28° 56′, so the time is 1h. 55′ 44″ post meridiem.

An Answer by Mr. Thacker.

Put radius = 1; d and c for the sine and cosine of the sun's declination; b and a = sine and cosine of the latitude given; x and y the sine and cosine of the hour from noon; z the arc itself: then by spherics $db + cay$ = sine of Bo the sine of the sun's altitude; which being multiplied by $n + z$ (n = semid. arc) the arc of time passed over from



sun-rising, gives $\overline{db + cay} \times \overline{n + z}$; which by the question is proportionable to the heat, and must therefore be a maximum; put into fluxions, is $ca\dot{y} \times \overline{n + z} + \dot{z} \times \overline{db + cay} = 0$; but by the nature of the circle $x : 1 :: -\dot{y} : \dot{z}$, ∴ $-\frac{\dot{y}}{x} = \dot{z}$; which put for $\dot{z}$ is $ca\dot{y} \times \overline{n + z} - \frac{\dot{y}}{x} \times \overline{db + cay} = 0$; and therefore $ca\dot{y} \times \overline{n + z} = \frac{\dot{y}}{x} \times \overline{db + cay}$, which

divided

divided by $\dot{y}$ gives $ca \times \overline{n+z} = \frac{1}{x} \times \overline{db + cay}$; this divided by ca will make $n+z = \frac{1}{x} \times \overline{\frac{db}{ca}} + y$.

From which equation it appears that the hotteſt time of the day is, when the arc of time paſſed over from ſun-riſing is equal to the rectangle of the tangents of the ſun's declination and latitude, plus the coſine of the hour or arc from noon, drawn into the co-ſecant of the arc paſſed over ſince noon. In numbers, the tangent of ſun's declin. 10° 53′ = 9·2839070 multiplied by tangent lat. 52° 30′ = log. 10·1150195 gives 9·3989265 = coſine 75° 29′ 40″ the arc of time from midnight, which ſubtracted from 180 leaves 104° 30′ 20″ the ſemi-diurnal arc = 12; to which 28° 56′ time after noon makes 133° 26′ 20″. This in decimals of degrees, multiplied by ·0174532, &c. (i. e. 360) 6·28318 (·01745) gives the arch of the circle 2·3263535. And tang. decl. × tang. lat. = ·2505642, plus coſ. arc 28° 56′ = ·8751832 from noon ($= n+z$) is 1·1257474 multiplied by the co-ſecant 28° 56′ = 2·0670056 gives 2·326 &c. equal to the other ſide of the equation, which is proved right. But the arch from noon is found thus: If v be put for the number of degrees paſſed over ſince noon; then will $\cdot 017453 \times \overline{104° \, 30' + v} = \text{co-secant } v \times \overline{\cdot 2505642 + \text{cos. } v}$; whence by a few trials, v may be found = 28° 57′, which in time is 1 h. 55′ 48″ afternoon.

If any one ſhould diſlike this gueſſing method, this following may be thought more valuable, viz.

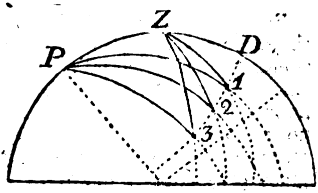
Put t = tangent of half the time from noon; then will $\frac{2t}{1+tt} = x$; and $\frac{1-tt}{1+tt} = y$; alſo, by the nature of the circle, $z = 2t - \frac{2t^3}{3} + \frac{2t^5}{5} - \frac{2t^7}{7}$, &c. whence $n+z = \frac{1}{x} \times \overline{\frac{db}{ca}} + y$ is $n + 2t - \frac{2t^3}{3} + \frac{2t^5}{5}$ &c. equal to $\frac{1+tt}{2t} \times \frac{db}{ca} \times \frac{1-tt}{1+tt} + 1 - tt$; and the equation ordered

$$\left.\begin{matrix} +2n \\ -2 \end{matrix}\right\} t \left.\begin{matrix} +4 \\ -\frac{bd}{ca} \end{matrix}\right\} t^2 + 2t^3 - \frac{4t^4}{3} + \frac{4t^6}{5} + \frac{4t^8}{7} + \frac{4t^{10}}{9}, \text{ \&c.} = \frac{db}{ca};$$

which ſeries reverted will give the value of t = tangent 14° 28′.

VIII. QUESTION 242 *answered by Mr.* T. Cowper.

Let $c =$ cosine of $Z1 = 57° 24' 52''$; $t =$ cosine of $Z2 = 55° 35' 19''$; $d =$ cosine of $Z3 = 53° 19' 15''$; $h =$ sine of $1P2 = 6° 3'$, its cosine $= p$; $m =$ sine of $1P3 = 12° 45'$, $n =$ its cosine; $x =$ sine $DP1$, $y =$ its cos. then $py - hx =$ cos. $ZP2$; $ny - mx =$ cosine $ZP3$. Then by theorem 1 in the solution to quest. 222, we



have $\frac{c + ty - t - cpy + chx}{y - py + hx} =$ cosine ZD, and $\frac{c + dy - d - cny + cmx}{y - ny + mx} =$ cosine ZD; which transposed and reduced, is $cpyy + tnyy + dyy - dpyy - cnyy - tyy + chx + tmx - dhx - cmx = cpy + tny + dy - dpy - cny - ty + chxy + tmxy - dhxy - cmxy$. Put $r = cp + tn + d - dp - cn - t$, or $= \cdot0001692$; and $s = ch + tm - dh - cm$, or $= \cdot00077322$; then the equation above, after substitution, will stand, $ryy + sx = ry + syx$, i.e. $sx - sxy = ry - ryy$, and dividing by $1 - y$, we have $sx = ry$; $\therefore \frac{x}{y} = \frac{r}{s}$ the tangent of $12° 20' 36''$: consequently the 1st observation was 12 h. 49′ 22″, the 2d at 1 h. 15′ 22″, the 3d at 1 h. 40′ 20″ *P. M.* And by the 1st and 2d theorem in the same 222d quest. is found the sun's meridian altitude $= 58° 15' 57''$, his septentrional depression $= 17° 20' 35''$; hence the latitude is $51° 58' 49''$, and ☉'s declination $= 20° 45' 36''$ north.

The same answered by F. R. S.

Let the quantities be represented as above; also let r and $s =$ sine and cosine of the latitude; and a and c the sine and cosine of the sun's distance from the pole.

Then in the 3 triangles, by the spheric theo. we have
$$\begin{cases}(1)\ er + asy = c,\\(2)\ er + aspy - ashx = t,\\(3)\ er + asny - asmx = d.\end{cases}$$

In the (1) $er = c - asy$, which substituted in the other two, make (4) $c - asy + aspy - ashx = t$, (5) $c - asy + asny - asmx = d$; per (4) $-asy + aspy - ashx = t - c = g$, $\therefore as = \frac{g}{-y + py - hx} = \frac{g}{-y \times \overline{1 - p} - hx}$: call $1 - p =$

$v =$

$v =$ versed sine 6° 30′, then will $as = \frac{g}{-yv - bx}$; by the (5) $-asy + asny - asmx = d - c = k$, $as = \frac{k}{-y + ny - mx} = \frac{k}{-y \times \overline{1 - n} - mx}$; and if $1 - n = w =$ vers. sine 12° 45′, then will $as = \frac{k}{-yw - mx}$; consequently $\frac{g}{-yv - bx} = \frac{k}{-yw - mx}$, or $\frac{g}{yv + bx} = \frac{k}{yw + mx}$, which out of fractions $gyw + gmx = kyv + kpx$, $\therefore \frac{x}{y} = \frac{kv - gw}{gm - kb} =$ tangent of the $\angle DP1 =$ 11° 20′ 36″ = time 49′ 22″. Whence the answer will come out as above.

I cannot but be persuaded that this curious problem will point out a way to be very useful in navigation, for determining the longitudes as well as latitudes: for, supposing at sea we knew neither, or had not the time of the day, but were furnished with a quadrant to take altitudes of the sun, and could find the difference in time between each observation, which a common pocket watch with a minute hand would give us very well, or with a second hand better. For though that watch or a clock was incapable of keeping true time at sea, yet it might very well measure a few minutes between one observation and another, in which space the error must be very inconsiderable. Now when this was done, by solving this problem, we get the latitude and the true times of the day, and then it would be no very difficult task to rectify the longitude pretty near. In order to this we shall deduce a theorem in words, by which any one that is but skilled in the common cases in trigonometry may put it in practice.

A Theorem for the Hour of the Day.

1. The difference between the first and second altitude, drawn into (*i.e.* multiplied by) the versed sine of the arch of time between the first and second observation; made less by the difference between the first and second altitude, multiplied into the versed sine of the arch of time between the first and third observations.

2. The difference between the first and second altitude, drawn into the right sine of the arch of time, between the first and third observations; minus the difference between the

the first and third altitude, drawn into the right sine of the arch of time between the first and second observations.

Lastly, Divide the former difference by the latter, and the quotient will be the tangent of the arch of the time from noon.

The same answered by Mr. J. May *the Proposer.*

Put $a =$ sine $57° 24' 52''$, $b =$ that of $35° 35' 19''$, $c =$ that of $53° 15' 16''$, the sine of 26 min. or $6° 30' = m$, cos. $= n$, $f =$ sine $12° 42'$ ($= 51$ min.) the time between the first and last observ. its cos. $= g$, radius $= 1$. Then put $a - b = h$, $a - c = i$, $r - n = s$, $r - g = t$; then the tangent of the hour angle from noon when the greatest altit. was taken will be $= \frac{ht - is}{mi - hf} r = \cdot 2190328 = 12° 21' 16''$, which in time is $49' 25''$, or after 12 o'clock; the second 1h. $15' 25''$; and the last at 1h. $40' 25''$, according to the altitude's decrease.

Now put the cosine of $12° 21' 16'' = d$; of $25° 6' 16''$ (the arc of 1h. $40' 25''$ time of the last observation) $= e$; put likewise $d - e = q$, $r - e = l$, and $r + d = p$.

Then the sine of the sun's southern altitude will be $= b + \frac{li}{q}$ (see his answer to quest. 222 Diary 1742) the degrees of which put $= w$. Likewise the sine of the sun's depression in the north will be $\frac{pi}{q} - a$; the degrees of it put $= u$; then the sun's declination will be $\frac{w + u}{2} = 20° 47'$ nearly; and the cosine of the latitude $\frac{w - u}{2} = 38° 4' 45''$, and $51° 55' 15''$ the latitude required.

IX. QUESTION 243 *answered by Mr.* J. Landen.

The bodies are A, B, C. Their weights 3, x, 27. } Then according to Mr. Keil's Introduct. Phys. we have $\frac{2a}{x + a} =$ the celerity wherewith the body B will approach C, and $\frac{4xa}{xx + xa + xc + ac} =$ the velocity of C after the impulse; the fluxion of which being made $= 0$, and reduced, we have $x = 9$, a mean proportional between A and C.

Mr.

Mr. Tho. Cowper's *Answer.*

The bodies and weights denoted as above, and putting 1 to express the velocity of *A*; from Dr. Keil's demonstration about the motions of elastic bodies is deduced this analogy: As the sum of the bodies : twice the weight of the moving or striking body :: the velocity of the striking body before percussion : the velocity of the quiescent body after it.

That is, $x+a : 2a :: 1 : \frac{2a}{x+a}$ = velocity of *B* after the stroke. Again, $x+c : 2x :: \frac{2a}{x+a} : \frac{4ax}{xx+cx+ax+ca}$ = the celerity of *C* after the stroke; which, per question, is a maximum, and the fluxion thereof $4ax^2\dot{x}+4acx\dot{x}+4aax\dot{x}-8ax^2\dot{x}-4acx\dot{x}-4aax\dot{x}=0$. Reduced, gives $x=\sqrt{ac}=9$ pounds.

Mr. *J. Watts*'s answer is in the same method. Mr. *William Honnor*, from Mac Laurin's Fluxions, p. 429, Mr. *Powle* the proposer, Mr. *J. Turner*, *Bironnos*, Mr. *S. Bamfield*, Mr. *Ash*, Mr. *Ric. Sowerby*, and *Philotechnus*, have answered this question.

The PRIZE QUESTION *answered by Mr.* A. Thacker.

Put $\left.\begin{array}{l} a=6 \text{ feet} \\ b=18 \\ c=8 \end{array}\right\}$ the height of the staves at $\left\{\begin{array}{c} A \\ B \\ C \end{array}\right\}$; the line $AB=33$ feet $=d$; the distance between the top of the staff *A* and bottom of the staff *B* ($=aB$) call *e*; the distance between the top of the staff *B* and bottom of staff *A* ($=bA$) call *f*; then will $\frac{\overline{c+a}\times\overline{e+f}}{2b+2a}+\frac{\overline{b+a}\times\overline{c-a}}{2e+2f}=m=$ 21·0789, the distance from the top of *C* to the bottom of *A*. Then (by 47 Euc. 1.) *AC* is found $=19{\cdot}501$, which call *x*; also $\frac{\overline{c+b}\times\overline{e+f}}{2b+2a}+\frac{\overline{b-c}\times\overline{b+a}}{2e+2f}=n=40{\cdot}216$, the length between the top of *B* and bottom of *C*; and hence *CB* is found $=35{\cdot}962=z$. Now putting $\frac{be-fa}{df+de}=h=$ 16111;

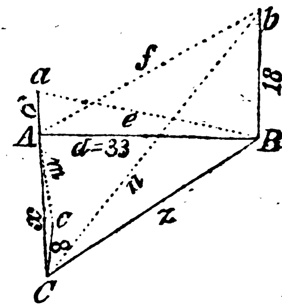
16111; $\sqrt{xx+aa} = s = 20'404$; $\frac{cs - am}{xm + xs} = k = '0454$;

and $\frac{dd + xx - zz}{2dx} = v = '13501$; then will

$\frac{2 - 2vv}{1 + bb + kk - 2bkv - vv} = 1'948315$, the versed sine of an arc which is double the latitude 161° 30′; whose half is = 80° 45′, the true latitude required.

Now calling the sine of the latitude p, then will $\frac{pc + pa}{s + m}$ = '33309 = the sine of 19° 27′, the sun's declination north.

Sir *I. Newton*, the author of this problem, finds the lat. 80° 45′ 20″, and declination 19° 27′ 20″, as may be seen in the 42d prob. of his Universal Arithmetic, in the English edit. 1720, p. 151, where the translator Mr. Raphson, Mr. Cunn, or the printer, committed a blunder, making the line *AB* = 30, instead of 33. By reason of which disappointment, the solution is here shorter than was designed; but the investigation of the theorem above we have printed in the first volume of Diary Questions [*Thacker's Miscel.*] If the line *AB* was = 30, then the latitude is 80° 4′ 7″, and declination 21° 7′ 4″. And Mr. *Ed. Cross* has found it 80° 4′ 11″, and declination 21° 7′ 48″.



The Prize of 10 *Diaries fell to Mr.* Ed. Cross.

Of the Eclipses in 1744.

Within the sphere of the earth's orbit will happen four eclipses this year; twice will the moon, in her wandering course, interpose and hide the splendour of the sun from falling on the earth, or its atmosphere; and twice will the earth, in its annual elliptic orbit, be so full in a line between the sun and moon, as to hinder her from receiving the light she borrows from the sun, to enlighten the earth by reflection.

1. Sun

1. Sun eclipsed April 1, at 10 at night, invisible.

2. Moon eclipsed April 15, at half an hour after 8 at night, visible.

Calculated by		Beg. h.m.		Mid. h.m.		End h. m.		Dig.	
From Astron. Carol. at Coventry		7	4	8	28	9	52	8	0
J. Randles of Wemm	London	7	5	8	30	9	58	8	9
	Rome	7	57	9	22	10	50		
	Wemm	6	55	7	53	9	48		
William Leighton,	London	7	8	8	32	9	57		
	Arlington	7	15	8	39	10	4		
S. Bamfield, Leadbet. Tab. Lond.		7	3	8	16	9	29	6	53
Will. Brown, Brent's Comp. A. Car. Cleobury		7	3	8	29	9	55	8	1
		7	1	8	25	9	49	8	0
W. Honnor,	Burchester	6	54	8	23	9	52	8	23
	Rushden	6	58	8	27	9	56		
Mr. Ral. Hulse, London		7	8	8	28	9	56		
Tho. Cowper, Wellingborough		7	9	8	35	10	1	8	2
Mr. Poole, Hereford		7	5	8	30	9	50	7	50
Mr. Betts, Oxford		7	7	8	35	10	3	8	35

3. Sun eclipsed September 25, invisible.

4. Moon eclipsed October 10, invisible. Mr. Hulse has given the calculation of this eclipse at Moscow: The beginning 11. 30. Middle 12. 46. End 1. 8.

New Questions.

I. Question 244, *by Mr.* J. Turner.

If a flexible chain, eighteen inches long,
On two pins horizontal was hung,
Whose distance asunder exactly shall be
A foot; its lowest descent then let's see.
A theorem that's general give, for to find
The areas of all such curves of that kind.

II. Question 245, *by Mr.* John Landen.

I have one hundred pieces of gold; some of which are pistoles, some guineas, and the rest moidores. Now if a pistole was worth 18s. 6d. a guinea worth 1l. 3s. and a moidore 1l. 10s. my hundred pieces would be worth just one hundred pounds. Quere, How many I have of each sort?

III. Ques-

III. QUESTION 246, *by Mr.* Peter Kay.

To find the center of oscillation of a pendulum, whose bob is composed of two equal and similar parabolical conoids, joined together at their bases; the thickness of the bob being three inches, the diameter of its greatest circle seven inches, and the distance of its center from the point of suspension 39·5 inches?

IV. QUESTION 247, *by Mr.* J. Betts.

A set of men and women were drinking together, and their reckoning came to just six guineas; towards the discharging of which, each man agreed to pay a certain sum, and each woman the square root of the same: Now it was found, if the number of men and women were mutually changed the one for the other, the reckoning would have come to half a Portugal piece less, or only to 4l. 10s. Again it was found, that each man paid as many shillings more than each woman, as there were women in company. It is required, what number were of each, and what each paid?

V. QUESTION 248, *by Mr.* William Daniel.

In an oblique-angled triangle (*EGF*) there is given the difference of the two sides, which compose the oblique angle (ED) $= 2$; the difference of the segments of the base (EB) $= 2·4$; and the oblique angle (EFG) $= 112° 37'$: It is required to find all the other parts of the triangle.

VI. QUESTION 249, *by Mr.* William Brown.

In the latitude 52° 30′, on the 10th of June (supposing it the longest apparent day) I asked a mathematical friend, what o'clock it was? who made me this puzzling answer: Count (says he) the hours from the visible time of the rising of the sun's center, and add their cube root to the square root of the hours to the apparent time of its setting; and it will give you the hour of the day. Quere, What o'clock was it?

VII. QUESTION 250, *by Mr.* John Hill.

There is a river, whose stream is divided into two parts; and after running some space, the waters are united; between which it has inclosed an island in the form of a geometrical

metrical ellipsis, whose transverse diameter is forty chains (according to Gunter) and conjugate = thirty chains. Upon the transverse diameter is built a farm or cottage house, 132 yards from the center; and as this piece of land is to be divided by straight hedges from the house to the water, one of them, which should be the shortest that can be made, is to convey the water from the river to fill a cistern by the cellar. It is required to find the shortest distance, and the position it will make with the transverse.

VIII. Question 251, *by Mr.* J. Powle.

To determine the law of the weights, which press each particle of a perfect flexible line, in such manner, as that it shall form a curve, whose equation is $ax = y^4$?

IX. Question 252, *by Mr.* T. Sandalls.

In an oblique-angled plain triangle, there is given the difference of the sides which include the angle of 112° = 20, and the perpendicular let fall from the angle on the base = 60: Required a theorem to determine the base and sides of the triangle?

X. Question 253, *by Mr.* J. Powle.

Granting the resistance, as the square of the celerity; in what law of density will a heavy body moving describe a curve, whose equation is $ax = y^3$?

XI. Question 254, *by* Diophantus.

Since the doctrine of triangles has an unbounded use and application in most parts of the mathematics, and the similarity of them generally had recourse to; let it be required to find eight right-angled plain triangles, whose hypothenuses are all equal; and shew a general method for determining the same.

XII. Question 255.

The various contrivances for measuring time have employed the curious in all ages; the true determining of which is a matter of no small importance in civil life; and perhaps I may surprize some, if I say, algebra is useful to know the time of the day by a clock, when it cannot be done otherwise; which is the reason for putting in this easy question, in order to convince others, the facetious Hudibras did not joke, when he says,

——And wisely tell what hour o'th' day
The clock does strike, by algebra.

The

The Question. Being at so large a distance from the dial-plate of a great clock, that I could not distinguish the figures; but as the hour and minute hands were very bright and glaring, I could perceive, that the minute hand pointed upwards to the right hand, at the same time the hour index pointed downwards to the left, so as both were in a right line, or diametrically opposite, and in such position, as that the elevation (I guessed) was some few degrees more than 50 above the horizon. Quere, The hour and minute of the day.

Prize Question *by Mr.* J. May, *jun. of Amsterdam.*

An architect, or master carpenter, in Holland, had (from that slender knowledge which usually attends mechanics) conceit enough to fancy, he could find the dimension of any piece of timber in a building, of which a design should be given: A burgo-master of the city of Amsterdam, intending to build a handsome house fronting the street, where his length was limited, because he would save the charges of a double roof and gutter, and at the same time put his best side outward, gives the said architect these dimensions, viz. That the building should be forty feet wide, and the front wall twelve feet higher than the back wall: Also, because too much of a large roof should not appear in view of the street, he will have the length of the rafters, from wall to ridge, on the back side of his house, just 37 feet; but the rafters on the front side to be of such a length, as may form the pitch, or steepness of the roof, the same on each side. The owner being frugal (not to say wise) orders the builder to sit down and count up the cost: But although he was skilful in numbers, and pretty well versed in some parts of geometry, yet he found the first would be so much adfected, and the latter only an approximation, that he was not able to know how high the roof would rise, nor the length of the rafters in front, and therefore was incapable of computing the timber and roofing. The burgo master surprized, probably thinking so famed an architect must be little less than a conjuror (when himself was none) resolves not to have his house begun until he can have the measures exact, and leaves him bare-breach'd, riding on the strange roof, although he is furnished with mathematical instruments, to describe curves and conic sections organically. But having heard of such things being effected by geometrical construction, he has, through the mediation of a friend, applied to the artists of Great Britain; and thinking the author of the Ladies' Diary deals in quibbles and quaint questions, hopes to see both methods in the next year's production.

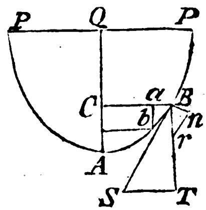
Questions

1745.

Questions answered.

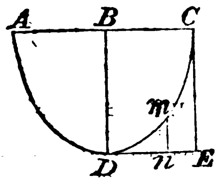
I. QUESTION 244 *answered by Mr.* N. Farrer.

LET $a =$ the force which keeps the chain in its position at A, $x = AC$, $y = BC$, $z = AB$; draw the tangent BS, and $BT = BA$ perpendicular to the horizon, and TS parallel thereto. Then will the line AB be sustained by three forces; for its gravity acts in the direction BT, it is drawn at A in an horizontal direction, by the force a, and it is sustained in B by the tension of the line SB; which three forces are consequently as BT, TS, and BS; as $BT = z : TS = a :: \dot{x} : \dot{y}$. Take $Br = Bb$, and draw rn parallel to BS, and Bn perpendicular thereto; then BT is the perpendicular force or weight of the line at B, $\dot{x}$ is the fluxion of the tension at B, whose fluent (x) is the tension or force in the direction nr; but in A where $x = 0$, this tension $= a$, ergo by correction the whole tension drawing in the direction of the curve is $a + x$; then $BS = a + x : z :: \dot{z} : \dot{x}$, $\therefore a\dot{x} + x\dot{x} = z\dot{z}$, its fluent is $2ax + xx = zz$, therefore $\dot{y} = \frac{a\dot{x}}{\sqrt{aa + xx}}$, and $y =$ hyperb. log. of $\frac{a + x + \sqrt{2ax + xx}}{a}$; which equation when $y = PQ$ will give $x = AQ = 6{\cdot}0314$ the lowest descent; hence $a = \frac{zz - xx}{2x} = 3{\cdot}6992$; the supplemental fluxion $x\dot{y} = \frac{ax\dot{z}}{\sqrt{aa + zz}}$ gives $az - aa \times$ hyp. log. $\frac{z + \sqrt{aa + zz}}{a}$, and the area $PAQ = xy - az + aa \times$ hyp. log. $z + \sqrt{aa + zz} = 25{\cdot}6916$.



The same answered by Mr. Cockson.

Put $DC=c$, $BC=b$, $Dn=x$, $mn=y$, and $Dm=z$. Then will $\dot{x}=\frac{a\dot{y}}{z}$, $\dot{z}=\sqrt{\dot{a}a+\dot{y}y}$; whence, the curve being the catenaria, $y=\sqrt{aa+zz}-a$, $\dot{y}=\frac{z\dot{z}}{\sqrt{aa+zz}}$; this put for $\dot{y}$ in the above value of $\dot{x}$, makes $\dot{x}=\frac{a\dot{z}}{\sqrt{aa+zz}}$; hence $y\dot{x}$ the flux. of the variable area Dmn is $\frac{a\dot{z}}{\sqrt{aa+zz}}\times\overline{\sqrt{aa+zz}-a}=a\dot{z}-\frac{aa\dot{z}}{\sqrt{aa+zz}}$, whose fluent is $za-ax=$ the area, and when $z=c$ (x being then $=b$) becomes $\overline{c-b}\times a$ the expression for the area CDE, $\therefore$ the area of the curve is 50·1888, and DB the lowest descent of the chain $=$ 6·0317 inches.



II. QUESTION 245 *answered by Mr.* Heath.

Let $x=$ the number of moidores, $y=$ guineas; then $b-y-x=$ the pistoles (putting b for 100 pieces); hence $60x+37b-37y-37x+46y=4000$ the number of six-pences; whence $x=\frac{300-9y}{23}$ or $y=\frac{300-23x}{9}$; from whence x and y are determined in whole numbers 6 and 18, also $b-y-x=76$.

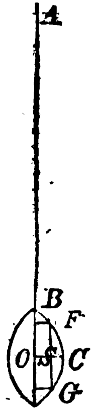
This question was also truly answered by Mr. *Brown*, Mr. *Farrer*, Mr. *Terey*, Mr. *Cross*, Mr. *Adams*, Mr, *Landen* the proposer, and several others.*

III. QUES-

* This question may also be solved after the manner of several others of the same kind already given in this work.

* III. QUESTION 246 *answered by Mr.* N. Farrer.

Let a represent the point of suspension. Now here is given $AO = 39\cdot5 = d$, $BO = 3\cdot5 = m$, and $CO = 1\cdot5 = n$: Let $y = SF = SG$, $x = CS$; then $x : yy :: n : mm$, $\therefore x = \frac{nyy}{mm}$; and then the fluent of $\overline{n-x}|^2 \times \overline{d-y}|^2 \times \dot{y}$ divided by the fluent of $\overline{n-x}|^2 \times \overline{d+y} \times \dot{y}$ gives $\frac{112dd - 70dm + 16mm}{112d + 35m}$, which let $= a$; again the fluent of $\overline{n-x}|^2 \times \overline{d+y}|^2 \times \dot{y}$ divided by the fluent of $\overline{n-x}|^2 \times \overline{d+y} \times \dot{y}$ gives $\frac{112d^2 + 70dm + 16m^2}{112d + 35m}$ $= b$; hence the distance of the center of oscillation from A the point of suspension is $\frac{aa + bb}{a+b} = 37\cdot35728$ inches.



Mr. *Cockson* and Mr. *Powle* have made the distance of the point of suspension from the center of oscillation $= 40\cdot0125$ inches; and Mr. *Wm. Hanbury* the same with the former.

IV. QUES-

---

* III. QUESTION 246.

The original solution above given to this question is wrong, for the center of oscillation ought to be farther below the point of suspension than the center of gravity is; but it is here made less than it.

Putting the quantities as in the notation above, and c for the distance of the center of oscillation from the point of suspension, then will c be $= d + \frac{\text{the flu. of } y^4\dot{x}}{2d \times \text{the flu. of } y^2\dot{x}}$ (by the nature of the center) $= d + \frac{\text{the flu. of } m^2x^2\dot{x}}{2d \times \text{flu. of } nx\dot{x}} = d + \frac{m^2x}{3dn} = d + \frac{y^2}{3d} =$ (when $y = m$ or $x = n$) $d + \frac{m^2}{3d} = 39\cdot5 + \frac{3\cdot5^2}{3 \times 39\cdot5} = 39\cdot5 + \cdot103376 = 39\cdot603376$ the distance required.

IV. QUESTION 247 *answered by Mr.* R. Gibbons.

Put $m =$ the number of men, $n =$ the number of women, and $x =$ what each woman paid; then per question each man paid xx. Hence $mxx + nx = 126$, $nxx + mx = 90$, and $xx - x = n$. Now by taking the value of m in the two first equations, we have $\frac{126 - nx}{x} = 90 - nxx$. From which and the third equation we get $x^5 - x^4 - x^3 + x^2 = 90x - 126$; whence $x = 3$ the shillings each woman paid, and $x^2 = 9$ what each man paid; consequently there were 12 men and 6 women.

This question was answered in the same manner by Mr. *Landen*, Mr. *Cross*, Mr. *Rubins*, Mr. *Brown*, Mr. *Adams*, Mr. *Dent*, Mr. *Cockson*, Mr. *Holliday*, and others.

V. QUESTION 248 *answered by Mr.* Heath.

Put $m =$ s. $\angle EFG = 112° 37'$, [see the following fig.] $x = BP = PG$, and $n = EB$ the diff. of the base's segments; $y = DF = FG$, and $ED = d$ the diff. of the sides containing the oblique angle. Now, per 47 Euc. 1, $EF^2 - EP^2 = PF^2 = FG^2 - PG^2$, i. e. $dd + 2dy + yy - nn - 2nx - xx = yy - xx$; whence $\frac{dd + 2dy - nn}{2n} = x$ by reduction. Again EG : s. $\angle EFG$ :: FG : s. $\angle FEG$, i. e. $2x + n : m :: y : \frac{my}{2x+n} =$ s. $\angle FEG$; and rad. : EF :: s. $\angle FEG : PF$, i. e. $1 : d + y :: \frac{my}{2x+n} : \overline{yy - xx}|^{\frac{1}{2}}$; whence $yy - xx = \frac{mmyy}{4xx + 4xn + nn} \times \overline{dd + 2dy + yy}$, or $\overline{yy - xx} \times \overline{4xx + 4xn + nn} = mmyy \times \overline{dd + 2dy + yy}$. In which if the value of x before found be substituted, an equation will come out in which there will be but one unknown quantity, viz. y.

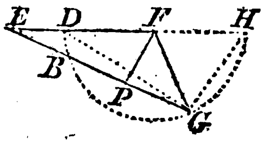
Answered by Mr. James Rubins.

As $EF - GF : EP - PG$ :: cos. $\frac{\angle E + \angle G}{2}$: cos. $\frac{\angle G - \angle E}{2}$ (per Thacker's Miscellany, theor. 18 p. 28.) Hence the sides are easily found by plain trigonometry, and are $EF = 12{\cdot}9$, $FG = 10{\cdot}9$, and $EG = 19{\cdot}874$.

The

The same answered by Mr. Arch. Scyth.

In the triangle EFG draw the line DG, then the angle $FDG = \frac{\angle G + \angle E}{2}$, and $BGD = \frac{\angle G - \angle E}{2}$; but in the triangle EGH, as $EG : EH :: s.\angle H : s.\angle HGE$ $(= \text{cos.}\,\frac{\angle G - \angle E}{2})$, and in the



$\triangle EGD$, as $s.EG : ED :: s.EDG : s.EGD\,(= s.\,\frac{\angle G - \angle E}{2})$; that is, putting $ED = a$, $EB = b$, $FG = \frac{1}{2}x$, $BG = y$, and c and d = sine and cosine of half the sum of the angles at the base, $y + b : x + a :: d : d \times \frac{x+a}{y+b}$ = cosine of half the difference; and $y + b : a :: c : \frac{ac}{y+b}$ = s. of half the difference; $\therefore dd \times \overline{a+x}|^2 + aacc = \overline{y+b}|^2$, or $\sqrt{daxx + 2add + aa} = y + b$: but $\overline{x+a} \times a = \overline{y+b} \times b$, $\therefore y + b = \frac{xa + aa}{b} = \sqrt{ddxx + 2addx + aa}$, $\therefore xx + 2ax = \frac{aabb - a^4}{aa - bbdd}$ and $x = -a \pm \sqrt{\frac{aabb - aabbdd}{aa - bbdd}} = \frac{abc}{\sqrt{aa - bb}} - a =$ 21·96; and therefore $\frac{abc}{2\sqrt{aa - bbdd}}$ = the greater side. $\frac{abc}{2\sqrt{aa - bbdd}} - a$ = the less, and $\frac{ac}{y+b}$ = ·056033 = s. 3° 12½′, so that the greater angle is 36° 54′, and the less 30° 29′.*

VI. QUES-

---

* A construction of this problem may be seen at prob. 8 of *Simpson*'s Algebra.

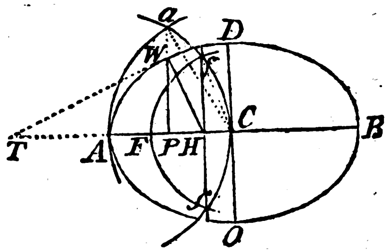
VI. QUESTION 249 *answered by Mr.* Farrer.

Let y = the hours from sun-rising, n his true time of rising, and m the length of the day; then the time to his setting is $m-y$, and per question $\sqrt[3]{y} + \sqrt{m-y} = \overline{n+y}$ the hour of the day: Put $y = x^3$, and we have $x + \sqrt{m-x^3} = n + x^3$, $\therefore m - x^3 = n^2 + x^6 + x^2 + 2nx^3 - 2nx - 2x^4$, which in numbers is $x^6 - 2x^4 + 8{\cdot}4002x^3 + x^2 - 7{\cdot}4022x = m^2 - n^2 = 2{\cdot}8997$; hence $x = 1{\cdot}0919$, and $x^3 = y = 1{\cdot}3083$, and $n + y = 5{\cdot}0029 = 5$ h. 0′ 10″44‴ the time required.

Mr. *Brown* the proposer, Mr. *Heath*, Mr. *Adams*, Mr. *F. Holliday*, Mr. *Gibson*, Mr. *Cockson*, and several others, have answered this question in the same method.

VII. QUESTION 250 *answered by Mr.* Wm. Lax.

Put $AC = t = 20$, $DC = c = 15$, $CH = d = 6$, $BH = a = 26$, $AH = b = 14$; and let $PH = x$. Then, per property of the curve, we have $tt : cc :: \overline{a+x} \times \overline{b-x} : cc \times \frac{ab + bx - ax - xx}{tt} = PW^2$, $\therefore cc \times \frac{ab + bx - ax - xx}{tt} + xx = HW^2$, which by the question is to be a minimum, $\therefore$ its fluxion $cc \times \dot{x}b - a\dot{x} - 2x\dot{x} + 2tt x\dot{x} = 0$; whence $x = \frac{cc}{2} \times \frac{a-b}{tt-cc}$; or (by writing $2d$ for $a-b$) $x = \frac{ccd}{tt-cc} = 169{\cdot}7$ yards; consequently $HW = 13{\cdot}36$ chains, making an angle with the transverse of 54° 45′.



Mr. N. Farrer's *Answer*.

Here is given $AB = 40$, $OD = 30$, $CH = 6$ chains: let $CB = m = 20$, $DC = n = 15$, $CH = d = 6$, and $PC = x$; then,

then, per conics, $mm : nn :: mm - xx : \frac{nnmm - nnxx}{mm}$ $= PW^2$, and $\frac{n}{m}\sqrt{mm - xx} = PW$; $x : m + x :: m - x : \frac{mm - xx}{x} =$ subtangent PT; then, per sim. triangles, $x - d : \frac{n}{m}\sqrt{mm - xx} :: \frac{n}{m}\sqrt{mm - xx} : \frac{mm - xx}{x}$, therefore $\frac{mmx - x^3 - dmm + dxx}{x} = \frac{nnmm - nnxx}{mm}$, therefore $m^4x - m^2x^3 - dm^4 + dmmxx = nnmmx - nnx^3$, that is $m^2x^3 - n^2x^3 - dmmxx + nnmmx - m^4x + dm^4 = 0$, which divided by $xx - mm$ gives $mmx - nnx - mmd = 0$, $\therefore x = \frac{mmd}{mm - nn} = 13\text{·}714286 = PC$, and $PH =$ 7·714286, $PW =$ 10·918c6, and $HW =$ 13·368406, making an angle with the transverse of 54° 45′ 23″ nearly.*

This question was also truly answered in the former method by Mr. *Tho. Cowper*, Mr. *J. Terey*, Mr. *John Landen*, Mr. *W. Dent*, Mr. *Adams*, Mr. *Fr. Holliday*, and Mr. *Gibbons*.

VIII. QUES-

---

* VII. QUESTION 250 *Constructed.*

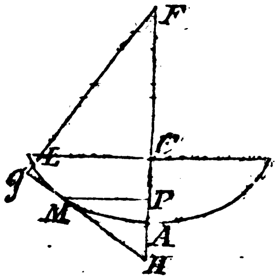
The point F being the focus, with the center C and radius CF describe an arc cutting $fHf \parallel OD$ in f and f; through f, C, f describe a circle; and with the center C and radius CA describe another intersecting it in a; then aWP being drawn $\parallel CD$ will give the point W in the curve from whence the shortest line WH must be drawn.

For, having drawn the chords Cf, Ca, by the nature of the circle it will be $CP : CH :: (Ca^2$ or$)$ $CA^2 : (Cf^2$ or$)$ CF^2; hence, by division, PH (or $CP - CH$) $: CH :: CD^2$ (or $CA^2 - CF^2$) $: CF^2$; $\therefore HW$ is $\perp$ the curve by prop. XV *Emerson*'s Conics, which it ought to be when it is shortest.

COROLLARY. Hence, when HW is $\perp$ the curve, CF^2 or $CA^2 - CD^2 : CA^2 :: CH : CP$. And from hence also the calculation is very easy; for $CP = \frac{CH \times CA^2}{CA^2 - CD^2} = \frac{6 \times 20^2}{20^2 - 15^2}$ $= \frac{6 \times 400}{175} = \frac{6 \times 16}{7} = \frac{96}{7} = 13\frac{5}{7}$.

VIII. QUESTION 251 *answered by* Amicus.

Draw the lines FH, Fg, Hg, and LC will represent the weights upon every particle of any curve: let $PM = \dot{y}$, $PH = \dot{x}$, $CF = a$, $AM = \dot{z}$, and by similar triangles, we shall have $\dot{y} : \dot{x} :: a : LC = \frac{a\dot{x}}{\dot{y}}$, whose flux. is $\dot{LC} = \frac{a\dot{y}\ddot{x} - a\dot{x}\ddot{y}}{\dot{y}\dot{y}}$, a general expression for all curves. In the present case we have $ax = y^4$, whose fluxion is $\dot{x} = \frac{4y^3\dot{y}}{a}$,



which substituting in the above equation $LC = \frac{a\dot{x}}{\dot{y}}$, and taking its fluxion we shall have $\dot{LC} = 12yy\dot{y}$; and if w expresses the weight that presses the particle M, we shall have $w\dot{z} = 12yy\dot{y}$, or $\frac{w\dot{y}}{a}\sqrt{aa + 16y^6} = 12yy\dot{y}$, or as the cube of the ordinate directly and tangent inversely. *Q. E. I.*

Answered by Mr. Farrer.

Draw the lines as per figure to the solution of quest. 244; and let a represent the tension in A, $x = AC$, $y = CB$, and $z =$ curve AB. Then we have, by the nature of the curve in the solution to question 244, $z\dot{y} = a\dot{x}$, $2ax + xx = zz$ and $a + x =$ the tension. Now per question $ax = y^4$, its fluxion $a\dot{x} = 4y^3\dot{y}$, $\therefore 4y^3\dot{y} = z\dot{y}$, and $4y^3 = z$ the gravity of the line or required law of the weights pressing every particle of the line; which is as the cubes of the ordinates.

IX. QUESTION 252 *answered by Mr.* T. Atkinson.

Given $AC - CB = 20 = 2n$, [see the fig. to quest. 248] $CE = 60 = m$, s. $\angle BCD$ or $68° = s$, its cosine $= c$; put $2y = BC + AC$, then $y + n = AG$, and $y - n = BC$; and per

per trig. $y+n : 1 :: m : \frac{m}{y+n} =$ s. $\angle CAE$, and $\frac{m}{y+n} : y-n :: s : \frac{syy-snn}{m} = AB$; also $1 : y-n :: c : cy-cn = CD$; but $AB^2 = AC^2 + BC^2 + 2AC \times CD$, i.e. $\frac{s^2y^4 - 2s^2n^2y^2 + s^2n^4}{mm} = 2y^2 + 2n^2 + 2cy^2 - 2cn^2$, or $y^4 - \frac{2s^2n^2 + 2m^2 + 2cm^2}{ss} \times y^2 = \frac{2m^2n^2 - 2cm^2n^2 - s^2n^4}{ss}$; or if for the coefficients of yy and the quantities on the other side of the equation be wrote $2A$ and B respectively, it will be $y^4 - 2Ay = B$, $\therefore y = \sqrt{A + \sqrt{B + A^2}} = 108{\cdot}4271$; hence $AC = 118{\cdot}4271$, $BC = 98{\cdot}4271$, and $AB = 180{\cdot}1279$.

The same answered by Mr. W. Kingston.

Let $AF = a = 20$, $EC = p = 60$, s and $c =$ sine and cosine of half the sum of the angles at the base, x and y the sine and cosine of half their difference; then will $sy + cx$ be the sine of the greater angle at the base, and $sy - cx$ the sine of the less; also $cy - sx$ and $cy + sx$ their respective cosines; then as $sy - cx : p :: 1 : \frac{p}{sy - cx} = AC$, and as $sy + cx : p :: 1 : \frac{p}{sy + cx} = CB$; hence $\frac{p}{sy - cx} - \frac{p}{sy + cx} = a = \frac{2cpx}{ssyy - ccxx}$, $\therefore 2cpx = assyy - accxx$: But if for yy be put its value $1 - xx$, and for cc, $1 - ss$, it will become $ass - axx = 2cpx$, $\therefore axx + 2cpx = ass$, substitute $2n = \frac{2pc}{a}$ and it will be $xx + 2nx = ss$; whence, by compleating the square, and extracting the root, we have $x = \sqrt{ss + nn} - n$, and the required sides as above.

Answered by Mr. A. Scyth.

Put $p = 60$, $d = 20$, s and $c =$ the sine and cosine of half the sum of the angles at the base; $y =$ the base, and $x =$ the sum of the sides; then will $y : x :: c : \frac{cx}{y} =$ the cosine of half the difference of the angles at the base, and $y : d :: s : \frac{sd}{y} =$ its sine, $\therefore ccxx + ssdd = yy$, and $xx = \frac{yy - ssdd}{cc}$;

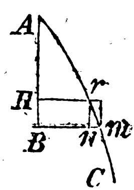
but

but $\frac{csx+csd}{y}$ = the sine of the greater, and $\frac{cxs-csd}{y}$ = the sine of the less; therefore $\frac{csx+csd}{y} : p :: 1 : \frac{py}{cs\times\overline{x+d}}$ = the lesser side, $\therefore \frac{py}{cs\times\overline{x-d}}$ = the greater, and their sum $\frac{2pyx}{sc\times\overline{xx-dd}} = x$, $\therefore xx = \frac{2py}{sc} + dd = \frac{yy-ssdd}{cc}$, or $yy - \frac{2pcy}{s} = ccdd + ssdd = dd$; which reduced gives $y = \frac{pc \pm \sqrt{ppcc+ssdd}}{s} = 180.122$; whence $\frac{x\pm d}{2} = \frac{\sqrt{dd + \frac{2p}{ssc}\times\overline{pc+\sqrt{ppcc+ssdd}}}}{2} \pm \frac{d}{2} = 118.42$ the greater side, or 98.42 the less.

This question was answered by Mr. *Brown*, Mr. *J. Boston*, *E. Sugget*, *Brancepeth*, Mr. *John Corbett*, Mr. *Fr. Holliday*, Mr. *Heath*, Mr. *Gibbons*, Mr. *E. Cross*, Mr. *Farrer*, and others.*

X. QUESTION 253 *answered by Mr.* N. Farrer.

Let *ArC* represent the curve described; draw the lines as in the figure, and let the velocity at *r* in the direction $rC = v$, $AH = x$, $rn = \dot{x}$, $Hr = y$, $mn = \dot{y}$, $rm = \dot{z}$, the required density as *D*, *c* the celerity, and law of resistance as ac^n; then $\dot{y} : v :: \dot{x} : \frac{v\dot{x}}{\dot{y}}$ = the velocity in the direction *nr*, its fluxion is $\frac{\dot{v}\dot{x} + v\ddot{x}}{\dot{y}}$ = the increase of velocity during



the time of describing *rm*; $\dot{y} : \dot{x} :: \dot{v} : \frac{\dot{v}\dot{x}}{\dot{y}}$ the part arising from

* To this prob. a Construction is given in prob. 78 of *Simpson's* Algebra.

from the resistance of the medium; therefore $\frac{v\ddot{x}}{\dot{y}}$ = the part arising from the force of gravity. The resistance is to the force of gravity as $-\frac{\dot{v}\dot{x}}{\dot{y}}$ to $\frac{v\ddot{x}}{\dot{y}}$, or as $-\frac{\dot{v}\dot{x}}{v\ddot{x}}$ to 1; but $\frac{v\ddot{x}}{\dot{y}}$, the velocity arising from gravity, being proportional to the time $\frac{\dot{y}}{v}$ of describing nm, may be expressed thereby; hence $\frac{v\ddot{x}}{\dot{y}} = \frac{\dot{y}}{v}$ or $vv\ddot{x} = \dot{y}\dot{y}$, in fluxions $2v\dot{v}\ddot{x} + vv\dddot{x} = 0$, or $-\frac{\dot{v}}{v} = \frac{\dddot{x}}{2\ddot{x}}$, which substituted in the foregoing proportion $-\frac{\dot{v}\dot{z}}{v\ddot{x}}$: 1, gives $\frac{\dot{z}\dddot{x}}{2\ddot{x}^2}$: 1 the ratio of the resistance to the gravity.

Again, since the absolute velocity is $\frac{v\dot{z}}{\dot{y}}$ the resistance by by supposition will be $a \times \overline{\frac{v\dot{z}}{\dot{y}}}\big|^{n}$; hence D as $\frac{\dddot{x}}{a\dot{z}^{n-1} \times \ddot{x}^{\frac{4-n}{2}}}$ which when $n = 2$, or the resistance as the square of the velocity will be $\frac{\dddot{x}}{\dot{z}\ddot{x}}$: But, per question, the equation of the curve is $ax = y^3$, $\therefore a\dot{x} = 3y^2\dot{y}$, $\ddot{x} = \frac{6y\dot{y}^2}{a}$, and $\dddot{x} = \frac{6\dot{y}^3}{a}$, $\therefore$ the density $\frac{\dddot{x}}{\dot{z}\ddot{x}}$ will be as $\frac{1}{\frac{y}{a}\sqrt{9y^4 + a^2}}$, or as the tangents reciprocally.

Answered

Answered by the Proposer.

Because the resistance is as the square of the celerity, $\frac{\dddot{x}}{\ddot{x}\sqrt{\dot{x}^2+\dot{y}^2}}$ (Simpson's Essays, p. 60) will express the required density: But by the equation of the curve ($ax=y^3$) we have $\dot{x}=\frac{3y^2\dot{y}}{a}$, $\ddot{x}=\frac{6y\dot{y}^2}{a}$, $\dddot{x}=\frac{6\dot{y}^3}{a}$, and $\sqrt{\dot{x}^2+\dot{y}^2}$ $=\frac{\dot{y}}{a}\sqrt{a^2+9y^4}$; therefore, by substitution, the required density becomes $\frac{6\dot{y}^3}{\frac{6y\dot{y}^3}{a}\sqrt{a^2+9y^4}}=\frac{a}{y\sqrt{a^2+9y^4}}$; or by a farther reduction $=\frac{1}{\sqrt{yy+9xx}}$; i.e. it is always as the tangent of the distance from the vertex to unity.

This question was answered in like manner by *Amicus*, Mr. *C. Cockson*, Mr. *J. Landen*, and Mr. *Hanbury*.

XI. QUESTION 254 *answered*.

Let $bb=$ the square of the common hypothenuse, and xx the square of one leg, then will $bb-xx$ be the square of the other leg. Suppose $b-nx=$ that leg whose square is $bb-xx$, then will $bb-xx=bb-2bnx+nnxx$, which equation reduced gives $x=\frac{2bn}{1+nn}=$ to one leg, and if instead of x in $b-nx$ we take its value last found, we shall have $\frac{1\backsim nn}{1+nn}=$ the other leg; and since n may be assumed at pleasure, not only eight, but any number of right-angled plain triangles whatsoever may be found; which will also appear by making the common hypothenuse the diameter of a circle.

XII. QUESTION 255 *answered by Mr.* J. Landen.

By considering the question, I find that the first time the hands are diametrically opposite after 6 o'clock, is the time of the day sought. Let x hours be the time from 6: now the minute hand going round once in an hour, the rounds it will be carried in the time x, may be represented by x; and the hour hand going round once in 12 hours, the part of the circumference it will be carried in the same time by $\frac{x}{12}$. It is evident that the index whose motion is swiftest, will outgo the slowest, one circumference in x hours; whence $x - \frac{x}{12} = 1$, and $x = \frac{12}{11}$; therefore the time required is 5 min. 27 sec. past 7.

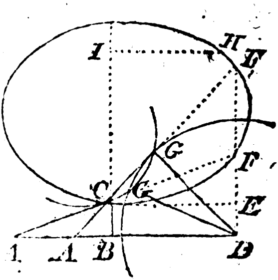
Answered by Mr. James Terey.

By the data it appears to be somewhat past 7. Let $x =$ the number of minutes past 7; then will $\frac{x}{12}$ be the distance of the hour hand from 7, and $25 - \frac{x}{12}$ the distance of the hour hand from 12; hence $25 + x - \frac{x}{12} = 30'$; $\therefore x = \frac{60}{11}$ $= 5\frac{5}{11}$ past 7, the time required.

This question was truly answered by *Lincolniensis*, Mr. *Jepson*, Mr. *Holliday*, Mr. *Kingston*, Mr. *Brown*, Mr. *Williams*, and others.

The PRIZE QUESTION *answered by Mr.* Ja. Terey.

Put BC, the height of the front wall above the back, $= 12 = d$, $BD = 40 = a$ the breadth of the house, $GD = GF = 37 = b$, and CG the required length of the front rafters $= x$; then $EF = \sqrt{xx + 2bx + bb - aa}$; and $AC : BC :: CF : FE$, i. e. $b - x : d :: b + x : \sqrt{xx + 2bx + bb - aa}$, therefore $db + dx = \overline{b - x} \times \sqrt{xx + 2bx + bb - aa}$; which equation, ordered,



makes $x^4 \begin{array}{l}-2bb\\-aaxx\\-dd\end{array} \begin{array}{l}+2baa\\-2ddb\end{array} x = \begin{array}{l}ddbb\\bbaa.\\-b^4\end{array}$ In numbers, $x^4 - 4482xx + 107744x = 513375$; whence $x = 6\cdot51224$ &c. or $x = 20\cdot21768$ &c. either of which determines the length of the front rafters.

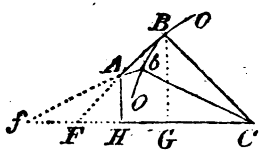
Geometrically thus: Let $AG = 37$ always pass through the point C, the point A sliding along the line AB; then will G the point at the other end of the line AG describe the curve $CGGL$. On D as a center with the radius $= 37$, describe a circle, and it will cut the said curve in the points G, G, which determines the length and position of the front rafters, as is evident by inspection.

Or let $AF = 2AG$ be moved as before, and it will cut the perpendicular DF in the points F and F, whence CG &c. is known as before.

N. B. If AF, CB, CI, and IH be called a, b, x, and y respectively; then will $\frac{xxaa}{xx + 2bx + bb} - xx = yy$ express the nature of all such curves.

The same answered by Mr. John Corbett.

Put $x =$ sine of the angle $AFH = GCB$, whose cosine is $\sqrt{1 - xx}$, radius 1; $BC = 37 = b$, $HA = 12 = c$, and $HC = 40 = d$: Then as $x : c :: 1 : \frac{c}{x} = FA$, and as $1 : b :: \sqrt{1 - xx} : b\sqrt{1 - xx} = GC$, for the triangles BGC and AHF are alike. As $1 : \frac{c}{x} (= FA)$



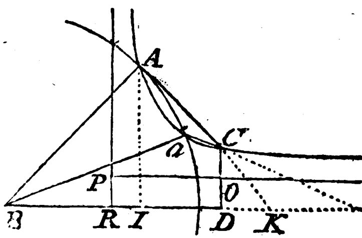
$:: \sqrt{1 - xx} : \frac{c}{x}\sqrt{1 - xx} = FH$; consequently $\frac{c}{x}\sqrt{1 - xx} + d = 2b\sqrt{1 - xx}$; which, reduced, gives $x^4 - \cdot324324x^3 - \cdot681519x^2 + \cdot324324x = \cdot0262965$. Two of the roots of which equation are $x = \cdot71507$ and $x = \cdot39403$, which in the tables answer to 45° 39′ and 23° 12′; and thence the side $AB = 20\cdot22$ and $AB = 6\cdot54$ are found, either of which lengths will answer the conditions of the question.

To effect this geometrically, Extend the line $HC = 40$ to any distance Ff, at one end of which H erect the perpendicular $HA = 12$; from the other end C with the extent of 37 describe

describe the arc oo; then lay a ruler upon the point A, and move it about till it cuts the arc oo and the line fH at a distance $= BC$; so shall BA or bA be the length of the roof sought $= 20{\cdot}22$ or $6{\cdot}54$ as above.

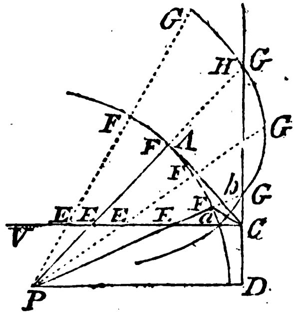
Answered by the Proposer.

On BD let fall a perpendicular from A, being supposed already found. Prolong BD and AC to K. Put BI or $IK = x$, and $AI = y$; then is $BK = 2x$, from which subtract $BD = a$, and there will remain $DK = 2x - a$. Now DK $(2x - a) : DC$ $(b) :: IK$ $(x) : AI$ (y), $\therefore 2xy - ay = bx$, an equation belonging to an hyperbola. Put x infinite, then is $y = \frac{1}{2}b$; and by putting y infinite, $x = \frac{1}{2}a$. Hence, if BD and DC are bisected, and the lines PO, PR drawn respectively parallel thereto, they will be two asymptotes to the said hyperbola. To find a point in the curve, put $x = a$ in the hyperbolic equation, then is $y = b$. Consequently C will be a point in the curve, through which draw the hyperbola AaC.



Per 47 Euc. 1, $BI^2 + AI^2 = AB^2$ $(= cc)$, or $xx = cc - yy$ (which is an equation belonging to a circle), therefore $x = \sqrt{cc - yy}$. Put the surd $= o$, then $x = o$, whence the center will be in B; whence also $y = c$ or $y = -c$, which determines the radius. Therefore, if on the center B, with the radius AB, be described the arc Aa, which cuts the above constructed hyperbola in the points A, a, and the lines AB and AC, or aB and aC, be drawn, they will be the sides of the roof required.

This question admits likewise of a very easy and elegant solution, by the help of another curve, and is thus performed: *BD* and *DC* being drawn as before, and likewise on the center *B* the arch *Aa*; lengthen the line *DC*, drawing *CV* parallel to *BD*, and suppose an infinite number of radii drawn from the center *B*, on which make *FG*, &c. $= FE$, &c. drawing through the points *G*, *G*, *G*, &c. the curve, whose equation is



$$y^4 + 2y^3b \begin{matrix} +xx \\ -4aa \\ +bb \end{matrix} y^2 + 2xxby + xxbb = 0,$$ and it will cut the line *DC* in *H* and *h*. Lastly, draw *BH* and *Bh*, which cut the above circle in *A* and *a*, as before.

The Prize of 10 *Diaries was won by Mr.* J. Corbett.

Of the Eclipses in 1745.

There will happen two eclipses this year; twice will the sun lose its light by the interposition of the moon's opaque body to some part of our terraqueous globe, in the following manner.

The first is on March 22, betwixt 2 and 3 of the clock in the morning, and therefore invisible in England, but conspicuous to our Antipodes, and much greater in the East Indies; especially in some of the Phillippine Islands, the sun will be vertically eclipsed.

The second happens Sept. 14, in the afternoon, but invisible at London, by reason of the paralax in lat. of the ☽ *a* ☉. Whence the moon is far depressed below the sun's limb, which proves it inconspicuous; but in some part of America it will be total and central; and near the meridian of port Royal, in Jamaica, it will happen in the following order:— Begins 14th day, 10h. 15m. 10s. in the morning, apparent time. Mid. 11h. 35m. 8s. Duration 2h. 34m. 4s. Ends 12h. 43m. 14s. Digits eclipsed 7°.

W. Leighton.

New

New Questions.

I. Question 256, by Mr. J. Turner.

In a 'pothecar's shop an old mortar I found,
Which being deem'd useless, was thrown on the ground.
The inside dimensions are plac'd * here below;
From whence its content in wine gallons I'd know.

* Given the perpendicular height of the mortar = 9 inches, bottom diameter = 6, top diameter = 12, and the curvature of the mortar's sides are supposed to be the apollonian parabola, whose vertex is a point on the uppermost edge of th mortar, or extremity of the top diameter.

II. Question 257, by Mr. T. Cowper.

The late phænomenon conspicuous here,
Each eve serene i'th' western hemisphere,
(When Sol withdrew his radiance from our sight)
With blazing tail and tremulating light,
Amongst those orbs in the concave expanse,
Which seem around this pensile world to dance;
Its nucleus first we in the æther saw,
'Twixt Pegasus and fair Andromeda;
From whence, by motion retrograde, it run
With gentle pace towards th' approaching sun,
Till course and declination so conspire,
Both eve and morn presents its sanguine fire:
This diff'rence only, that its streaming tail
Descends direct below th' horizon's vail,
But in the morn unfolds its orient light
In oblique glances, to the wond'ring sight;
Whose length'ning train, glowing in azure skies,
Fills gazing mortals with immense surprize.
Whether they in elliptic orbits run,
By gravitation, round the central sun?
Or but as transient fiery balls appear,
Thrown off in tangents from the solar sphere?
That, the Newtonian system doth regard;
This, the late the'ry of a modern bard.
As themes uncertain, leave we them behind,
As yet inscrutable to human kind,
Perhaps reserv'd for future years to find.
Soon as Aurora with refulgent beams,
Obscur'd each lesser constellation's gleams,

The cyprian ſtar her ſcintillating rays
Near the horizon ſplendidly diſplays;
* Fifty-three minutes paſt † apparent time,
I likewiſe ſaw the comet eaſtward ſhine;
Whoſe nucleus (by a common quadrant view'd)
Had five degrees one-third of altitude:
Its diſtance from bright Venus (taken true)
Was fifty-ſix degrees and ſix-tenths too.
Deduct refraction from its height before:
By ſpherics hence the comet's place explore.

* In lat. 52° 20′ N. Feb. 12, 1744. † Paſt 5.

III. QUESTION 258, *by Mr.* J. Powle.

Given 1 + 2 + 3 + 4, &c. continued to *x* terms; to find 1 + 4 + 9 + 16, &c. the ſum of the ſquares of thoſe numbers.

IV. QUESTION 259, *by Mr.* Landen.

A cannon ball projected from the ground, in a direction making an angle of 14° 29′ with the horizon, fell at the feet of a perſon ſome diſtance off the very moment he heard the report of the piece: Quere, how far he was from the place of projection?

V. QUESTION 260, *by Mr.* N. Farrer,

In an oblique-angled triangular grove, one of whoſe ſides is 20 chains, and the angle oppoſite thereto 78° 45′, if a perpendicular be let fall from each angle to its oppoſite ſide, they interſect at a fountain within the grove, whoſe neareſt diſtance to the given ſide is 8·19 chains: Quere its diſtance from each of the other ſides, by a ſimple equation?

VI. QUESTION 261, *by Mr.* C. Cockſon.

There are two ponds of water of the ſame quality and depth, under the ſame meridian, one in the lat. of $a - 16b$* north, and the other $8b = 5125$ miles due north from it. In the year 1743-4, the 16th of January, at three of the clock in the morning, the thickneſs of the ice in the ſouthermoſt was 6 inches. Quere the latitude, and thickneſs of the ice of the northermoſt pond at the ſame time?

* $\frac{\sqrt{aaa} + \sqrt[4]{aaa} - \sqrt[3]{aa}}{b} = a.$

VII. QUES-

VII. Question 262, *by Mr.* Powle.

To determine the aſymptotes of a curve whoſe equation is $x^5 - y^5 + axy = 0$.

VIII. Question 263.

Let AFK be the conchoid of Nicomedes, [*ſee the fig. to the prize queſt. for* 1738] and BC the aſymptote whoſe length is 60, and P the pole; a line drawn from P perpendicular to the aſymptote, to the curve at A, is 40; alſo from the pole to the aſymptote is 20: Required the length of the curve line AFK, with the analytical inveſtigation?

IX. Question 264, *by* Amicus.

To determine the greateſt area that can be encloſed by a parabolic curve of the ſecond kind, whoſe (equation is $ax^2 = y^3$, and) length 100 feet, and an ordinate rightly applied to its greateſt axis?

Prize Question *by Mr.* N. Farrer.

A bragging young gauger pretending to ſhew
The content of a caſk from what's given below,
Occaſion'd this wager—Five guineas to two:
He's try'd all his ſkill, but all will not do;
So begs the aſſiſtance, fair ladies, of you.

The length of the caſk is 31·907 inches, bung diameter 14 inches, and is the lower fruſtum of two equal conoids, generated by the rotation of a curve about its axis, whoſe equation is $y^7 - 1000000x = 0$.

A Paradox, *by* Amicus.

It has been aſſerted by a late celebrated mathematician, that if a veſſel formed by the rotation of an hyperbola round one of its aſymptotes be filled with water, and a hole made in the bottom of this veſſel, (let the hole be ever ſo large, and the depth of the veſſel ever ſo ſmall, it will take an infinite time to be exhauſted. Quere how this can be?

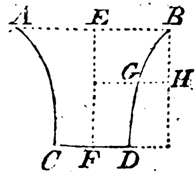
Questions

1746.

Questions answered.

I. QUESTION 256 answered by Mr. J. Landen.

LET $ABCD$ represent the mortar, generated by the revolution of the parabolic curve BD, about the axis EF, parallel to BH. Call EB, b; BH, x; GH, y; and put $3\cdot1416 = p$. Then will $p\dot{x} \times \overline{bb - 2by + yy}$ be the fluxion of the required solidity; which by putting for y and yy, their values $x^{\frac{1}{2}}$ and x (found by the equation of the curve) will be



$p\dot{x} \times \overline{bb + x - 2b\sqrt{x}}$, and its fluent $pbbx + \frac{1}{2}pxx - \frac{4}{3}pbx\sqrt{x}$; which, when $x = 9 = EF$, will be $466\cdot5276$ inches $= 2\cdot0108$, &c. wine gallons. Q. E. F.

This question was also solved by Mr. *Heath*, Mr. *Farrer*, Mr. *Powle*, *Bironnos*, Mr. *Ash*, Mr. *R. Williams*, Mr. *Arch. Scyth*, and Mr. *Bamfield*.

II. QUESTION 257 answered by Bironnos.

The place of Venus at the given time is ♑ 20° 50′ 49″, her latitude 1° 22′ 7″ N. declination 20° 27′ south, and is 44° 29′ 45″ short of the meridian: Hence there is known, $Z\odot$, the comet's true zenith distance $= 84° 48' 35''$, [*see the fig. to Q.* 220] ZP, the co-latitude of the place, $= 37° 40'$, $\angle ZPO = 44° 29' 45''$, OP the distance of Venus from the pole $= 110° 27'$, and $O\odot$ the comet's distance from Venus $= 56° 36'$. In $\triangle ZPO$ is known ZP, PO, and $\angle P$: Find the $\angle ZOP = 25° 35' 49''$, and $ZO = 82° 25' 41''$. In the $\triangle ZO\odot$, are known the three sides: Find the $\angle ZO\odot = 88° 44' 54''$: From which take the $\angle ZOP$, and there remains the $\angle PO\odot = 63° 9' 5''$. Then, in the $\triangle PO\odot$, is known PO, $\odot O$, and $\angle PO\odot$: Find the $\angle OP\odot = 48° 59' 54''$, and $P\odot = 80° 44' 20''$. Therefore the comet's

declination

declination is 9° 15′ 40″. N. and right ascen. 341° 19′ 34″. Hence the comet's longitude is ♓ 16° 25′ 26″, and latitude 15° 54′ 54″ N.

III. QUESTION 258 *answered by Mr.* J. Powle.

Put $S = 1 + 4 + 9 + 16$, &c. $\dot{S}$ = the next succeeding term in the series to the *last*, or the increment of S when the number of terms are $x + 1$. Then, because the first term and common difference are each unity, $x + 1 = \sqrt{\dot{S}}$, which in the series $\underset{''}{x}\underset{'}{x}\overset{'}{x}\overset{''}{x}$ &c. answers to x, $\therefore \dot{S} = \underset{'}{x}\underset{'}{x}$, or $\underset{'}{x}x + \underset{'}{x}$. Whence $S = \frac{1}{3}\underset{'}{x}xx + \frac{1}{2}\underset{'}{x}x =$ (by substitution) $\frac{1}{3} \times x + 1 \times x - 1 \times x + \frac{1}{2} \times x . x + 1 = \frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{1}{6}x$. *Q.E.F.*

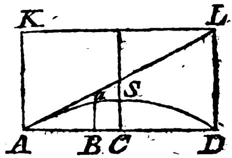
Answered by Bironnos.

Let $Ax^{n+1} + Bx^n + Cx^{n-1} + Dx^{n-2}$ &c. .... $+ L = 1^n + 2^n + 3^n$ &c. to x^n; then will $\overline{Ax+1}^{n+1} + \overline{Bx+1}^{n} + \overline{Cx+1}^{n-1}$ &c. .... $+ L = 1^n + 2^n + 3^n$ &c. to $x^n + \overline{x+n}^n$; from which subtract the former, and there remains $A \times \overline{x+1}^{n+1} - x^{n+1} + B \times \overline{x+1}^n - x^n + C \times \overline{x+1}^{n-1} - x^{n-1}$ &c. $= \overline{x+1}^n$. By expanding the several powers of $x+1$ we get $A = \frac{1}{n+1}$, $B = \frac{1}{2}$, $C = \frac{n}{3.4}$, $D = 0$, $E = \frac{n \times n - 1 \times n - 2}{2.3.4.5.6}$. Consequently $1^n + 2^n + 3^n$ &c. $= \frac{x^{n+1}}{n+1} + \frac{x^n}{2} + \frac{nx^{n-1}}{3.4} - \frac{n \times n - 1 \times n - 2}{2.3.4.5.6}x^{n-3}$ &c. Which when $n = 2$ will become $\frac{2x^3 + 3x^2 + x}{6}$, the theorem required.

This question was answered by Mr. *Farrer*, Mr. *Jepson*, Mr. *Peter Wood*, Mr. *Arch. Scyth*, Mr. *Landen*, Mr. *J. Ash*, Mr. *Williams*, and lastly by Mr. *Heath* in an elegant and general manner.

IV. QUES-

IV. QUESTION 259 *answered by Mr.* N. Farrer.



Let ASD represent the path of the projectile, thrown from A, with a velocity that will carry it in a perpendicular ascent to K; or in the direction AL, to L; and let this celerity carry it through the distance d in the time u, and the distance run through by a falling body in that time $= w$. Put any distance $AB = y$, s and e the sine and cosine of the angle of direction LAD; then will $\frac{usy}{de}$ be the time in which the projectile runs through the curve Aa, and $w \times \overline{\frac{usy}{de}}|^2$ the distance descended by a heavy body in that time; $\therefore Ba = \frac{seddy - wuussyy}{ddee}$; and when this $= CS$, then $y = \frac{sedd}{w}$, and the time of description $= \frac{sdes}{w}$. Put 1142 the feet sound moves in 1 second $= q$; then $q : 1 :: \frac{sedd}{w} : \frac{sdu}{w}$, therefore $d = \frac{qu}{e}$; consequently $\frac{sqquu}{ew}$ = the required distance. Now let $u = 1''$, then $w = 16\frac{1}{12}$ feet; hence $AD = 20895{\cdot}106$ feet $=$ 3 m. 7 f. 21 p. and 2 yards nearly, and the time of description $\frac{sdu}{w} = 18{\cdot}29$ seconds.

Mr. Tho. Cowper's *Answer.*

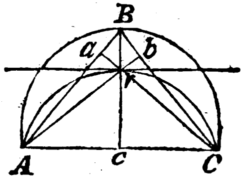
Let AL be the line of direction, and AD the distance of the person from the place of projection. Put t = tangent of the $\angle DAL$; $d = 16\frac{1}{12}$ feet, the perpendicular descent of heavy bodies in 1 second; $b = 1142$ feet, the velocity of sound in the same time; and x = the time the ball was in motion. Then $bx = AD$, and $dxx = DL$; but as $1 : t :: bx : tbx = DL$, $\therefore x = \frac{tb}{d}$; hence $\frac{tbb}{d} = AD = 20945{\cdot}59$ feet, the distance sought.

This

This queſtion was ſolved by Mr. *Heath*, Mr. *Powle*, Mr. *Jepſon*, Mr. *Bamfield*, Mr. *Davies*, Mr. *Terey*, *Bironnos*, Mr. *Williams*, Mr. *T. Garrard*, Mr. *J. Aſh*, and Mr. *Landen* the propoſer.

* V. QUESTION 260 *anſwered by Mr.* Heath.

It is evident by the data that the triangle muſt be iſoſceles. In the △ *ABC*, the ∠*A* = ∠*C*, or the ∠*B* muſt be the given angle; and either way the ∠*ArB* = ∠*CrB*, or ∠*ArC* is given = 101° 15′ oppoſite to the ſide given = 20 chains, on which deſcribing a ſegment of a circle to contain it, and drawing the parallel diſtance 8·19 (or rather 8·2 as it ſhould be) chains = *cr*, and it will be found a maximum (as is alſo proved by trigonometry), therefore the data are *rc*, *Ac* = *cC*, and ∠*B*, ∠*A* = ∠*C*, and ∠*BAb*; whence follow by trigonometry *Ar* = *Cr* = 12·925, *ar* = *rb* = 2·5215 (*AB* = *BC* = 15·64) chains, required.



VI. QUES-

V. QUESTION 260.

The meaning of this prob. is thus: In a triangle we have given the baſe (*AC*), the vertical angle (*ABC*), and the diſtance (*cr*) of the baſe from the common point of interſection of three lines drawn from the three angles perpendicular to the oppoſite ſides; to determine the triangle.

We are not at liberty to ſuppoſe the triangle iſoſceles, for that would be introducing a condition too much into the problem; and whether it be iſoſceles or of any other form, can only appear from the conſtruction or calculation.

CONSTRUCTION. On the given baſe deſcribe two ſegments of circles, the one *ABC* to contain the given vertical angle, and the other *ArC* to contain its ſupplement; parallel to *AC*, and at the given diſtance of the point from it, draw a line to cut or touch *ArC* in *r*; then through *r* draw *crb* ⊥ *AC*, and *B* will be the vertex of *ABC* the triangle required.

For, through *r* drawing *Arb* and *Crc*, ſince by the conſtruction *AC* is the given baſe, *ABC* the given vertical angle, and *rc* the given diſtance, we have only to prove that the angles at *a* and *b* are right angles. Now by the conſtruction it appears that *ArC*, *ABC* are ſegments of equal circles, and that the two ſegments together

VI. QUESTION 261 *answered by Mr.* N. Farrer.

Given $\frac{\sqrt{aaa} + \sqrt[4]{aaa} - \sqrt[3]{aa}}{b} = a$. In which write $y^{12} = a$, and it is $y^{10} + y - 640{\cdot}625y^4 - 1 = 0$, whence $y = 2{\cdot}936056$, and $y^{12} = a = 410367$; from which subtract $16b = 10250$ and there remains 400117 miles $= 6668° 37'$, from which subtract $360 \times 18 = 6480° 0'$, and there remains $188° 37'$. Hence $188° 37' - 180° 0' = 8° 37'$ S. the latitude of of the southermost pond; which taken from $85° 25'$, leaves $76° 48'$ N. the latitude of the northermost. In lat. $8° 37'$ S. sun's depression $= 34° 35'$, continuance under the horizon 8h. 48m. In lat. $76° 48'$ N. sun's depression $= 27{\cdot}48$; continuance under the horizon 91 days. And, supposing the intensity of cold as the sines of the sun's depression multiplied by the time of his continuance under the horizon, and the solidity or thickness of ice in the subtriplicate ratio thereof, we have $\sqrt[3]{{\cdot}2081} : \sqrt[3]{42{\cdot}441} :: ({\cdot}5926 : 3{\cdot}488 ::)$ 6 inches : 35·31 inches, the thickness of the ice of the northermost pond.

VII. QUES-

---

gether make up a whole circle; then the $\angle ABr$ or $aBr = \angle ACr$ as standing on equal segments Ar, and the opposite angles arB, crC are also equal; ∴ the third angles are equal, that is $\angle a = \angle c =$ a right angle. In the like manner b is proved to be a right angle.

SCHOLIUM. Another method of construction might be by first finding the point r as before, through which draw the indefinite lines Arb, Cra, and perpendicular to them the lines CbB, AaB. ——And then we should have to prove that these last two lines and cr, produced, meet in the same point B, and that ABC is $=$ the given angle.

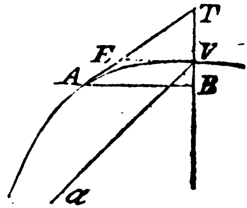
COROLLARY 1. The equal opposite angles Ara, Crb, being the supplements of the $\angle ArC$, are each $=$ the $\angle ABC$, which is also the supplement of ArC by the construction.

COROLLARY 2. Hence also the $\angle BAr = BCr$, and the four triangles BAb, aAr, BCa, bCr are all similar.

THE METHOD OF CALCULATION will be, first to calculate the angles rAC, rCA, of the $\triangle ArC$, by prob. V. *Simpson*'s Algebra, and then the segments Ac, Cc; then to each of these two angles adding the $\angle BAb$ or $BCa =$ the comp of $\angle ABC$, there will be had the $\angle BAC$ and BCA; from which and the segments Ac, Cc, the two hypothenuses AB, BC are easily got, and will come out 15·386 and 16·127, the angles at A and C being $52° 16'$ and $48° 59'$.

* VII. QUESTION 262 *answered by Mr.* N. Farrer.

Here is given the equation of the curve $x^5 - y^5 + axy = 0$.



Let $zx = y$, then $x = \sqrt[3]{\frac{az}{z^5 - 1}}$. $y = \sqrt[3]{\frac{az^4}{z^5 - 1}}$, and $\frac{y\dot{x}}{\dot{y}} = BT = \frac{4z^5 + 1}{z^5 + 4} \times \sqrt[3]{\frac{az}{z^5 - 1}}$, $\therefore VT = \frac{4z^5 + 1}{z^5 + 4} - 1 \times \sqrt[3]{\frac{az}{z^5 - 1}}$; and when the flowing quantity becomes infinite, the tangent AT will become an asymptote, in which case $VT = 4\sqrt[3]{\frac{a}{z^4}}$ and $\dot{x} : \dot{y} :: 1 + 4z^5 : 4 + z^5 :: VT : EV = \frac{16 + 4z^5}{1 + 4z^5} \times \sqrt[3]{\frac{a}{z^4}}$; hence the position of the asymptote is determined.

VIII. QUES-

---

* VII. QUESTION 262.

This question may be much better performed from the original equation alone without any substitution. Thus in the given equation $x^5 - y^5 + axy = 0$, supposing x to be infinite, the term axy will vanish in comparison of x^5 or y^5, and then $x^5 - y^5 = 0$, and $x = y$; that is, at an infinite distance the abscissa is = the ordinate, and therefore the asymptote, or tangent at the infinite distance, must make an angle of 45° with the abscissa. Again the given equation in fluxions gives $\dot{x} = \frac{5y^4 - ax}{5x^4 + ay}\dot{y}$, hence the subtangent BT or $\frac{y\dot{x}}{\dot{y}}$ is $= \frac{5y^5 - axy}{5x^4 + ay} =$ (by expunging y^5) $\frac{5x^5 + 4axy}{5x^4 + ay}$, and consequently $VT = BT - x = \frac{3axy}{5x^4 + ay}$ = (when $x = y$ = infinite) $\frac{3a}{5x^2} = 0$, and therefore the asymptote passes through the vertex V and makes an angle of 45° with VB. And the form of the curve is as represented in the above figure, where Va is the asymptote.

VIII. QUESTION 263 *answered by Mr.* Heath.

To rectify the conchoid of Nichomedes generally. Let $b = PB$, [*see the fig. to the Prize Q. for* 1738] $a = BA$ ($= b = 20$), $y = EF$ any ordinate, and $x = MD = BE$. Then because DF is always equal to BA, $EF = \frac{b+x}{x}\sqrt{aa-xx}$ and $AE = a - x$, its corresponding abscissa. In fluxions $-\dot{x} \times \frac{x^3 + baa}{xx\sqrt{aa-xx}} = \dot{y}$, whence $\sqrt{\dot{y}\dot{y} + \dot{x}\dot{x}} = ax^{-2}\dot{x} \times \sqrt{bbaa + 2bx^3 + x^4} \div \sqrt{aa-xx} = ax^{-2}\dot{x} \times : ba + \frac{x^3}{a} + \frac{x^4}{2ba} - \frac{x^6}{2ba^3}$ &c. $\times : \frac{1}{a} + \frac{x^2}{2a^3} + \frac{3x^4}{8a^5}$ &c. the fluent of which terms collected is $\frac{bx}{2a} + \frac{x^2}{2a} + \frac{x^3}{6ba} + \frac{bx^3}{8a^3} + \frac{x^4}{8a^3} + \frac{x^6}{16a^5}$ &c. $- \frac{ba}{x} - \frac{x^5}{20ba^3} - \frac{x^7}{112ba^5} - \frac{3x^9}{114ba^7}$ &c. but when $y = 60$, $x = 8{\cdot}6$ fere; consequently when $-x$ in the corresponding abscissa $= 8{\cdot}6$, or $x = -8{\cdot}6$, then the fluent or curve $= 43{\cdot}4$; but when $x = a$, and $x = -a$ in the abscissa, the fluent or curve $= 19{\cdot}6$, which added to the former is 63 fere, the length of the curve required.

IX. *The* 185*th or* 264*th* QUESTION *solved by Mr.* Heath.

By the equation of the curve $axx = y^3$ (a being as yet unknown, but considered as fixed) we get $\sqrt{\dot{x}\dot{x} + \dot{y}\dot{y}} = \dot{y}\sqrt{\frac{9y+4a}{4a}}$ for the fluxion of the curve, whose fluent (corrected) is $\frac{8a}{27} \times \overline{\frac{9y+4a}{4a}}\Big|^{\frac{3}{2}} - \frac{8a}{27} = c = 50$. Whence $aa + \frac{27cc - 36yy}{16 \times \overline{c-y}}a = \frac{27y^3}{16 \times \overline{c-y}}$. And

$$a = \frac{\sqrt{27 \times 64cy^3 - 16y - 72ccyy + 27c^4 + 36yy - 27cc}}{32 \times \overline{c-y}}.$$

The fluxion of the area of the semi parabola $= y\dot{x} = \frac{3yy\dot{y}\sqrt{y}}{2\sqrt{a}}$, whose fluent $\frac{3\sqrt{y^5}}{5\sqrt{a}}$ must be a maximum, and conse-

consequently $\frac{y^5}{a}$; therefore

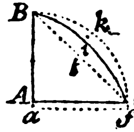
$$\frac{cy^5 - y^6}{\sqrt{3 \times 64cy^3 - 16y^4 - 72ccyy + 27c^4} + 12yy - 9cc}$$ will be a maximum. Put into fluxions, &c. $\overline{5c - 6y} \times$

$$\overline{\sqrt{3 \times 64cy^3 - 16y^4 - 72ccyy + 27c^4} + 12yy - 9cc} - 3 \times \frac{06cyy - 32y^3 - 72ccy}{\sqrt{3 \times 64cy^3 - 16y^4 - 72ccyy + 27c^4}} + 24yy \times \overline{c - y}$$

$= 0$; here $y = 34{\cdot}89$ by a new method of solving equations, and consequently $a = 35{\cdot}543$ fere, and the area of the whole parabola formed thereby (which is now the greatest) $1447{\cdot}4$ fere.

In answer to the objections by *Amicus*, a, in the equation, is as much a variable quantity as x or y, till it is determined. And in the equation $\frac{8a}{27} \times \overline{\frac{y + 4a}{4a}}|^{2} - \frac{8a}{27} = c$ (where $y = \frac{a^{\frac{1}{3}} \times \overline{27c + 8a}|^{\frac{2}{3}} - 4a}{9}$) it has a variable relation to y, when $\frac{y^5}{a}$ or $\frac{y}{a^{\frac{1}{5}}}$ is to be determined a maximum; and substituting this way for the value of y, $\frac{a^{\frac{1}{3}} \times \overline{27c + 8a}|^{\frac{2}{3}} - 4a}{9a^{\frac{1}{5}}}$, or $a^{\frac{2}{15}} \times \overline{27c + 8a}|^{\frac{2}{3}} - 4a^{\frac{4}{5}}$, will will as properly express the maximum, as if it had been denoted by relative y's; hence by making the fluxion of it $= 0$, we get by reduction $aa - \frac{9c}{16}a = \frac{27cc}{16^2}$, where the value of $a = \frac{3c\sqrt{21} + 9c}{32} = 35{\cdot}5433$ &c. whence $y = 34{\cdot}8909$ &c. and $\frac{6y^{\frac{5}{2}}}{5a^{\frac{1}{2}}} = 1447{\cdot}4001$, the area of the greatest parabolic space, as before. Also when $c = 10$, then $x = 6{\cdot}913$, $y = 6{\cdot}978$, and $a = 7{\cdot}1087$ &c. And when $c = 1000$, $x = 691{\cdot}3$, $y = 697{\cdot}82$, and $a = 710{\cdot}87$; by which it is proved that when the figure is a maximum, the abscissa and ordinate will be nearly equal.

In the present case, where $c = 50$, let $x = y$, then the equation $axx = y^3$ becomes $axx = xxy$ when the semi-parabola is nearly the greatest; and consequently $a = y = x$, at that time $= \dfrac{27c}{13^{\frac{3}{2}} - 8} = 34{\cdot}7292$ &c. from what is done above. Whence 1447·34 will be the area of the parabola very nearly a maximum. And therefore in all questions of this nature, the area may be computed by taking the ordinates and abscissas equal; the error being inconsiderable in the maximum. Draw the parabola $ABiCA$ correct, and it will approach the form of a quadrant $aBkCfa$, as near as the inequality of the curve BiC permits; and its double will ever be inscribed in a segment of a circle something less than a semicircle: but if the point C be made to pass through f, it will be inscribed exactly in a semi circle, and the area of the semi-space $ABiCA$ will vary from the true maximum, but by an exceeding small quantity, as is evident from above. When $ABkCA$ is a maximum, the space $ABtCA$, or $\triangle ABC$ is a maximum, which is when $AB = AC$: for, by the equation $x = \dfrac{y\sqrt{y}}{\sqrt{a}}$, and $yx = \dfrac{yy\sqrt{y}}{\sqrt{a}}$ or $\dfrac{y^5}{a}$ is a maximum as proved before: but since the relation of y and a can be only had from the rectification of the curve BiC, and its equation, with the length of curvature given, the inequality of the *(fiddlestick)* space $BiCtB$, or unequal curvature of BiC, involves a necessity of some little inequality betwixt AB and AC, when $ABiCA$ is most capacious.*



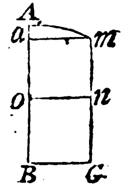
The

---

* This question is the same as the 185th, the solution of which was not compleated. Both the above two methods of solution bring out true answers, but the latter is much the easier. By Mr. *Emerson*'s method explained at the solution of the prize question for 1741 the same conclusions are also easily obtained.

The PRIZE QUESTION *answered by* Mr. R. Heath.

Projecting the curve $AmnG$, which is easily done by making $y = 1, 2, 3, 4, 5,$ &c. and thence finding the x's by the equation $\frac{y^y}{1000000} = x$; whence it will appear that when the semi-bung diameter $y = 7$ inches, that x will be but ·823543 which the half length of the cask should be taken out of, and proves an answer to be impossible: but correcting the data, and making the semi-bung diameter $y = 8 = BG$, then $x = 16·777216 = AB$; whence taking 15·9535 (or 15·953673) the semi-length of the cask, and there remains ·823543 $= Aa$; whence $am = 7$ inches, or the head diameter = 14, bung diameter = 16, and length 31·907346 inches; and being near the form of a cylinder, the mean diameter = 15 inches, and content of the semi-cask by the rotation of $BoamnG$ about aoB = 2819·25 &c. or 10 ale gallons, and the whole cask 20 gallons.



The prize of 10 *Diaries was won by Mr.* Heath.

The PARADOX *answered by Mr.* Tho. Sparrow.

Since the velocity of the fluid is always in the sub-duplicate ratio of the height of its surface above the hole, 'tis evident that when that height is infinitely small, the velocity must be so too, *i. e.* in effect, *Nothing*: Consequently the water can never be exhausted.

Of the Eclipses in 1746.

Calculated by Mr. Ralph Hulse.

To the inhabitants of our terraqueous globe there will happen four eclipses, two of each luminary. The 1st of the moon, February 4th, but invisible at London, as ending 22m. 44s. before the moon rises. The 2d of the sun, on the 4th of March, early in the morning, but invisible in the horizon of London. The 3d is a visible eclipse of the moon, the 19th of August, 3 quarters past 10 at night, and visible at London, according to the following calculation, viz. Beg. 10h. 36m. Mid. 12h. 8m. End 1h. 24m. Total duration 2h. 38m. Digits eclipsed 8deg. 22m. The 4th is of the sun, September 4th, in the afternoon, but invisible.

New Questions.

I. QUESTION 265, *by Mr.* Heath.

Myfterious things, we always find,
Are moft amufing to mankind;
When once familiar they appear,
We look for more another year.
Juft fo, when men are plainly known,
We're weary of acquaintance grown;
We hug the ftrange, and leave the true,
And ftill are feeking fomething new.
Ladies, how comes this ftrange inconftancy,
So vifible in you, as well as me.

The QUESTION.

If the hind wheel of a coach be feven feet in diameter, and a tack be driven into the middle of the fpoke (or radius) ftanding next the ground, and a nail touch the ground at the end of the faid fpoke (or radius) when the coach fets out to travel: Quere how many miles will the tack and nail travel refpectively in driving the coach from London to Exeter; allowing the diftance between thofe two places to be 200 miles? What will be the nature of the curves they defcribe? And their pofition, or height of tack and nail from the ground, at the end of the journey.

II. QUESTION 266, *by Mr.* Farrer.

Surveying a triangular field *ABC*, and ftanding at the corner *C*, I took the angle included between the fide *BC* and a line drawn from the angle *C*, to a houfe fituated within the field, and found it 78° 10′. I then proceeded to meafure the fhorteft diftance to the oppofite fide *AB*, and having meafured 20 chains, I obferved the houfe and the angle *A* in a right line; then meafured on 10 chains to the fide *AB*: I likewife obferved that the fides *AC* and *BC* were equal, and the houfe equally diftant from the angles *A* and *C*. Quere the area of the field?

III. QUESTION 267, *by Mr.* Powfe.

Given $35x + 43y + 55z = 4000$, to find all the poffible values of x, y, z, in whole numbers, and to fhew the method of inveftigation?

IV. QUES-

IV. QUESTION 268, *by* Fortunatus.

Let the sorts of faces to be thrown on seven dice by four persons, at a single throw each, be as follow, viz. by *A*, a^4bcd; by *B*, a^2bcdef; by *C*, $a^2b^2c^2d$; by *D*, a^3b^2cd: Quere their respective chances of winning? And what throw, as to sorts of faces, has the greatest number of chances for coming up, at a single throw, of all the sorts which can be thrown on the said number of dice?

N. B. The number of the same and different letters represent so many of the same and different sorts of faces, viz. so many aces, duces, trays, cators, &c. of the same and different sorts.

V. QUESTION 269, *by Mr.* J. Ash.

A gentleman has a piece of ground, whose three sides are an abscissa, semi-ordinate, and curve, the equation of which is $ax = y^3$; he has taken from thence the biggest oblong garden which he could possibly enclose, whose area is 141·84243 poles; and he finds the abscissa longer than the semi-ordinate by 5 poles. It is required from thence to find the area of the whole enclosure, its perimeter, and the sides of the garden (taken out of it) separately?

VI. QUESTION 270, *by* Rhinoceros.

The perpendicular of a triangular field is 200 poles; the line equally bisecting the angle opposite to the base, drawn to the base, is 250 poles; and the distance from the said obtuse angle to the middle of the base is 295 poles: Quere the sides and area of that triangular field, with the geometrical construction of the same.

VII. QUESTION 271, *by Mr.* Christ. Mason.

Lately young Chloe struggling to be coy,
And still prolong her Strephon's wish'd-for joy,
Did artfully a stratagem contrive,
Herself to stint, her Strephon still deprive:
But he yet pressing with the urgent *when*
Shou'd he be made the happiest of men;
Your when, quoth she (if you can make't appear)
That night the twilight's shortest in the year.
Pray lend your aid the nuptial night to fix:
The latitude is fifty, forty-six. *i. e.* 50° 46′

VIII. QUES-

VIII. QUESTION 272, by F. R. S.

In what law of gravity will a projectile describe a curve express'd by the equation $ax^2 = y^3$, in a non-resisting medium?

IX. QUESTION 273, by Mr. Farrer.

Quere the area of a right-angled triangle whose hypothenuse is x^{3x}, and the two legs x^{2x} and x^x?

X. QUESTION 274, by Mr. Clarke.

The thickness of a ring belonging to a ship's anchor is nine inches in circumference, and the outward circumference shewing the width of that ring is 50 inches: Quere the solid content, and weight thereof?

XI. QUESTION 275, by Filius Diophanti.

To find three numbers, that when each is subtracted from the cube of their sum, a cube number shall remain?

XII. QUESTION 276, by Mr. J. Landen.

It is required to find the periodic time of a pendulum describing a conical surface; the perpendicular height of the described cone being 200 inches?

XIII. QUESTION 277, by Crocus Metallorum.

What annuity, to continue as many years as its pounds, can I purchase for the square of its pounds ready money, allowing me 5l. per cent. per ann. compound interest for my bargain?

XIV. QUESTION 278, by Hurlothrumdro.

Required the ratio of the diameter of the bore to the length of a piece of cannon (or other fire arms) to make it capable of throwing a ball the farthest possible; supposing the diameter of the ball nearly equal to the diameter of the bore, with a proportionable weight of powder, and the metal of the piece formed sufficient to sustain the effect?

PRIZE

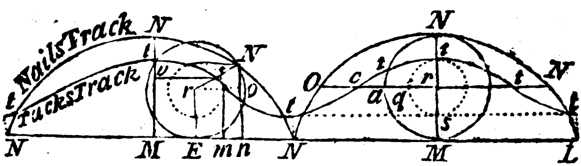
PRIZE QUESTION *by Mr.* W. Chapple.

A gentleman has a circular garden, whose diameter is 310 yards, in which is contained a circular pond, whose diameter is 100 yards, so situated in respect of each other, that their peripheries will inscribe and circumscribe an infinite number of triangles [*i. e.* whose sides shall be tangents to the pond, and angles in the fence of the garden.] He being disposed to make enclosures for different uses, and farther ornaments on his scheme begun, in order thereto applies himself to the artists of Great Britain for the dimensions of the greatest and least triangles that can be inscribed and circumscribed as aforesaid? and the nearest distance of the peripheries of the garden and pond? and for a demonstration of the truth of his pond's situation?

1747.

Questions answered.

I. QUESTION 265 *answered by the Proposer Mr.* Heath.



THE track of the nail will be the curve *LNNONN* &c. which is a cycloid, and that of the tack the curve *ttt* &c. both which curves are thus rectified. Put $a = NM = 7$, $x = Nr$, $y = Or$, $v = ar = \sqrt{ax - xx}$, $z =$ circular arch $Na = Oa$ (per nature curve), $\because z + v = y$, $vv = ax - xx$,

$\dot v = \frac{a-2x}{2v} \times \dot x$, $\dot z = \frac{a\dot x}{2v}$, also $\dot y = \dot z + \dot v = \frac{a-x}{v}\dot x =$

a − x

$\frac{a-x}{\sqrt{ax-xx}}\dot{x}$, hence $\sqrt{\dot{x}\dot{x}+\dot{y}\dot{y}}=\dot{x}\sqrt{\frac{a}{x}}$, whose fluent is $2\sqrt{ax}$ = arch NO; therefore when $x=a$, arch $NON=2a$, consequently $NONNtL=4a=28$ feet, the nail describes in one revolution of the wheel. The nature of the curve described by the tack is expressed by $cq=2$ arch qt, which referred to the foregoing symbols, will be $y=2z+v$, parts of the lesser circle. Consequently, by substitution in this case, the fluxion of the inner curve ctt will be $\frac{\dot{x}}{2}\sqrt{\frac{9aa-8ax}{ax-xx}}$; whose fluent, by series, is $\sqrt{ax}\times:3+\frac{x}{18a}+\frac{7xx}{216aa}+\frac{613x^3}{3\times7\times6^4a^3}$ &c. which when $x=ts=3{\cdot}5=a$, is 11·696 fere (but this fluent may be otherwise found). Hence the tack describes 23·392 feet in one revolution, whilst the nail describes 28, and the axis 21·9911485 &c. $=7\times3{\cdot}141592653$ the wheel's circumference; by which the wheel will revolve 48019·31995 times in travelling 200 miles.——But 21·9911485 : 28 :: 200 : 254·648 miles, travelled by the nail. And 21·9911485 : 23·392 :: 200 : 212·74 miles travelled by the tack pretty nearly; the small difference being only what EN (·31995 part of a revolution) differs in proportion with N, N, and t, t, part of curves described by nail and tack at the end of the journey, in the position of N, t, r, making an $\angle NrE$ $=·31995\times360=115°\ 11'$, or $Nro=25°\ 11'$ with the horizon. Whence $Nn=4{\cdot}988$ feet, and $tm=4{\cdot}244$ the height of nail and tack from the ground.

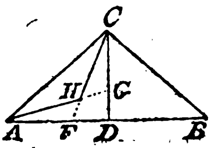
N. B. The number of whole revolutions multiplied into the whole curves aforesaid, and the rectifications of the last parts being respectively added, will exactly shew the distances described by the nail and tack, very nearly as before. Or this question might be resolved by a curve described upon the curve of a great circle of the earth, which solution would come out not much different.

Mr. *Bamfield* has curiously described these curves, and given an exact solution of their lengths; and so has Mr. *I. Waine*; which are the only true answers. Mr. *Bamfield* has found a point of inflection and retrogression to be in the inner curve, when the stroke and tack are horizontal.

II. QUE-

II. QUESTION 266 *answered by Mr.* I. Waine.

Put x = sine, and y = cos. $\angle CAD = \angle CBD$, and s and c for those of the given $\angle FCB$. Then by elements of trigonometry, $sx - cy$ = sine, and $sy + cx$ = cos. $\angle FCD$: But $\angle ACD - FCD = \angle ACF$ whose tang. is $\frac{syy - sxx + 2cxy}{cxx - cyy + 2sxy}$, which per quest. = tang. $\angle CAG$. But $AD : GD$ :: radius : tang. $\angle GAD = \frac{x}{3y}$; whence $\frac{2xy}{3yy + xx}$ = tang. $\angle CAD - \angle GAD = \frac{syy - sxx + 2cxy}{cxx - cyy + 2sxy}$; reduced, $\frac{x^4}{y^4} + \frac{6xx}{yy} - \frac{8cx}{sy} = 3$; solved, $\frac{x}{y}$ = ·80907 = tang. of 38° 58′ 30″; whence AD = 37·0796, and consequently the area = 111·239 acres. *Q.E.F.*



Mr. *Kingston*'s answer is the same.

Mr. Ash *answers this* Question *thus.*

Call the tang. of the given angle m; CD, a; and GD, c; also put x for tang. $\angle ACD$. Then tang. $\angle ACB$ will be $\frac{2x}{1 - xx}$, and that of $\angle ACH$ (when $\angle ACB$ is obtuse) $\frac{2x + m - mxx}{2mx + xx - 1}$. But (by trig.) $\frac{c}{ax}$ = tang. $\angle DAG$, $\therefore \frac{ax - cx}{axx - c}$ = tang. $\angle HAC = \angle ACH$; consequently $\frac{ax - cx}{axx - c} = \frac{2x + m - mxx}{2mx + xx - 1}$; solved, x = 1·2359 = tang. of 51° 1′ fere. Whence the area of the field = 111·23 acres.

Mr. *John Turner* has elegantly solved this question; so has Mr. *Anth. Baker*, and some others.

Mr. *Cuth. Cockson* informs us that this question is taken from Ronayne's Algebra, p. 273, being case 1 of prob. 12. We would do the proposer all the honour due to so distinguished a genius, but yet we desire to have sent what is new, as well as curious; being rather desirous that the Ladies' Diary should be a pattern for, than an imitation of others.

III. QUES-

III. QUESTION 267 *answered by Mr.* J. Waine.

For z in the given equation $(35x + 43y + 55z)$ substitute its least value, viz. 1, and we have $y = 91\frac{32}{43} - \frac{35}{43}x$, therefore $\frac{32 - 35x}{43}$ must be some whole number; multiplied by 2 and divided, is $1\frac{21}{43} - \frac{27x}{43} - x$, $\therefore \frac{21 - 27x}{43} \times 2$ will be a whole number $= \frac{42}{43} - \frac{11x}{43} - x$, $\therefore \frac{42 - 11x}{43} \times 4$ is also some whole number $= 3\frac{39}{43} - \frac{x}{43} - x$, whence $\frac{39 - x}{43}$, or rather $\frac{x - 39}{43} = m$, a whole number, $\therefore x = 43m + 39$, which value of x substituted for x in $y = 91\frac{32}{43} - \frac{35x}{43}$, gives $y = 60 - 35m$; whence $x = 43m + 39 = 82$ or 39, $y = 60 - 35m = 60$ or 35, $z = 1$. And thus by assuming $z = 1, 2, 3, 4$, &c. we obtain all the possible values of x, y, and z, viz. z admitting of 65 different values, and x and y of 89.

Mr. *John Turner* confirms the same by working out all the numbers. Mr. *Farrer* has exhibited a concise method for finding those numbers; and Mr. *Landen* is very explicit in finding the same. Mr. *Cuth. Cockson*, Mr. *Flitcon*, and Mr. *John Williams* likewise answered this question.

The following Table is a Compendium of Mr. *Turner*'s, deduced from $x = \frac{4000 - 43y - 55z}{35}$, and also $= \frac{20z + 8y - 10}{35}$, by finding a Submultiple of 35.

z	y	x	z	y	x	z	y	x
1	25 60 —	82 39 —	4	— 35 70	— 65 22	7	10 45 80	91 48 5
2	5 40 75	105 62 19	5	15 50 85	88 45 2	8	25 60 —	71 28 -
3	20 55 —	85 42 —	6	30 65 —	68 25 —	9	5 40 75	94 51 8
And so on to $z = 65$								

IV. QUES-

IV. QUESTION 268 *answered by Mr.* Heath.

No solution has appeared in any author to questions of this nature, which will admit of several varieties still to be proposed. Mr. *Kay*'s question was the first proposed, in a particular case, and the general method of solution is exhibited in the following examples, not hit upon before, that I have seen, by any.

Sorts of Faces.	N° of Chances.	Combinations.		Permutations of each comb.
a^2	6	Sides revolve		
ab	30	$= \frac{6.5}{1.2}$ Prog. of equal indices.	×	$\frac{2.1}{1.1}$ dec. dice / decrease of highest indices of faces.
6^2 all ch. on 2 dice	= 36			
a^3	6			
a^2b	90	$= 6.5$	×	$\frac{3.2.1}{2.1.1}$
abc	120	$= \frac{6.5.4}{1.2.3}$ Prog. eq. ind.	×	$\frac{3.2.1}{1.1.1}$
6^3 all ch. on 3 dice	= 216			
a^4	6			
a^3b	120	$= 6.5$	×	$\frac{4.3.2.1}{3.2.1.1}$
a^2b^2	90	$= \frac{6.5}{1.2}$	×	$\frac{4.3.2.1}{2.1.2.1}$
a^2bc	720	$= \frac{6.5.4}{1.2}$	×	$\frac{4.3.2.1}{2.1.1.1}$
$abcd$	360	$= \frac{6.5.4.3}{1.2.3.4}$	×	$\frac{4.3.2.1}{1.1.1.1}$
6^4 all ch. on 4 dice	= 1296			
a^5	6			
a^4b	150	$= 6.5$	×	$\frac{5.4.3.2.1}{4.3.2.1.1}$
a^3b^2	300	$= 6.5$	×	$\frac{5.4.3.2.1}{3.2.1.2.2}$
a^3bc	1200	$= \frac{6.5.4}{1.2}$	×	$\frac{5.4.3.2.1}{3.2.1.1.1}$
a^2b^2c	1800	$= \frac{6.5.4}{1.2}$	×	$\frac{5.4.3.2.1}{2.1.2.1.1}$
a^2bcd	3600	$= \frac{6.5.4.3}{1.2.3}$	×	$\frac{5.4.3.2.1}{2.1.1.1.1}$
$abcde$	720	$= \frac{6.5.4.3.2}{1.2.3.4.5}$	×	$\frac{5.4.3.2.1}{1.1.1,1.1}$
6^5 all ch. on 5 dice	= 7776			

Faces	Chances.	Combinations.		Permutations of each comb.
a^6	6			
a^5b	180	$= 6.5$	×	$\frac{6.5.4.3.2.1}{5.4.3.2.1.1}$
a^4b^2	450	$= 6.5.4$	×	$\frac{6.5.4.3.2.1}{4.3.2.1.2.1}$
a^4bc	1800	$= \frac{6.5.4}{1.2}$	×	$\frac{6.5.4.3.2.1}{4.3.2.1.1.1}$
a^3b^3	300	$= \frac{6.5}{1.2}$	×	$\frac{6.5.4.3.2.1}{3.2.1.3.2.1}$
a^3b^2c	7200	$= 6.5.4$	×	$\frac{6.5.4.3.2.1}{3.2.1.2.1.1}$
a^3bcd	7200	$= \frac{6.5.4.3}{1.2.3}$	×	$\frac{6.5.4.3.2.1}{3.2.1.1.1.1}$
$a^2b^2c^2$	1800	$= \frac{6.5.4}{1.2.3}$	×	$\frac{6.5.4.3.2.1}{2.1.2.1.2.1}$
a^2b^2cd	16200	$= \frac{6.5.4.3}{1.2.1.2}$	×	$\frac{6.5.4.3.2.1}{2.1.2.1.1.1}$
a^2bcde	10800	$= \frac{6.5.4.3.2}{1.2.3.4}$	×	$\frac{6.5.4.3.2.1}{2.1.1.1.1.1}$
$abcdef$	720	$= \frac{6.5.4.3.2.1}{1.2.3.4.5.6}$	×	$\frac{6.5.4.3.2.1}{1.1.1.1.1.1}$
6^6 all ch. on 6 dice	= 46656			
a^7	6			
a^6b	210	$= 6.5$	×	$\frac{7.6.5.4.3.2.1}{6.5.4.3.2.1.1}$
a^5b^2	630	$= 6.5$	×	$\frac{7.6.5.4.3.2.1}{5.4.3.2.1.2.1}$
a^5bc	2520	$= \frac{6.5.4}{1.2}$	×	$\frac{7.6.5.4.3.2.1}{5.4.3.2.1.1.1}$
a^4b^3	1050	$= 6.5$	×	$\frac{7.6.5.4.3.2.1}{4.3.2.1.3.2.1}$
a^4b^2c	12600	$= 6.5.4$	×	$\frac{7.6.5.4.3.2.1}{4.3.2.1.2.1.1}$
a^4bcd	12600	$= \frac{6.5.4.3}{1.2.3}$	×	$\frac{7.6.5.4.3.2.1}{4.3.2.1.1.1.1}$
a^3b^3c	8400	$= \frac{6.5.4}{1.2}$	×	$\frac{7.6.5.4.3.2.1}{3.2.1.3.2.1.1}$
$a^3b^2c^2$	12600	$= \frac{6.5.4}{1.2}$	×	$\frac{7.6.5.4.3.2.1}{3.2.1.2.1.2.1}$
a^3b^2cd	75600	$= \frac{6.5.4.3}{1.2}$	×	$\frac{7.6.5.4.3.2.1}{3.2.1.2.1.1.1}$
a^3bcde	25200	$= \frac{6.5.4.3.2}{1.2.3.4}$	×	$\frac{7.6.5.4.3.2.1}{3.2.1.1.1.1.1}$
$a^2b^2c^2d$	37800	$= \frac{6.5.4.3}{1.2.3}$	×	$\frac{7.6.5.4.3.2.1}{2.1.2.1.2.1.1}$
a^2b^2cde	75600	$= \frac{6.5.4.3.2}{1.2.1.2.3}$	×	$\frac{7.6.5.4.3.2.1}{2.1.2.1.1.1.1}$
a^2bcdef	15120	$= \frac{6.5.4.3.2.1}{1.2.3.4.5}$	×	$\frac{7.6.5.4.3.2.1}{2.1.1.1.1.1.1}$
6^7 all ch. on 7 dice	= 279936	Sum, &c. *ad infinitum.*		

Hence

Hence the respective chances of *A*, *B*, *C*, and *D* winning are obvious, as 12600, 15120, 37800, and 75600, the same as 1, 1·2, 3, and 6 exactly. And a^3bbcd and $aabbcde$ have equal, and the greatest number of chances for coming up, viz. 75600. Hence also is inferred that the best throw for winning will be when the sorts are within a place or two of being all different. *Q.E.F.*

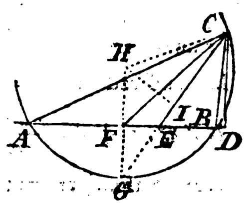
V. QUESTION 269 *answered by Mr.* John Turner.

Put $x = HG$, $y = GF$, $b = AD$, $b - x = GD$, [*see fig. page* 109] a = parameter, the equation of the curve being $ax = y^3$, $\therefore y = \sqrt[3]{ax}$, and $\overline{b-x} \times \sqrt[3]{ax}$ is to be a maximum; which in fluxions, &c. gives $-\sqrt[3]{ax} + \frac{b-x}{\sqrt[3]{aaxx}} \times \frac{a}{3} = 0$; hence $3x = b - x$, and $x = \frac{1}{4}b$ exactly. To proceed, put $z = HD$, $\frac{z}{4} = HG$, $z - 5 = DA$, $\frac{3z}{4} = GD$, $m = 141{\cdot}84243$; then $GF = \sqrt[3]{\frac{az}{4}}$, $DA = \sqrt[3]{az} = z - 5$, $d = \sqrt[3]{4} = 1{\cdot}5874$, and per quest. $\sqrt[3]{\frac{az}{4}} \times \frac{3z}{4} = m$. Hence $\sqrt[3]{az} = \frac{4dm}{3z} = z - 5$; $3zz - 15z = 4dm$; here $z = 20$ exactly. And $a = 168{\cdot}75$; also the length of the curve $HFA = 27{\cdot}014$. So that the fences of the garden are $GF = 9{\cdot}4494$, and $GD = 15$: Moreover $HD = 20$, $DA = 15$. And the area $HFADH = \frac{3}{4} HD \times DA = 225$ square poles, or 1 acre, 1 rood, 25 perches.

This question was concisely solved by Mr. *Farrer*. And also solved by Mr. *J. Waine*, and by Mr. *Kingston* the same; the Rev. Mr. *Baker*; the proposer, Mr. *Ash*; and others.

VI. QUESTION 270 *answered by Mr.* Farrer.

At the point *D*, on the line *AD* raise the perp. $CD = 200$ poles; make $CE = 250$, and $CF = 295$; produce *CF* to cut the perp. *FG* in *G*, draw *CH* making the $\angle GCH = \angle CGH$; then upon *H*, as a center, with the rad. *HG* describe the arch *CBGA*. Join *CB* and *CA*, and the $\triangle ABC$ is that required.



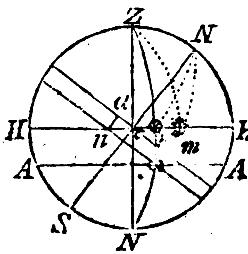
CALCU-

CALCULATION. Having CD, CE, and CF given, find $ED = 150$, $FD = 216\cdot85$, and $EF = 66\cdot15$. But $ED : DC :: EF : GH = 89\cdot14$; $\because GE = 111\cdot42$; and $GF : GE :: GI : GH = 225\cdot89$; hence $AB = 359\cdot58$, $BC = 203\cdot41$, and $AC = 441\cdot21$. The area 35958 square poles, or 224 a. 2 r. 38 perches.

Mr. *Ash* has elegantly constructed and solved this question, and so has Mr. *John Turner*, Mr. *Joseph Orchard*, Mr. *John Hunter*, (and Mr. *James Tercy*, who objects to the solution of question 260 by Mr. *Heath*, as supposing him not to know that three perpendiculars let fall from the angles of a triangle would intersect, in one point, within or without the triangle; his expression of data not inferring such thing.) It is also analytically solved by Mr. *Richard Gibbons*, Mr. *Baker*, Mr. *J. Waine*, Mr. *Kingston*, and others.

VII. QUESTION 271 *answered by Mr.* J. Turner.

If N represent the north pole, Z the zenith, H☼ an arch of the horizon, A☼A a parallel of the sun's depression 18° below it; and ☼⊙ a parallel of declination descended; then the angle ⊙N☼ will be a minimum: but 'tis also evident that when the crepusculum is the shortest, the distance ☼⊙ descended in the parallel of declination will be a minimum: being the arch of a lesser circle cutting the same azimuth in the points of setting and end of twilight. And this will be when the motion of the sun is most perpendicular, and therefore descends the fastest to 18°, or the parallel of A⊙A; which he does by touching at the points ☼ and ⊙ in the same azimuth Z☼⊙ with his setting, instead of making an oblique angle with the setting azimuth, as in the position of ☼m⊙ with Za☼. This being admitted, put $y =$ cos. N⊙ $= N$☼; m and n sine and cos. Z⊙ $= 108°$, or sines of 72° and 18°; p and q sine and cos. $NZ =$ comp. lat. and rad. unity $= Z$☼.



Then per spherics $\frac{y}{p} =$ cos. NZ☼, and also $\frac{nq - y}{mp} =$ cos. NZ⊙, which (for the above reasons) are $=$ to each other, i. e. $\frac{y}{p} = \frac{nq - y}{mp}$, there-

fore

fore $y = \frac{nq}{m+1}$, which is a general theorem for all questions of this nature. And the sun's declination in the present case is found by it $= 7^\circ\ 2'\ 42''$ southerly, answering to Feb. 19, or Sept. 30; $\because m+1 : n :: q : y$.

N. B. If the lat. is north, the declination must be south; and vice versa.

Mr. Farrer *answers thus.*

Let ☉ be the point in the circle bounding twilight 18° below the horizon, where the shortest twilight happens (without saying why the bounding point falls in the same azimuth circle with the sun's setting.) Then in the triangle bc☉, ∠☉ $= 90^\circ$, $\angle c =$ co-lat. ☉$b = 9^\circ$, c☉ $= cn$, whence rad. : tang. ☉b :: sine lat. : sine arc $= 7^\circ\ 3'$ fere, the sun's declination when Strephon is allowed to wed his Cloe.

Mr. *J. Turner* (from Dr. Gregory's Elem. of Astron.) confirms the same proportions.*

VIII. QUESTION 272 *answered by Mr.* Landen.

Let ArC represent the curve; AB the axis thereof; Bm and Hr ordinates indefinitely near each other. [*See fig. to* Q. 253.] Call AH, x; Hr, y; and the gravity G. Then since the velocity in the direction Hr is always the same, that in the direction rn will be $\frac{\dot{x}}{\dot{y}}$, whose fluxion $\frac{\ddot{x}}{\dot{y}}$ ($\dot{y}$ being constant) will be as $G \times \dot{y}$; that is, as the force by which the body is accelerated at r drawn into the time of describing rm. Hence putting $G \times \dot{y} = \frac{\ddot{x}}{\dot{y}}$ we have $G = \frac{\ddot{x}}{\dot{y}\dot{y}}$. In which expression if for $\ddot{x}$ we put its value $\frac{15\dot{y}\dot{y}\sqrt{y}}{4\sqrt{a}}$, found by the equation of the curve ($axx = y^5$) we get $G = \frac{15\sqrt{y}}{4\sqrt{a}}$; i. e. the gravity in this case must be in a subduplicate ratio of the ordinates, or in a subquintuplicate ratio of x, the distance of the ordinate from the vertex.

Mr. *Farrer* attempted this solution by another method, as also did Mr. *Ash*.

IX. QUES-

* A construction to this question is given at question 564.

* IX. QUESTION 273 *answered by Mr.* I. Ash.

Let $y = x^x$, then (per 47 E. 1, and per qu.) $y^2 + y^4 = y^6$, $y^2 = \frac{\sqrt{5}+1}{2} = 1{\cdot}618034$, $y = 1{\cdot}272019$, and the area required $= 1{\cdot}02908$.

Mr. *Richard Gibbons* solved this question in the same elegant manner. The Rev. Mr. *Baker* likewise gave a curious solution, and so did Mr. *Farrer* the proposer, and others.

X. QUESTION 274 *solved by Mr.* Landen.

Put $d =$ diff. of the given circumferences, $r =$ rad. of the lesser circumference, and $x =$ any abscissa of the circle whose radius is r. Then $d \times 2\sqrt{2ax - xx} \times \dot{x}$ is the fluxion of half the required solidity. But the fluent of $2\sqrt{2ax - xx} \times \dot{x}$, when $x = r$, is the area of the semi-circle, whose radius is r; therefore, the area of the circle shewing the thickness of the ring, multiplied by d, the diff. of the given circumferences, will give the solid content of the ring $= 264$ inches fere. Whence the weight thereof, according to Dr. Wiberd, is nearly $= 77$ pounds avoirdupois.

Mr. *John Turner*, and several others, have proved the solidity of the ring to be equal to a cylinder whose length is equal to the middle circumference, and the area at the base equal to the area of its circular section, or of the circle whose diameter expresses the thickness; most agreeing in the solidity to be $= 264$ &c. inches, and the weight 73 pounds, &c. according to Ward's proportions.

N. B. There are several methods of investigating the fluxion of this ring, whose fluents respectively give the solid content as above.

XI. QUES-

---

* IX. QUESTION 273.

Of this triangle, the perpendicular from the right angle on the hypothenuse is $= 1$. For it is $=$ the double area divided by the hypothenuse $=$ the product of the two legs divided by the hypothenuse $= \frac{x^x \times x^{2x}}{x^{3x}} = \frac{x^{3x}}{x^{3x}} = 1$.

XI. QUESTION 275 *answered by Mr.* J. Hampson.

The numbers are $\frac{12851}{85184}$, $\frac{19467}{85184}$, and $\frac{18954}{85184}$; these fractions are in lower terms than those given by Dr. Wallis from Dr. Pell, where the method of solution may be seen.

Mr. Turner *gives this Answer from Dr. Wallis's Algebra.*

1st, $a=\frac{494424}{2352637}$; 2d, $b=\frac{472696}{2352637}$; 3d, $c=\frac{448000}{2352637}$, whose sum $=\frac{1415120}{2352637}=\frac{80}{133}$. The cube of their sum $=\frac{512000}{2352637}$, from which numbers taking severally the values of a, b, and c, there will remain these three cubes, viz. $\frac{17576}{2352637}$, $\frac{39304}{2352637}$, and $\frac{64000}{2352637}$, whose roots are $\frac{26}{133}$, $\frac{34}{133}$, and $\frac{40}{133}$. *Q. E. F.**

† XII. QUESTION 276 *answered by Mr.* I. Ash.

Suppose the sine of the vertical angle of the cone ·0507; then by trigon. the side or length is found $=3944{\cdot}77$ inches or $328{\cdot}73$ feet $=$ length of the pendulum's string. Hence per

* This is the same with question 51, which see for a solution.

† XII. QUESTION 276.

The principle used in the above solution is not general for any angle at the vertex, but only for that one particular angle there used, as may be seen in the Prop. of *Keil* there referred to. But the times of gyration do not depend on the length of the string but only on the altitude of the cone, they being universally as the square roots of the altitudes; and when the altitudes are equal, the times will be equal also, whatever the lengths of the pendulums may be.

By Prop. IX. *Emerson*'s Centrip. Forces, the proportion is universally thus, as $\sqrt{16\frac{1}{12}}$ feet : $\sqrt{400}$ inches (twice the given ⊥) :: $3{\cdot}1416$: $3{\cdot}1416\sqrt{\frac{400}{193}}=\frac{62{\cdot}832}{\sqrt{193}}=4{\cdot}5228$ seconds, the time required.

(per Keil's Introd. Theor. 11, p. 302) the time of one revolution is equal to the time of the perpendicular fall of a heavy body from a height equal to the pendulum's length, ∵ $16\frac{1}{12}$: $1''$ :: 328·73 : 20·439442″ the square time, whose root = 4·521 seconds required.

This question was elegantly solved by Mr. *Farrer*, Mr. *Turner*, Rev. Mr. *Baker*, and others, which agree with the proposer's solution.

XIII. QUESTION 277 *answered by Mr.* Heath.

Square is printed instead of *Square Root* in this question; correcting which, and putting $a =$ the pounds of an annuity, $r = 1·05$ the amount of a pound and its interest for one year at 5 per cent. $t =$ the year's continuance; then $\frac{a}{r-1} - \frac{a}{r^a \times \overline{r-1}} = z$, the present worth. And if the conditions of the questions be substituted therein, the equation becomes $\frac{a}{a-1} - \frac{a}{r^a \times \overline{r-1}} = \sqrt{a}$. Whence $1·05^a = \frac{\sqrt{a}}{\sqrt{a}-·05}$; here $a = 1·034$ fere = 1 l. 0s. 7 d. the annuity required.

N.B. As the question was printed, the final equation is $1·05^a = \frac{1}{1-·05a}$, where a is evidently = 0, or the least money possible.

~~* XIV. *Observation*~~ *on* QUESTION 278, *by the* Editor.

This question was proposed with an intent to improve gunnery, of which there are several things wanting. In particular, a treatise on the subject by an experienced hand: For to be treated on by any other person, will only be compiling of matters already known. And in order for this improvement, and the solution of this question, experiments should be made in a general way, which we have not yet received.

The

---

* XIV. QUESTION 278.

The Editor observes above that this question was proposed with an intent to improve practical gunnery, though it does not appear from

The PRIZE QUSTION *answered by Mr.* R. Heath.

The biggest two circles, within one another, admitting of an infinite number of triangles to be drawn with their angles and sides terminating in their outer, and touching their inner peripheries, are those which are concentric, and their diameters exactly as 2 to 1; in which case the triangles will be all equal and equilateral. This is so evident as to appear upon the slightest examination. And if the diameters be in proportion less than 2 to 1, then no triangle whatsoever can be drawn as aforesaid. But if the proportion be greater than 2 to 1 (as 310 to 100, or 3·1 to 1 as in the present case) their peripheries being eccentric and at a proper distance, will admit of an infinite variety of triangles to be drawn in and about them, from the isosceles △ *ACC* to that of *BDD*, which are the greatest and least triangles: Because the area of every triangle so drawn, being equal to the sum of all the sides

---

from the nature of the science that it would at all have answered that purpose. It is well known that *short pieces* are requisite at *sea*, both for the convenience of working them in an engagement, and on account of the small space they must stand in when the ports are closed. The *land-service*, on the contrary, requires *long pieces*, particularly in the attack of a place, in order to preserve the embrasures from the blast of the powder, which short pieces would soon destroy, besides the danger of setting fire to a fascine battery. Not only the *lengths* of pieces are *limited* by the nature of the service, but also the *diameters of the bores*; for pieces which carry balls from 24 to 42 pounds comprehend the limits of *battering* cannon, and those from 3 to 12 pounds limit *field* pieces. Experience proves that balls of a less weight than 24 pounds are insufficient to make a moderate breach; and that pieces carrying 42 pound balls, or upwards, become unmanageable from their great weight; so that in general the 32 pounder is the most common battering piece. When the field piece exceeds 12 pounds, it, in like manner, becomes too unwieldy for *that* service.

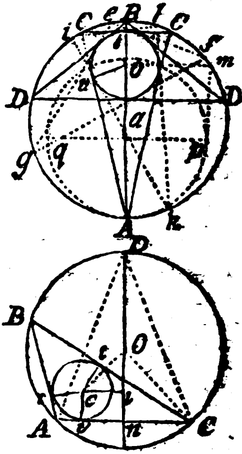
Mr. *Robins* is the only author, that I know of, who has solved a proposition exhibiting the relation between the *velocity* of the ball and the *dimensions* of the piece. Another author, who proceeds in a very different manner, makes the *velocity* always increase with the *length* of the piece.

If any person however think it worth his while to go through the calculation of this problem, he may easily do it, making the expression for the velocity found by prop. 7 of *Robins*'s Principles of Gunnery, a maximum, then its fluxion being taken, &c. there will be determined the relation between the diameter and length.

sides into half the given radius of the inscribed circle, the more the sides of any triangle are situated about the center, or diameter, or the farther removed from them, the greater or less will the periphery, and consequently area, of that triangle be: the sides of the triangles *ACC* and *BDD* being the most near and remote to such a situation.

For the property of drawing triangles as aforesaid, in and about eccentric circles, there is this demonstration.

First to inscribe the isosceles, or one △, *ACC* in the greater circle, which at the same time shall circumscribe the lesser circle. Put m = rad. greater circle = Aa, n = rad. lesser = Ov, and $x = AO$; then $\sqrt{xx-nn} = Av$; per sim. △s, $Av : AO :: At : AC = \frac{x+n}{\sqrt{xx-nn}} \times x$; again, $AO : Av :: AB : AC = \frac{2m\sqrt{xx-nn}}{x}$; hence $\frac{x+n}{\sqrt{xx-nn}} \times x = \frac{2m\sqrt{xx-nn}}{x}$, and $x - m = \sqrt{m^2 - 2mn} = ao$, shewing the distance of the centers to be least, or o, when $2n = m$; both circles being then concentric, and radius of the greater = twice the radius of the less. Whence 12·260074 = Bt the nearest distance of the peripheries. And area △ *ACC* = 18242·89 yards, the greatest; and that of △ *BDD* = 16799·82 &c. the least.



The distance of these centers of the circles being known, as $d = cO$ capable of having one triangle drawn as aforesaid: Suppose any chord $AC = x$ touch the inner circle at v, then it is plain another chord can touch it somewhere at t, and if another can touch it at r, the property is proved. But, in general, the $\angle ABC = \angle ADC = \angle nOC$ its sine $= \frac{x}{2m}$; and $\angle rBc = \angle cBt = \angle nDC$ (per 33 and 20 Euc. 3) $= \frac{x}{2\sqrt{2mn + m\sqrt{4mm - xx}}}$; the comp. or $\angle rcB = \angle tcB = \angle ncD$, its sine $= \frac{x}{\sqrt{2mm \times m\sqrt{4mm - xx}}}$; whence, by trig. $rB = Bt =$

2mm

$\frac{2nm + 2n\sqrt{4mm - xx}}{x}$. Again $iO = \frac{1}{2}\sqrt{4mm - xx} - n$, and (per 47 Euc. 1) $ci = vn = \sqrt{dd - \overline{\frac{1}{2}\sqrt{4mm - xx} - n}|^2}$; hence $vC = \frac{1}{2}x + vn$, $Av = Ar = \frac{x}{2} - vn$, and AB and BC, also $\angle vCc = \angle cCt$, and $\angle vCt$ are expressed. Say $\frac{x}{2m}$ (s. $\angle ABC$) : x (AC) or 1 : $2m$ :: s. $\angle vCt : AB$, which gives an equation shewing the general value of x, whilst two other chords are tangents likewise; and triangles ilk, gef, will revolve as in the 1st fig. Hence, if any chord or side of a triangle is given, the others follow by trigonometry; having first found the distance of the centers aforesaid. This property of drawing triangles about circles I discovered some years ago, as may be seen in the Monthly Oracle; though the proposer has greatly deserved in a long account of it from Scilly, printed in a book called the Quarterly Miscellanea Curiosa.

Mr. *Landen* puts b = rad. lesser circle, a = diam. greater, and x = altitude isosceles triangle, and gets $x = \frac{a+2b}{2} \pm \sqrt{\frac{aa - 4ab}{4}} = 297{\cdot}33$ or $112{\cdot}67$ &c. Whence he infers this construction [*see fig.* 1]. From the center of the garden, along the diameter, set off the diameter of the pond, find a center (at n) betwixt that point and the other end, describing a circle Apq, draw pm parallel, and mo perpendicular to AB, and the point o will be the center of the pond.

Mr. *Ash* finds the distance of the centers by a theorem like the former; and so does Mr. *Bamfield*, who has given a concise and elegant solution.

The Prize of 10 *Diaries was won by Mr.* R. Gibbons.

The Eclipses calculated for 1747, *shewing in what Parts of the World they will be visible. By Mr.* Ralph Hulse.

1. On January 29, at 3 in the afternoon, the sun will be eclipsed in ♒, 21 deg. vertical to Brazil, lat. 14 deg. south, long. 50 deg. west. This eclipse will be very small, and visible near the antartic circle.

2. Feb.

2. Feb. 14, at 5h. 2m. in the morning the moon will be eclipsed visible and total at London: Beginning 3h. 15m. Middle 5h. End 6h. 45m. Total duration 3h. 30m. Digits eclipsed 2c½.

Calculated by		Begin. h. m. s.	Mid. h. m. s	End h. m. s.	Dur. h. m. s.	Dig. h. m.
Mr. C. Cockson, by Leadbetter's Tables,	London	3 13 20	5 1 30	5 50 29	3 36 9	19 43
	Durham	3 8 18	4 56 28	5 45 27	† End	of
	Lambton	3 8 10	4 56 20	5 45 19	Total	Dark.
	Cornwen	2 56 9	4 44 17	5 33 16		
-----New Tables from Sir Isaac Newton's Theo.	Durham	3 2 10	4 44 9	5 40 8		
	Lambton	3 2 2	4 44 1	5 40 0	3 40 18	19 54
	Cornwen	2 50 1	4 32 0	5 27 59		
By Mr. Bulman,	London	3 2 0	5 5	7 5 *	* End	of the
	Edinburgh	2 50	4 53	6 53	Eclipse.	
	Dublin	2 34	4 37	6 37	4 3	19 58
	Carlisle	2 51	4 54	6 54		
	Rochester	3 3	5 6	7 6		
Mr. Cowper, London		3 24 34	5 13 19	7 2 4	3 37 30	19 43
Mr. Farrer, Sunderland		2 41	4 30	6 19	3 38	20 12

3. Feb. 28, at 5 in the morning, the sun will be eclipsed 1 digit in ♓ 20 deg. He is then vertical to the Indian sea, between Bornio and Java, lat. 4 deg. south, long. 110 deg. and will be seen in Greenland, and the places adjacent.

4. July 26, at 8 in the forenoon, the sun will be 1 digit eclipsed in ♌, 13 deg. vertical to Arabia Fœlix, lat. 17 deg. long. 41 deg. east, visible in the north frozen sea, lat. 80 deg.

5. August 9th, at 10 in the forenoon, the moon will be totally eclipsed, viz. 27 deg. in ♒. She is then vertical to Mardelzur, lat. 12 deg. S. long. 146 deg. W. visible to the Japan and Philippine islands, to all the west ocean between Asia and America, from S. to N. including Jamaica, Cuba, Carolina, and Virginia.—And the horizon of the visible disk passes through Pensylvania.

6. August 24, at 9 at night, when the sun will be eclipsed in ♍, 12 deg. This will be a very small eclipse, and visible only in the unknown southern parts of the world.

New

The 2d Eclipse was observed at *St. Angelo, Paragua*; whose lat. is 28° 17′ south, and long. west of the Ferro isle 36° 30′.

The end of the eclipse 15 h. 16 m. 4 s.

The 5th Eclipse was observed at *St. Maria Major*, lat. 27° 51′ south, and long. 37° 20′ west of the Ferro isle.

The beginning of the eclipse	14 h.	15 m.	14 s.
Total obscuration — —	15	53	16
Beginning of the emersion	17	34	48

New Questions.

I. QUESTION 279, *by Mr.* Landen.

A charming brisk maid has assur'd me, and said,
Since I'm such a fine mathematician,
(Laying puzzling aside, for the joys of a bride)
She will wed me——but on this condition:
That I first shall unfold what * pieces of gold
Her father has for her in store:
And these I must find from the data subjoin'd,
And then I'm to puzzle no more.

* The pieces are half-guineas, guineas, moidores, and three-pound-twelves. The whole number is 4000. And if v, x, y, and z be put for the number of each sort respectively, $v^4x^3y^2z$ is a maximum. Quere what is the lady's fortune?

II. QUESTION 280, *by Mr.* John Williams.

Going along a river's side, on an even and direct road ABC, I observed a tower on the other side of the river, whose angle of altitude at A was 5° 24′; going farther on to B, 100 yards, the angle of altitude was 6° 27½′; and intending, again, to take an observation when directly opposite to the tower, but was prevented by an island in the river (over-grown with furz), I then came to C, 400 yards from B, where I found the angle of altitude was 8° 36′. Quere the tower's height?

III. QUESTION 281, *by Mr.* J. May, *jun. of Amsterdam.*

It is required to find (by a general theorem) the number of fractions of different values, each less than unity, so that the greatest denominator be less than 100?

IV. QUESTION 282, *by the Rev. Mr.* Anth. Baker.

A gentleman would have a silver punch-bowl made in the form of a parabolic conoid, containing exactly two gallons, but being frugally inclined, desires first to know what ought to be its inside dimensions so that, cæteris paribus, it may require the least quantity of silver possible?

V. QUESTION 283, *by Mr.* N. Farrer.

Quere the axis and parameter of a parabola and semi-ellipsis, when the latter is circumscribed by the former, whose ordinate is equal to the conjugate axis, and abscissa equal to the semi-transverse: both curves having the same focus, the difference of their parameters 2, and the length of the parabolic curve being 28·68?

VI. QUESTION 284, *by Mr.* Cuth. Cockson.

Quere with what part of a cylindrical stick should a person strike, to give the greatest blow; the length of the arm being 20 inches, and that of the stick 50?

VII. QUESTION 285, *by Mr.* Landen.

I am about building a house, the breadth whereof I design shall be 24 feet, and the perpendicular height from the ground to the ridge 42 feet. The ends, which are to point directly east and west, will be sheltered by neighbouring tenements; but the sides will be exposed to the fury of the north and south winds. I therefore would be satisfied what the angle of the ridge must be, and how high the side walls, that the wind blowing from either of those quarters shall have the least effect on the building?

VIII. QUESTION 286, *by Mr.* J. Ash.

If a parabolic conoid, whose altitude is 9, and base 6 inches, be cut by a right line at some distance from, but parallel to its axis, what is the solidity and convex surface of that segment, or part cut off, when the height of the plane of that section is 5 inches?

IX. QUESTION 287, *by Mr.* Cuth. Cockson.

Given $a^3\dddot{y} + \ddot{y}\ddot{y}x^2 - aa\dot{y}\dot{y} = 0$, the equation of a curve, whose radius of evolution at the vertex is $140 = a$; to find the value of y, the abscissa, when its corresponding semi-ordinate $x = 50$?

X. QUESTION 288, *by Mr.* John Hampson.

Required to find three numbers, that when each is severally added to the cube of their sum, their respective sums shall be a cube number?

XI. QUES-

XI. QUESTION 289, *by Mr.* Heath.

On what days of the year do our ſhadows move floweſt and faſteſt in London? And at what times do they move floweſt and faſteſt on any day?

XII. QUESTION 290, *by Mr.* Bulman.

Being at a town in Kent, I obſerved three objects on the other ſide of the river Medway, (a caſtle, wind-mill, and ſpire) whoſe diſtance from one another are known: From the caſtle (the neareſt object ſeen) to the ſpire, is 10 furlongs; from the caſtle to the wind-mill 23 furlongs; and from the wind-mill to the ſpire is 25 furlongs. I alſo obſerved the town angle between the caſtle and ſpire $= 28° 34'$, and the town angle between the caſtle and wind-mill $= 57° 45'$. What diſtance did I ſtand from each of thoſe objects? And give a geometrical conſtruction of the ſame.

XIII. QUESTION 291, *by Mr.* J. Aſh.

A lady of important ſpeculation,
Would gladly know her age from this * equation.

$$* \; 1{\cdot}05^{A} = \frac{\sqrt{A}}{1 - {\cdot}05\sqrt{A}}.$$

XIV. QUESTION 292, *by Mr.* Heath.

The pounds power of their Napier's logarithm be
Equal that logarithm power of ſhillings left to me.
From myſtic words, artiſts, the truth extract,
And tell what is the legacy exact?

XV. QUESTION 293, *by Mr.* Bulman.

A ſpheroidal ullage lies upon the ground with the bung uppermoſt, from whence to the ſurface of the liquor (which is exactly the height of the upper ends of the caſk) is 9 inches, and its diagonal either way from the bung to the lower ends $= 55$ inches, its ullage is a maximum: Quere the content of the caſk and ullage, brother gaugers?

XVI. QUESTION 294, *by Mr.* Chr. Maſon.

In an evening lately, hearing the noiſe of guns at a ſmall diſtance off at ſea, I ſtraight repaired to the Strand, where I loomed a man of war's tender giving chace to a French privateer; I perceived a flaſh of a gun from the tender S. W.

by S. at 33 half ſeconds of time heard the report: Eight minutes after I obſerved another flaſh at S. S. W. from the ſame, and at 25 half ſeconds heard the report. The chace kept her courſe, making equal way; in 35 minutes more was drove on ſhore, being but two minutes of time a-head of the tender. What was the bearing and diſtance between my ſtation, the tender, and pilf'ring poltroon when ſtranded?

PRIZE QUESTION *by Mr.* Bamfield.

If the diameter of Syſiphus's cylindrical ſtone be two feet, which he continually rolls upon the ſurface of a ſemi-globular mountain, half a mile high: Quere what ſpace will a ſpot on the convex ſurface of that ſtone travel through in rolling directly up and down the ſaid mountain? And what will be the time of its deſcent from the top, by the force of gravity?

1748.

Queſtions anſwered.

I. QUESTION 279 *anſwered by Mr.* J. Turner.

LET $v =$ the half guineas, $x =$ guineas, $y =$ moidores, and $z = 3$l. 12s. By the queſtion $v + x + y + z = 4000 = b$: Now $v^4 x^3 y^2 z$ being a maximum, or (expunging z) $b - v - x - y = \frac{1}{v^4 x^3 y^2}$. In fluxions $-\dot{v} - \dot{x} - \dot{y} = \frac{-2\dot{y}}{y^3 x^3 v^4} - \frac{3\dot{x}}{y^2 x^4 v^4} - \frac{4\dot{v}}{y^2 x^3 v^5}$; whence $\dot{y} = \frac{2\dot{y}}{y^3 x^3 v^4}$; $\dot{x} = \frac{3\dot{x}}{y^2 x^4 v^4}$; $\dot{v} = \frac{4\dot{v}}{y^2 x^3 v^5}$: And $\frac{1}{v^4 x^3 y^2} = \frac{y}{2} = \frac{x}{3} = \frac{v}{4} = b - v - x - y$. Conſequently $x = \frac{3}{2}y$; $v = \frac{4}{3}x = 2y$: Alſo $\frac{1}{2}y = b - 2y - \frac{3}{2}y - y$; or $y = \frac{1}{5}b$; $z = \frac{1}{10}b$; $x = \frac{3}{10}b$; $v = \frac{4}{10}b$; and the lady's fortune is 4620l. Hence this

GENERAL RULE.

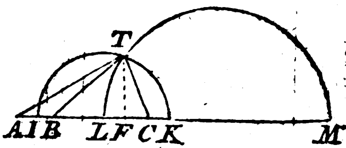
Make the ſum of the exponents a denominator, and each particular exponent (of the quantities) the numerator of a fraction,

fraction, these respectively multiplied into the whole number of pieces, will shew the particular number of pieces of each sort.

Mr. *James Waine* has solved this question curiously and concisely, as also did Mr. *Landen* (the proposer), Mr. *Kingston*, Mr. *Jepson*, Mr. *Bamfield*, Mr. *Farrer*, Mr. *Collingridge*, Mr. *Garrard*, Mr. *Dun*, Mr. *Cowper*, Mr. *Cockson*, &c. Though it has been observed that this question is taken from Emerson's Fluxions (p. 128), yet the new mode of language and application of it, is a merit which must be acknowledged.

II. QUESTION 280 *answered by Mr.* James Terey.

Put x = tower's height, $AC = 500 = b$, $AB = 100 = d$, $BC = 400 = a$. Co-sec. $\angle A = t$, of $B = v$, of $C = y$. Then (rad. = 1) $tx = AT$, $vx = BT$, and $yx = CT$. Now (geometrically) making $t : v :: AI : IB :: AK : KB$, and describing the semi-circle ITK;



also $v : y :: BL : LC :: BM : MC$, describing the semicircle LTM; the intersection T, being the place of the tower, for which see Universal Arith. prob. 26) let down $TF \perp AC$. Per fig. $b : tx + yx :: tx - yx : \frac{ttxx - yyxx}{b} = AF - FG$; also $BT^2 - BF^2 = AT^2 - AF^2$; whence $attxx + dyyxx - bvvxx = abd$, and $x = \sqrt{\frac{abd}{att + dyy - bvv}} = 44{\cdot}4609$ &c. yards, the tower's height. *Q.E.F.*

The same answered by Mr. J. Ash.

Call the co-secants of the three angles of altitude a, b, and c; AB, s; BC, $4s$; the tower's height, x. By trigon. (T being the top) $AT = ax$; $BT = bx$; $CT = cx$; cos. $\angle CBT = \frac{ccxx - bbxx - 16ss}{8sbx}$; cos. $\angle ABT = \frac{ss + bbxx - aaxx}{2sbx}$; which two last expressions made equal, $x = \sqrt{\frac{20ss}{cc - 5bb + 4aa}} = 44{\cdot}4557$ yards, the tower's height required.

This question was also answered by Mr. *J. Turner*, Mr. *Waine*, Mr. *R. Hall*, Mr. *Kingston*, Mr. *Jepson*, Mr. *Bamfield*, Mr. *Pitches*, Mr. *Hampson*, Mr. *Walker*, and Mr. *Collingridge*.

III. QUESTION 281 *answered by Mr.* Heath.

The number of fractions of different and like values, each being less than unity, and the greatest denominator being any number, (from 2 upwards) will appear by the following series, continued to $\frac{98}{99}$, &c.

Sum = 27 = fractions of diff. values.		Sum = 9 = fractions of like value.
6	$\frac{8}{9}$ $\frac{7}{9}$ $\frac{6}{9}$* $\frac{5}{9}$ $\frac{4}{9}$ $\frac{3}{9}$* $\frac{2}{9}$ $\frac{1}{9}$	2
4	$\frac{7}{8}$ $\frac{6}{8}$* $\frac{5}{8}$ $\frac{4}{8}$* $\frac{3}{8}$ $\frac{2}{8}$* $\frac{1}{8}$	3
6	$\frac{6}{7}$ $\frac{5}{7}$ $\frac{4}{7}$ $\frac{3}{7}$ $\frac{2}{7}$ $\frac{1}{7}$	0
2	$\frac{5}{6}$ $\frac{4}{6}$* $\frac{3}{6}$* $\frac{2}{6}$* $\frac{1}{6}$	3
4	$\frac{4}{5}$ $\frac{3}{5}$ $\frac{2}{5}$ $\frac{1}{5}$	0
2	$\frac{3}{4}$ $\frac{2}{4}$* $\frac{1}{4}$	1
2	$\frac{2}{3}$ $\frac{1}{3}$	0
1	$\frac{1}{2}$	0
27	36	9

Here it is evident, that the greatest numerator, or greatest denominator less unity, will be the number of terms, which will always be equal to the last term; therefore $\frac{n+1}{2} \times n =$ the sum of the series, or the number of different and like fractions in all cases. The fractions of like value with some of the rest, are all those not in their lowest terms: As those of different value are all the incommensurable ones: For the more speedy determining of which, all the incommensurable denominators, from the least in the series to the greatest, must be taken, viz. 3, 5, 7, 11, 13, &c. to 97, which are 25; and the sum of their different fractions will be the series 2, 4, 6, 10, 12, &c. to 96; each row being one less than the denom. To these if the different fractions (in lowest terms) of the commensurable denominators 2, 4, 6, 8, 10, &c. to 99, be added, the sum will be all the different fractions. But as there is more trouble than art to discover them, I shall leave it to persons of leisure to pursue the computation, the method being here planned out.

Mr. *Ash*, upon the same principles, computes the number of different fractions to be 3055; but doubts the truth.

N. B. By the above theorem, the whole number of fractions = 4851 (as Mr. *Bamfield* made it) from which subducting those of like values, all the different fractions will remain.

A General

A General Method of solving this Question *from the Diary for* 1751, *by Mr.* Flitcon.

Each diff. let. in 1st scheme denotes a diff. prime.

	Steps	Denominators	The several quotients arising by dividing any denominator, by: Each single prime	The prod. of each 2	Product of each three	Product of each four	Numb. of all fract. in each series.
	1	x	-1				n
$1 \times x$	2	x^2	$-x$				xn
$2 \times x$	3	x^3	$-x^2$				x^2n
	4	vx	$-v$ $-x$	$+1$			n
$4 \times v$	5	v^2x	$-v^2$ $-vx$	$+v$			vn
$5 \times x$	6	v^2	$-v^2x$ $-vx^2$	$+vx$			vxn
	7	vxy	$-vx$ $-xy$ $-yv$	$+y$ $+v$ $+x$	-1		n
	8	$vxyx$	$-vxy$ $-xyz$ $-yzv$ $-zvx$	$+vz$ $+xy$ $+yz$ $+zv$ $+yv$ $+xz$	$-z$ $-v$ $-x$ $-y$	$+1$	n

II.

Denominator	Component primes	Diff. fractns	Denominator	Component primes	Diff. fractns	Denominator	Component primes	Diff. fractns	Denominator	Component primes	Diff. fractns	Denominator	Component primes	Diff. fractns	Denominator	Component primes	Diff. fractns	Denominator	Component primes	Diff. fractns
2	P	1	16	—	8	30	2·3·5	8	44	—	20	58	2·23	28	72	—	24	86	2·43	42
3	P	2	17	P	16	31	P	30	45	—	24	59	P	58	73	P	72	87	3·29	56
4	—	2	18	—	6	32	—	16	46	2·23	22	60	—	16	74	2·37	36	88	—	40
5	P	4	19	P	18	33	3·11	20	47	P	46	61	P	60	75	—	40	89	P	88
6	2·3	2	20	—	8	34	2·17	16	48	—	16	62	2·31	30	76	—	36	90	—	24
7	P	6	21	3·7	12	35	5·7	24	49	—	42	63	—	36	77	7·11	60	91	7·13	72
8	—	4	22	2·11	10	36	—	12	50	—	20	64	—	32	78	2·3·13	24	92	—	44
9	—	6	23	P	22	37	P	36	51	3·17	32	65	5·13	48	79	P	78	93	3·31	60
10	2·5	4	24	—	8	38	2·19	18	52	—	24	66	2·3·11	20	80	—	32	94	2·47	46
11	P	10	25	—	20	39	3·13	24	53	P	52	67	P	66	81	—	54	95	5·19	72
12	—	4	26	2·13	12	40	—	16	54	—	18	68	—	32	82	2·41	40	96	—	32
13	P	12	27	—	18	41	P	40	55	5·11	40	69	2·23	44	83	P	82	97	P	9
14	2·7	6	28	—	12	42	2·37	12	56	—	24	70	2·5·7	24	84	—	24	98	—	2
15	3·5	8	29	P	28	43	P	42	57	3·19	36	71	P	70	85	5·17	64	99	—	6
		71			198			314			416			564			666			774

Third columns collected.		The sum of all the different fractions
	71	
	198	
	314	
	416	
	564	
	666	
	774	
	3003	

N. B. P signifies each prime number in the second column in scheme 2, with a new number to each in the 3d column, deduced from the series in the first scheme, according to the general method: the different denominators being placed in the first column all along.

For the multipliers, or powers of each. Against 3 in the 1st column, stands 2 in the 3d column, each being drawn into 3 gives 6 to be set in the 3d column against 9 in the first collection. Against 6 in the 1st col. is set 2·3 in the 2d col. whence (by 4th step) is found 2 for the 3d collection. Also for its multiples, into either, or both its parts, as $2\times 3 = 6$, drawn into 6 and 2 gives 12 (by 6th step) to be set in the 3d col. against 36 in the first: dotting through all the second columns all such multiples, by which the fractions are found. Proceeding thus, the whole number of different fractions are truly determined in a short time.

IV. QUESTION 282 *answered by Mr.* J. Turner.

When the convex superficies is the least, the conoid must come nearest to the form of a semi-sphere (being under the least surface of all solids of given solidity formed by the rotation of a curved space about its axis) which is when the abscissa and semi-ordinate of the generated parabola are nearly equal: the conoid then being nearly most capacious. Putting $b = 462$ inches in two wine gallons, $p = 3.1416$, $y =$ ordinate $x =$ abscissa of the parabola. Then the solidity of conoid $= b = \frac{pyyx}{2} =$ (when $y = x$) $\frac{py^3}{2}$; where $y = 6.65$ inches. But accurately, $\frac{p}{6a} \times \overline{aa + 4yy}|^{\frac{3}{2}} - \frac{paa}{6}$ is the superficial content of the same (*see p. 202 of Mr. Emerson's Fluxions*). Now if the value of yy be substituted therein, and the fluxion of it made $= 0$, then $a = 6.1$; $y = 6.34$; $x = 6.94$; and the convex surface $= 235.5$ square inches.

The same answered by Mr. Ash.

The conoid's convex surface is $\frac{p}{3a} \times \overline{aa + 4yy}|^{\frac{3}{2}} - \frac{paa}{3}$, a minimum ($p = 1.5708$); ∵ $\overline{\frac{aa + 4yy}{a}}|^{\frac{3}{2}} - aa$ is also a minimum. Put $cc =$ double solidity $= 4$ wine gall. $qq =$ area of a circle whose rad. $= 1$. Then, by the curve, $ax = yy$, and $qqyyx = cc$, whence $4yy = \frac{4c\sqrt{a}}{q}$; and substitu-

ting

ting for $4yy$ in the min. it becomes $\overline{\frac{qaa - [illegible]}{a}}\Big|^{\frac{3}{2}} - aa$.

Whence by fluxions, $x = \overline{\frac{15}{24} + \sqrt{\frac{11cc}{192qq}}}\Big|^{\frac{3}{2}} = 6{\cdot}03$; whence semi-axis $= 6{\cdot}98$, and semi-ordinate $= 6{\cdot}49$ maxime.

It was elegantly solved by the proposer, also by Mr. *Landen*, Mr. *Farrer*, Mr. *Dixon*, Mr. *Waine*, Mr. *Kingston*, Mr. *Bamfield*, &c.

V. QUESTION 283 *answered by Mr.* Ash.

'Tis inconsistent with the nature of the parabola and semi-ellipsis that the latter should be circumscribed by the former, when the ordinate = conjugate, and axis = semi-transverse (which Mr. *Turner* also observes); ∵ semi-ellipsis must be read for parabola, & contra. Then from the properties of the curve and conditions of the question, the parameter of the parabola is found $= 2$. Suppose $AH = c = 12{\cdot}5$; the corresponding curve $Ar = 13{\cdot}90375$ &c. ∴ the remaining part rm $(= z)$ of the curve $= 0{\cdot}43625$. [*See fig. to Q. 253.*]

Put $y = nm$, $x = nr$, and we shall have $\dot{y} = \frac{a\dot{x}}{\sqrt{4ac + 4ax}}$, and $\overline{\dot{x}^2 + \dot{y}^2}\Big|^{\frac{1}{2}} = \dot{z} = \sqrt{1 + \frac{a}{4c + 4x}} \times \dot{x}$; whose fluent is $x + \frac{ax}{8c} - \frac{aax}{128cc} - \frac{axx}{16cc} + \frac{aaxx}{128c^3}$ &c. $= z$; or $ax - bx^2 + cx^3$ &c. $= z$ [by substituting a for $1 + \frac{a}{8c} - \frac{aa}{128cc}$; b for $\frac{-a}{16cc} + \frac{aa}{128c^3}$; &c.] And by reversion $x = \frac{z}{a} + \frac{bz^2}{a^3} + \frac{2bb - ac}{a^5}z^3$ &c. $= 0{\cdot}4289$; which added to AH makes $12{\cdot}9289$ for the required axis of the parabola. *Q. E. F.**

VI. QUES-

---

* V. QUESTION 283.

As the method of finding the parameter is not inserted, I shall here supply it, and at the same time give another very easy method of solving the latter part of the question.

Since then $ax = yy$ by the nature of the parabola; putting $y =$ the ordinate, $x =$ the abscissa, and $a =$ the parameter of the parabola; and, by the nature of the ellipse, $x : y :: 2y : \frac{2yy}{x} =$ parameter

VI. QUESTION 284 *answered by Mr.* J. Turner.

Put $a = 70$, the length of the arm and stick; $b = 20$, the length of the arm; then the distance of the center of percussion of the stick from the upper end of the arm will be $= \frac{2aa + 2ab + 2bb}{3a + 3b} = 49.63$ inches, and from the farthest end of the stick $= 20.37$ inches (vide Stone's Flux. p. 177).

The same answered by Mr. J. Ash.

Let $20 = c$ represent the length of the arm, and put $x =$ the distance of the center of percussion from the hand. Then $\overline{c + x}x \times \dot{x} =$ fluxion of the momentums; and $\overline{c + x}x \times \dot{x} =$ that of the forces; the fluent of which divided by that of the momentums is $\frac{6cc + 6cx + 2xx}{6c + 3x} =$ the distance of the center of percussion of the part xp, from the point of suspension; which (when $x = 50$) will be $= 49.6296$ &c. Consequently the part of the stick required is 20.37037 &c. inches from the top.

Mr. *Walker*, Mr. *Kingston*, and Mr. *Cuth. Cockson* gave a solution to the same.

VII. QUES-

---

parameter of the ellipse $= a + 2$ by the question; or $ax + 2x = 2yy$; subtract the former equation from this, then $2x = yy = ax$, and therefore $a = 2$ the parameter of the parabola.

The parameter being thus found, by p. 309 my *Mensuration*, the length of the double curve will be $y\sqrt{1 + yy}$ + hyp. log. of $y + \sqrt{1 + yy} = 28.68$, by the question, $= c$; call the hyp. log. of $y + \sqrt{1 + yy}$, v; then $y\sqrt{1 + yy} = c - v$. Now it is easy to perceive that $y = 5$ nearly; then since a small difference in the value of y will make a still smaller and inconsiderable difference in the value of v, and when y is supposed 5 then $v = 2.312456$, and then the equation $y\sqrt{1 + yy} = c - v$ becomes $y\sqrt{1 + yy} = 26.367564 = d$, and $y = \sqrt{\sqrt{dd + \frac{1}{4}} - \frac{1}{2}} = 5.08$. With this value of y find a new value of $v = 2.3286281$, and thence of $d = 26.3513719$; and this same theorem will give $y = 5.08501$, which is very exact.

Then $x = \frac{yy}{a} = \frac{yy}{2} = 12.92806$.

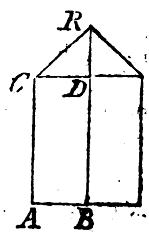
VII. QUESTION 285 *answered by Mr.* Landen *the Proposer.*

Put $a = 42$ the height of the building; $b = 12$, half the breadth; $x =$ perpendicular height of the roof. The force of all the particles of the air impinging against the wall, to blow upon the building, will be as $aa - 2ax + xx$. The effect of any particles striking against the roof (found by mechanics) is to the effect the same would have had, striking directly against an upright plane, as the square of the sine of the angle of incidence to the square of radius; therefore the force of all the particles against the roof, to blow down the building, will be as $\frac{2ax^3 - x^4}{bb + xx}$. And by the question $\frac{2ax^3 - x^4}{bb + xx} + aa - 2ax + xx$ must be a minimum. Let the fluxion of it be put $= 0$, then $x = \frac{b\sqrt{bb + 4ax} - bb}{2a}$ $= 10{\cdot}4076$ feet: Whence the height of the side-walls must be $31{\cdot}5924$ feet, and the ridge angle 98° 7′.

Mr. *Tho. Cowper* answered this question exactly as above. Mr. *Jepson* makes the ridge angle $= 90°$, and side walls $=$ 30 feet, as does Mr. *Ash*, but by a method like the proposer's first consideration, who then made the ridge angle and side walls agree the same. Mr. *Farrer*, by a different method, makes the side walls $= 32{\cdot}566$ feet, and the ridge angle $= 103°$ 39′.

Mr. Jepson*'s Solution is as follows.*

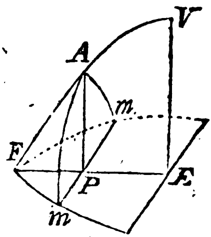
Let $BR = 42 = a$, $AB = CD = 12 = b$, $CR = y$, $DR = x$, then $AC = BD = a - x$. By p. 255 of Emerson's Fluxions, the resistance of the plane DR, to the resistance of the plane CR, is as yy to xx (supposing the wind to blow in a direction parallel to the plane of the horizon); but the resistance of the plane DR is $= cx$ (putting $c =$ the length of the building) and (per 47 Euc. 1) $yy = bb + xx$, therefore the resistance of the plane CR is $= \frac{cx^3}{bb + xx}$, and that of the side wall AC $= ca - cx$, $\therefore \frac{x^3}{bb + xx} + a - x$ is a minimum; which in fluxions, &c. gives $bbxx - b^4 = 0$, and $x = b = 12$. Hence the height of the side walls $= 30$ feet, and ridge angle 90°, which will always be the same, let the height and length of of the building be what they will.



VIII. QUES-

VIII. QUESTION 286 *answered by the Proposer Mr.* Ash.

Let $x = FP$, $n = 2FE = 6$. By the nature of the circle we get $\sqrt{nx - xx} = Pm = \sqrt{PA}$ by the nature of the parabola (because 9 the altitude of the whole parabola = 3^2 the square of its semi-ordinate, and all the perpendicular sections are sim. parabolas); and $\frac{4a}{3} \times \overline{nx - xx}|^{\frac{3}{2}} =$ = area mAm; consequently $\frac{4}{3} \times \overline{nx - xx}|^{\frac{3}{2}} \dot{x}$ = flux. of the solidity; whose fluent is $\frac{4\sqrt{nx}}{3} \times : \frac{2nx^2}{5} - \frac{6x^3}{14} + \frac{6x^4}{72n}$ &c. = 6·1122 the solid content.



Again since $nx + xx = Pm^2 = AP$, whence the fluxion of the curve is $\dot{x}\sqrt{mm + 4mx + 4xx + 1}$, or putting cc for $nn + 1$) $\dot{x}\sqrt{cc + 4nx + 4xx}$; and because $2\sqrt{mx + xx}$ = chord of the arch mFm; and $\sqrt{mx + 2xx}$ = chord of half that arch; therefore (by Huygens's Theor.) the arch $\dot{m}Fm = \frac{8\sqrt{4x + 4xx} - 2\sqrt{nx - xx}}{3}$ nearly. Consequently $\frac{8\sqrt{4x + 2xx} - 2\sqrt{nx - xx}}{3} \dot{x}\sqrt{cc + 4nx + 4xx}$ = fluxion of the surface. The fluent of which (when $x = 1$) is (if no mistake be made) = 16·34 fere. *Q.E.F.*

Mr. *Farrer*, by a different method, only finds the solidity of the segment (without the content of the surface) which he makes but 6·845 cubic inches.

IX. QUESTION 287 answered by Mr. *Emerson*, at p. 39 and 40 of his Fluxions, from whence it was taken; y being there $= \frac{x^2}{2a} + \frac{x^4}{24a^3} + \frac{x^6}{720a^5} - \frac{x^8}{44800a^7}$ &c. = 9·0238 &c.

Mr. *Farrer*, Mr. *Kingston*, Mr. *Waine*, and Mr. *Collingridge* confirm the same.

We wish the invention of our worthy contributors may be more prolific, than to send us questions already solved, in authors, to hand.

X. QUES-

* X. QUESTION 288 *answered by Mr.* Hampson *only.*

1. Answer, $a = \frac{23625}{157464}$, $b = \frac{1538}{157464}$, $c = \frac{18577}{157464}$.

2. Answer, $a = \frac{18954}{132651}$, $b = \frac{4184}{132651}$, $c = \frac{271}{132651}$.

† XI. QUESTION 289 *answered by Mr.* Heath *only.*

It is evident, that, when the sun's motion is most vertical, his increase of azimuth is least, and, consequently of altitude greatest; and when his motion is most horizontal, his increase of azimuth greatest, and of altitude least.

At

* This question may be solved after the manner of the 51st or 275th.

† XI. QUESTION 289 *otherwise solved by the Rev. Mr.* Cha. Wildbore.

'Tis manifest from my solution to the 654th diary question, that at all places within the polar circles, the velocity of the shadow of the summit of an object, increases from passing due north (or south in the Antartic) till it attains the maximum there determined, after which the velocity continually decreases till noon, when it is the least possible. And with regard to the different greatest velocity on different days, 'tis evident that the expression for the maximum $PE^2 \times PE \times rS + Pb - rP \times PE \times rS - Pb + rP$ will continually increase, by increasing rS or diminishing the declination; consequently the greatest velocity on the longest day will be less than the maximum on any other day: the said maximum increasing till its time and that of rising coincide, when it will be infinite.

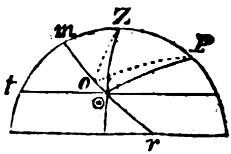
There is likewise a time in problems of this kind, when the velocity of the shadow, after being the greatest, decreases the fastest, which may be found by taking the second fluxion of the above maximum $= 0$. But in our latitude both these times coincide at sun-rising, when the velocity being always infinite, decreases, and that more and more slowly till noon, when it is always the least possible. And the same may much more be said of the velocity with which the altitude alone increases. As to the velocity with which the azimuth alone increases, what Mr. Simpson has done upon the subject is right; for it must increase the slowest

 when

At 12 each day, the ſun's motion is parallel to the horizon, or leaſt vertical, being then perpendicular to the meridian, or vertical circle: therefore the circular motion of ſhadows, in general, is at noon the faſteſt.

To find when the increaſe of azimuth is leaſt on any day, and in any place.

Put $p = s. \odot P$, the ſun's diſtance from the pole; $d =$ its coſ. or ſine of declination; $c = s. ZP$; $b =$ coſ. or s. of latitude; $y = s.$ azim. $\odot ZP$. Now if $r \odot om$ be the ſemi-diurnal arc of a leſſer circle, it is evident that the ſun's motion is moſt vertical when angle $o \odot Z$ is leaſt, or its coſ. $Z \odot P$ greateſt (the leſſer circle coinciding, at the point $\odot$, with a



greater,

---

when the fluxion of the hour angle P bears the greateſt ratio poſſible to that of the azimuth Z, the reaſon for which is no more, than if any ſmall diſtance $\dot{x}$ be divided by the time $\frac{\dot{x}}{v}$ in which it is gone over, it will give v the velocity of uniform motion at that time. Therefore the fluxion of the azimuth, divided by that of the time, muſt give the velocity with which the azimuth increaſes at that time, or $\frac{\text{coſ.} \odot}{s. Z \odot} =$ (vide my ſolution above referred to) $\frac{Pb . PE - rP}{rS . EO . aO}$ or (becauſe $aO = \frac{EO}{EP}$) $\frac{Pb . PE^2 - rP . PE}{PE^2 - 1}$ or $\frac{Pb - rP . PE}{PE^2 - 1}$ is a minimum. Which will evidently be leſs, the greater rP the ſine of declination is taken; and therefore the longer the days are, the leſs will the ſloweſt increaſe of azimuth be, and conſequently it will be leſs on the longeſt day than on any other day. Moreover, when the ſun is in the equinoctial, the minimum becoming $\frac{Pb}{PE^2 - 1}$, will be leſs as PE is greater, and therefore will be the leaſt poſſible when PE is greateſt, and the altitude $= 0$, or at ſun-riſing; therefore on the equinoctial days the azimuth will increaſe the ſloweſt at riſing due eaſt. Conſequently when the declination is ſouth, the minimum will be before ſun-riſing, and therefore in that caſe the azimuth will increaſe faſter and faſter from ſun-riſing till noon, when it will in all caſes ncreaſe the faſteſt, but on the ſhorteſt day ſlower than on any other day, and on the longeſt faſter.

greater, at right angles with $\odot P$); say, $p : y :: c : \frac{cy}{p} =$ a maximum, which is evidently when $y = 1$, the sine of 90°. Whence, the azimuth increases slowest and altitude fastest when the sun is due east, on all days, and in all places whatsoever; and therefore shadows in general move slowest when the sun is due east; or at sun-rise (the most easterly) in the southern declinations.

In the maximum $\frac{cy}{p}$, when p is least (the sine of 66° 30′ or 113° 30′) the azimuth on those days increases the slowest of all other days, and altitude fastest: Whence, our shadows move slowest of all on the longest and shortest days of the year: the sun due east, or at rising. But when $p = 1$, the increase of azimuth, on that day, is least slow (at sun due E.) and the increase of altitude the least fast; and as the azimuth increases its swiftness till noon, therefore our shadows move fastest of all at noon, on the day of the year when the sun is in the equinoctial.

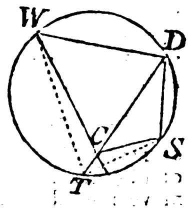
By erecting a wire perpendicular to an horizontal plane, and marking the circular increase of the shadow at equal distances of time, the truth of the foregoing will appear by inspection. And by this circular motion, and the shortening one of the shadow, or by the increase of azimuth and altitude together, the summit of it is made to describe a curve, whose nature is determinable, according to the latitude of the place and sun's declination.

N. B. The increase of azimuth being always as the angle $o \odot Z$, and the increase of altitude as its comp. the $\angle o \odot t$, the sum of both increases (viz. of azim. and alt.) will be at all times and all places alike; *i. e.* the sum of the circular and shortening motions of shadows on a horizontal plane will be every where equal.

The increase of an azimuth is but the fluxion of it, and the slowest increase of an azimuth is but the least fluxion of that fluxion of azimuth; therefore if the angle of time be substituted for (which flows equally) and an expression be raised of the azimuth angle (by trigon. and series) the fluxion of that fluxion made $= 0$ will exhibit an equation, shewing the azimuth properties as aforesaid. *Q. E. F.**

XII. QUESTION 290 *answered by Mr.* I. Ash.

The $\triangle CSW$ being formed, make $\angle WSD = 57° 45'$, and $\angle DWS = 28° 34'$; then circumscribe $\triangle SWD$ with a circle: draw DT through C, and the point T is the town's situation. From whence, by trigonometry, $TC = 0\cdot65707$, $TW = 23\cdot3439$, and $TS = 10\cdot5721$ fere.



Mr. *James Tercy* has given a concise and elegant construction in the same manner; and so has Mr. *Farrer*. And it is solved by Mr. *Bamfield*, Mr. *T. Cowper*, Mr. *Collingridge*, Mr. *Walker*, Mr. *Hampson*, Mr. *Pitches*, Mr. *Elgar*, Mr. *Warne*, Mr. *Kingston*, &c.

XIII. QUESTION 291 *answered by Mr.* Ash.

By trials A is found between 48 and 49; put $\overline{7-x}|^2 = A$, $1\cdot65^{\overline{7-x}|^2} = \frac{7-x}{0\cdot65+0\cdot5x}$ per quest. (which substitution is necessary to make the following series converge). Let $c = 0\cdot5$, $n = 7$, $q =$ hyp. log. 7, $m = \cdot65$, and $p =$ its log. also $a =$ log. $1\cdot05$; then (by log. series) $\overline{n-x}|^2 \times a =$

$$q - \frac{x}{n} - \frac{x^2}{2n^2} - \frac{x^3}{3n^3} \text{ \&c.} + p - \frac{cx}{m} - \frac{ccxx}{2mm} - \frac{c^3x^3}{3m^3} \text{ \&c.}$$

And by reversion, $x = 0\cdot030409$; $\because \overline{n-x}|^2 = A = 48\cdot5752$ &c. the lady's age.

Mr. *Turner* exactly solved the same by a table of logarithms (which is the easiest and quickest way). Mr. *Farrer*, Mr. *Dixon*, Mr. *Waine*, and Mr. *Hall* solved it by another method.

XIV. QUESTION 292 *answered by Mr.* J. Turner.

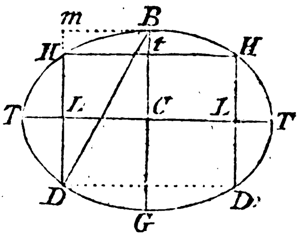
Put $x =$ the pounds of the legacy; then $\overline{\text{l.}\,x}|^x = \overline{20x}|^{\text{l.}\,x}$ per question; or putting $b = 2\cdot30258$ &c. $\overline{b\,\text{l.}\,x}|^x = \overline{2cx}|^{b\,\text{l.}\,x}$ by common logarithms: And by a few trials $x = 15\cdot615 =$ 15l. 12s. 3d. required.

Mr. *Farrer* solved the same by series; as did several others: But as the solving of all sorts of exponential equations by the tables of logarithms is preferable to all other methods, the general method of solution shall be exhibited in some future Diary.

XV. QUES-

XV. QUESTION 293 *answered by Mr.* Heath.

As the contained liquor always bears a proportion considerably greater than the vacuity, in the several variations of the form of the cask (which supposing but 2 to 1) the greater the vacuity, the greater will be the quantity of liquor, and consequently the sum of both; therefore when the ullage is a maximum, the whole cask will be also a maximum.



First, *to find the content of spheroidal frustums.* Let $t = TT$, the transf. $b = BG$, the conjugate of the gener. ellipsis; and $x = CL$. By the property of the curve, $tt : bb :: \frac{tt}{4} - xx : \frac{bb}{4} - \frac{bbxx}{tt} = HL^2$; consequently $bb - \frac{4bbxx}{tt} = HD^2 = hh$, (whence $tt = \frac{4bbxx}{bb - hh}$), and $n\dot{x} \times \frac{bbtt - 4bbxx}{tt}$ = flux. fruft. $BGDHB$ (n being ·7854) whose fluent is * $nx \times \frac{bbtt - 4bbxx}{3tt}$; in which substituting for tt, we have $nx \times \frac{2bb + hh}{3}$ = the content $BGDHB$, which holds true when it is the segment or frustum of a sphere.

THEOREM. Multiply the length of any spheroidal cask into the sum of twice the square of the bung diameter added to the square of the head diameter, and that product multiplied by ·2618 will give the content in inches; or divided instead by 1077·158, or 882, 3529 will give the content in ale or wine gallons: or multiply instead by ·00092837, or ·0011333, &c.

N. B. If x be put for TL; then the flux. segment $DTH = n\dot{x} \times \frac{bbtt - 4bbxx}{tt}$, whose fluent gives the same theorem as above for the content.

Now if $c = Bt$, $d = BD$, and x be put $= Dm$; then $x - c$ = head diam. $x + c$ = bung diam. $2\sqrt{dd - xx}$ = length of the cask, and $\frac{3cc + 2cx + 3xx}{\text{Divisor}} \sqrt{dd - xx}$ =

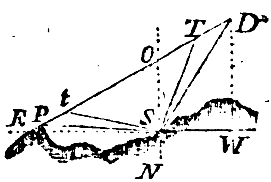
 the

the content, a maximum. In fluxions and reduced, $9x^3 + 4cxx + 3ccx = 6ddx + 2cdd$, where $x = 44\cdot1615$. Whence head diam. $= 35\cdot1615$, bung diam. $= 53\cdot1615$, length $= 65\cdot0567$ inches, and content $= 419\cdot313$ ale gallons. Again, put $x = Bt$; the section or surface of ullage, HtH, is an ellipsis, whose conjugate $= 2\sqrt{bx - xx}$; and tC being $\frac{b}{2} - x$ $= HL$, if $y = LC$, then $tt : bb :: \frac{tt}{4} - yy : \frac{bb}{4} - bx + xx$, and $2y = \frac{2t}{b}\sqrt{bx - xx} =$ transverse axis. The area section is $4nt \times \frac{bx - xx}{b}$, and flux. segment HBH $4nt\dot{x} \times \frac{bx - xx}{b}$, whose fluent $4ntx \times \frac{3b - 2x}{6b}$ is its content (which when $b = t$ becomes $4nxx \times \frac{3t - 2x}{6}$ for the spherical segment, as the other segment * becomes $nx \times \frac{3tt - 4xx}{3}$, proved also in Diary p. 45, 1747). But, by above, $t = \frac{2bx}{\sqrt{bb - hh}}$ $= 87\cdot4207$; whence the content of the frustum, or segment $HBH = 34\cdot99$ gallons; consequently $384\cdot322$ ale gallons of liquor remain in the cask. *Q.E.F.*

Mr. *Waine* was curious in his method of substitution for finding the content of the whole cask, and solution of the same; but sent no theorem for the ullage. The Rev. Mr. *Baker* sent the same theorem for finding the content of the ullage, with the above; but came too late to be inserted.

XVI. QUESTION 294 *answered by Mr.* T. Cowper.

Let D and T be the places of the tender at the first and second observations; t and P the places of the tender and privateer when the latter is stranded; and S the station of the observer. By the quest. $\angle TSD = 11° 15'$, and found moving 1142 feet per second, $SD = 18843$ feet, and $ST = 14275$; whence by trigon. $TD = 5586$, and $\angle STo = 41° 9\frac{1}{4}'$. Now the interval between the first and second flashes at D, and T, being $8'\ 16\frac{1}{2}''$, and and from the flash at T, till the privateer was drove ashore

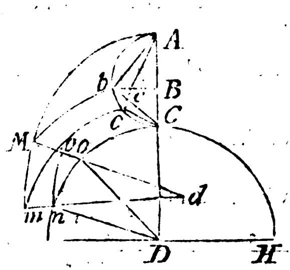


at

at P, by the tender at t, 35′ 12½″; by proportion of motion, Tt will be found 23767·22, and $tP = 1350·09$ feet. Whence $\angle oSt = 80° 31'$, the bearing of the tender about E. by S. dist. $St =$ 3·04061 miles; also the $\angle oSP = 84° 10'$, the privateer's bearing about E. ½ S. dist. $SP =$ 3·2514 miles required.

The PRIZE QUSTION *answered by Mr.* J. Landen.

Let Md be perpendicular to the epicycloid AMm, and md another perpendicular supposed infinitely near the former; then the concentric arches Mb, mc, being described from the center D of the immovable circle; the chords Cb, Cc of the moveable one will be respectively $= Mo$, mn.



From the center d (the point where the perpendiculars Md, md touch the evolute) describe the small arch nv; and the small right lines be, ce, being considered as small arches, described from their respective centers A and C; the small right-angled triangles bce, vno, will be similar and equal; bc being $= no$, and $ce = nv$; ∴ $Cc : dn :: \angle ndv : cCe$. But $ndv \doteq nDo + cCe$. And calling AC, $a = 2$; CD, $b = 2640$ feet; AB, x; we get $nd = \frac{b\sqrt{aa-ax}}{a+b}$; now, as $\frac{b\sqrt{aa-ax}}{a+b}$ (nd) : $\frac{a\dot{x}}{2\sqrt{ax}}$ $(vn = ce)$ :: $\frac{a+2b}{a+b} \times \sqrt{aa-ax}$ $(mn + nd)$: $\frac{a+2b}{2b\sqrt{ax}} \times a\dot{x}$ (Mm) the fluxion of the epicycloid; the fluent whereof is $\frac{a+2b}{b}\sqrt{ax} =$ half the space a spot on the convex surface of the stone will go through in one revolution (when $x = a = 2$) $=$ 4.001515, &c. Consequently 8·00303, &c. feet is the whole space described at each revolution; and the stone revolving exactly 1320 times, going up and down the mountain, the whole space gone through by the spot will be 10564 feet, or 2·00075, &c. miles.

Removing the stone to the next point from the vertex, the gravity will make it descend along the side of the mountain, till its velocity, in an horizontal direction, is the greatest it can acquire; when it will fly off, and descend to the bottom, in the curve of a parabola. Putting $a = 2640$ feet, the mountain's mountain's height, $x=$ dist. perp. descended, $s=16\frac{1}{12}$ feet, a space perp. descended in the 1st second of time, then $\sqrt{s}$: $2s$ :: $\sqrt{x}$: $2\sqrt{sx}$ the velocity in the curve, which is to the velocity in an horizontal direction, as the tangent is to the ordinate, i. e. $\frac{a}{a-x} : 1 :: 2\sqrt{sx} : \frac{2\sqrt{s}}{a}\sqrt{aax-2axx+x^3}$ the velocity in a direction parallel to the horizon, whose fluxion reduced is $xx-\frac{4ax}{3}+\frac{aa}{3}=0$, where $x=\frac{1}{3}a$ the distance descended when the stone ceases to touch the mountain.

The velocity $2\sqrt{sx}$ applied to $\frac{a\dot{x}}{\sqrt{2ax-xx}}$, the fluxion of the arc, gives $\frac{a}{2\sqrt{s}}\times\frac{\dot{x}}{x\sqrt{2a-x}}$, the fluxion of time, whose fluent is $\frac{\sqrt{a}}{2\sqrt{2s}}\times$ hyp. log. $x+\frac{\sqrt{a}}{2\sqrt{2s}}\times : \frac{x}{2.2.a}+\frac{3x^2}{2.2.4.2^2a^2}+\frac{3.5x^3}{3.2.4.6.2^3a^3}$ &c. $=31''$ (when $x=\frac{1}{3}a$). If to which 6'' 18''', the time of descending through the parabolical arch, be added, the sum is 37'' 18''', the whole time of descent required; and the stone will fall 326 feet from the foot of the mountain. *Q.E.F.*

Mr. Stone observes in his Mathematical Dictionary, that 'as semi-diam. resting circle : sum diameters of both circles :: double sine of half the arc, touching the circle at rest : length of the part of the epicycloid described by a point in the revolving circle; if upon the convexity of the resting circle. But if upon the concavity; as semi-diam. resting circle, to the difference of diameters of the touching circle.'

In a semi-revolution, 180° of the moving circle touched the circle at rest; the double sine of its half arc $=2$ (its rad. being unity); ∵ 2640 : 5282 :: 2 : 4·001515, &c. = half the epicycloidal curve, as before.

Mr. *Nicholas Dixon* sent us (from cor. 2 prop. 49 lib. 1 Principia Newtonian.) the same proportions, who gave the space run through by the spot 10564 feet; computing it from 1320 revolutions of the stone. Our ingenious correspondent Mr. *Ash* gave the fluxion and fluent of the curve, by two different methods; confirming the truth of the above solutions. He refers to Mr. Stone's Fluxions (p. 127 cor. 1) in his

his first and best method; finding the fluent of the semi-curve $\frac{2bx+2ax}{b}$; $b=$ rad. resting circle; $a=$ rad. moving circle; and $x=$ a variable chord, till it becomes $=$ diam. moving circle, or $2a$.

Mr. *William Heunsell*, and Mr. *Cottam*, are true in the distance travelled by the spot.

The Prize of 10 *Diaries was won by Mr.* Landen.

The Eclipses calculated for 1748, *by* Mr. Bulman.

1. The sun eclipsed Jan. 19, at 3 h. 27 m. in the morning. Consequently invisible to our part of the globe.

2. The moon eclipsed Feb. 3, at 11 h. 53 m. morn. invisible.

3. A great visible solar eclipse July 14, at 10 h. 43 m. morn.

Calculated by		Beg. h. m.	Mid. h. m.	End h. m	Dur. h. m.	Dig.	
Mr. Turner, by a geom. construct. and calculation for	Hull	9 0	10 32	12 6	3 6	$10\frac{3}{4}$	☉'s upper limb.
	London	——	10 34	——	——	$10\frac{1}{8}$	
	Paris	——	10 49	——	——	$9\frac{3}{8}$	
	Boston N. E.	——	4 53	——	——	$9\frac{7}{8}$	
	Dundee	——	10 19	——	——	Central.	
	Stockholm	——	12 14	——	——	$9\frac{3}{8}$	☉'s low. limb.
	Petersburgh	——	1 15	——	——	$8\frac{1}{2}$	
Mr. Bulman,							
By Astro. Carol. London		9 13	10 43	12 17	3 4	10 20	
Leadbet. Tables, Idem		9 6	10 40	12 9	3 13	10 26	
Astro. Carolina, Rochester		9 14	10 44	12 18	3 4	10 21	
Leadbet. Tables, Idem		9 7	10 41	12 20	3 13	10 28	
Mr. Hawkins,	London	9 3	10 40	12 17	3 14	10 31	
Mr. Cockson,	London	8 56	10 31	12 6	3 10	10 18	
	Lambton	8 50	10 31	12 12	3 22	11 14	
	Durham	8 50	10 31	12 12	3 21	11 14	
	Feckenh.	8 46	10 28	12 10	3 24	11 6	
Mr. Cowper,	London	8 57	10 28	12 1	——	10 1	
	Northamp	9 1	10 32	12 6	——	10 4	
Mr. Farrer, Sunderland		9 15	10 48	12 24	3 9	11 38	
Late Mr. Beighton, Coven.		9 8	10 38	12 12	3 4	10 16	

Mr. *Turner* curiously observes that the penumbra, in the great eclipse of the sun, first touches the earth's disk, in lat. 44° 15′ north, and long. 75° 47′ west from London. That the

the central eclipse first enters upon the river St. Lawrence, a little to the southward of Quebec; from whence directing its course about E. N. E. it passes over St. Lawrence Bay, the northermost part of Newfoundland, and the Atlantic ocean; and arriving in lat. 57° 32′ and long. 18° 30′ west (where it approaches the nearest to the north pole) it then passes about E. S. E. traversing the western isles of Scotland, over the midst of Barra, and the north part of Mull; so over Fort William and Dundee; then crosses the German ocean, and passes over Lubec, Glogaw, and Breslaw, being centrally eclipsed in the meridian in lat. 52° 5′ N. and long. 15° E. in which same longitude, and in lat. 18° 3′ N. the lower limb of the moon just touches the upper limb of the sun in the meridian. Then the central shadow enters Poland, goes near Cracow, and crossing the southern parts thereof, enters Turkey in Europe, and crosses over the Euxine sea, the Grand Signior's Asiatic dominions, Persia, and the estates of the Great Mogul; and finally quits the earth near Tranquebar, upon the coast of Coromandel, in lat. 10° 54′ N. long. 77° 49′ E. Lastly, in long. 55° 30′ E. under the equator, the penumbra quits the earth's disk, and the eclipse totally ends at the top of the sun's vertical diameter, whilst his last rays are hiding under the horizon.

The breadth of the annular shade is about 166 geographical, or 192 English miles; and the velocity of which, over the earth's disk, 40 statute miles in a minute.

But the velocity whereby the shade recedes from any given place on the earth's surface, is less than that with which it passes over the disk: Because while the shadow moves from west to east, all the places of the earth are carried by its rotation the same way, which following the motion of the shadow with a slower pace, they diminish the velocity of it, moving from them; making it move not above 30½ statute miles per minute.

Hull, April 11*th*, 1737. *J. Turner.*

Mr. *Bulman* says according to the Chaldean saros, that this eclipse will return again July 25, 1766; and that it is the greatest visible solar eclipse which will happen before March 21, 1764. Mr. *Sam. Owen*, of Kent, sent an exact calculation for London. Some other persons, besides those inserted in the tables, sent calculations of this year's eclipses, which for want of being correct are omitted. And we must acknowledge the favour of Mr. *Cowper* in offering us a correct and annual calculation of the lunations; as we are sorry his lunations for last year were omitted for those which were grossly false and inconsistent.

4. The

4. The moon visibly eclipsed July 28, at 11 h. 34 m. at night.

Calculated by		Beg. h. m.	Mid. h. m.	End h. m.	Dur. h. m.	Dig.
Mr. Bulman,	London	10 24	11 34	12 44	2 20	4 37
	Edinburgh	10 12	11 22	12 32		
	Dublin	9 56	11 6	12 16		
	Carlisle	10 13	11 23	12 33		
	Rochester	10 25	11 35	12 45		
Mr. Hawkins,	London	10 32	11 37	12 41		4 45
	Hanover	11 12	12 17	1 21		
	Oxford	10 27	11 32	12 36		
Mr. C. Cockson,	London	10 32	11 36	12 40		
	Lambton	10 45	11 49	12 53	2 8	4 40
	Durham	10 45	11 49	12 53		
	Feckenham	10 38	11 42	12 46		
Mr. T. Cowper,	Wellingbor.	10 6	11 13	12 19		5 near
Mr Farrer,	Sunderland	10 43	11 48	12 52	2 9	5 2

Mr. Hawkins has confirmed the above eclipses, and given (very judiciously) the appearance of the solar eclipse on Jan. 19, in lat. 43° 38′ S. long. 64° 8′ E. And also that of the moon's eclipse for Feb. 3, at Boston, Carthagena, and Hispaniola.

New

The best observations of the Eclipse of July 14, are as here below.

Places	Observer	Beginning	End
		h. m. s.	Between
Marlborough House	Dr. Bevis	9 3 48	12 h. 9 m. 15 s. and 12 9 35 Ap.t.
Lufwick, North-ampt. lat. 52° 27½′	Mr. Mark Day	9 1 0	12 5 25 Ap.t.
Madrid	A. N. Grexhow	8 49 6	True time
Aberdour Castle, N. Britain	Lord Morton, Mr. Short, and Mr. le Monnier	8 51 18	11 48 18

Aberdour Castle is in lat. 56° 4′, and long. 6° 15′ west of Edinburgh College.——The Eclipse was observed to be annular in N. Britain, Berlin, Francfort, and many other places.

A COMET

Was this year observed, and an account of its motion taken at Pekin in China, in the months of April, May, and June.

New Questions.

I. QUESTION 295, *by Mr.* Heath.

With guineas and moidores the fewest, which way
Three hundred and forty-one pounds can you pay?
If paid ev'ry way 'twill admit of, what sum
Do the pieces amount to?——My fortune to come.

II QUESTION 296, *by* Amanda.

To find the least number of guineas, which being divided by 6, 5, 4, 3, and 2, respectively, shall leave 5, 4, 3, 2, and 1, respectively remaining?

III. QUESTION 297, *by Mr.* John Turner.

What is the day of the month, and hour of the day, at Rochester, in Kent, (at which time a couple are to be married this year) when the degrees of time from noon, to the degrees of time from sun-rise, are in the proportion of 3 to 1, and the respective sines of those degrees in the proportion of 9 to 4?

IV. QUESTION 298, *by Miss* Manlove.

All the different ways possible, in which a gentleman can place his servants, combining them by 1, 2, 3, &c. at a time are 960799; what number of servants does he keep?

V. QUESTION 299, *by Mr.* Bulman.

Three to two, top and bottom, a tub's width I've found,
Seven yards the diag'nal, as it stands on the ground,
With its gallons the most; but in regions below,
Where 'twill take in three more, brother conjurers, show.

VI. QUESTION 300, *by* Rodomontado.

What is the least degree of velocity with which an iron ball of 12 pounds weight must be projected from the surface of our earth, at an angle of 40° elevation, whereby it shall not return.

VII. QUES-

VII. Question 301, *by Mr.* Landen.

If an infinite number of perpendiculars be let fall from one end of the diameter of a ſemicircle, upon an infinite number of tangents drawn about it, and a curve paſſes through all thoſe angular points, what will be the length of that curve? The area of the ſpace included betwixt it and ſemi-circle? With the dimenſions of the greateſt ordinate, when the ſaid diameter is = 20 inches?

VIII. Question 302, *by* Upnorenſis.

Obſerving a horſe tied to feed in a gentleman's Park, with one end of a rope to his fore foot, and the other end to one of the circular iron rails, incloſing a pond, the circumference of which rails being 160 yards, equal to the length of the rope, what quantity of ground, at moſt, could the horſe feed?

IX. Question 303, *by Mr.* John Turner.

If the axis of the penumbral cone, falling upon the diſk the earth, makes an angle with the earth's diameter at the ſurface of 24°, (the angle at the cone's vertex being 32′ 46″) and from a point in that axis, at the diſtance of 58·5 ſemi-diameters of the earth from the vertex, it is 64 ſemi-diameters to the earth's center, how much of the earth's ſurface is included in the penumbral ſhadow?

X. Question 304, *by Mr.* John Hampſon.

Required to find three ſuch fractional numbers, that when each is leſſen'd by the cube of their ſum, three cube numbers ſhall remain?

XI. Question 305, *by* Roſamond.

To find two (or more ſuch pair of amiable, but unequal) numbers, that each ſhall be mutually equal to the ſum of the aliquot parts of the other? And alſo to find the leaſt number, whoſe aliquot parts ſummed up, ſhall exceed it by 7?

XII. Question 306, *by Mr.* Heath.

If the long-diſputed prize money, between the officers of the ſhip Centurion and Glouceſter, had been divided in the proportion of two numbers [each of which being raiſed to a power expreſſed by the natural logarithm of the other, ſhall

 be

be equal to the ſum and difference of thoſe numbers]. What would be the odds of advantage allotted to Lord Anſon's officers belonging to the ſaid ſhip the Centurion?

XIII. QUESTION 307, *by Mr.* Landen.

Let a ball of heavy metal be laid upon one end of an horizontal plane, of an indefinite length, round which end let the plane be made to revolve downwards, with ſuch an uniform motion, that the angle of inclination may increaſe at any given rate: It is required to find what length the ball will deſcend along the plane, before it acquires ſuch a velocity as will cauſe it to fly off, and ceaſe touching the plane?

XIV. QUESTION 308, *by* Upnorenſis.

A lady paid twice as much a-piece for geeſe, as ſhe paid for ducks; and twice as much a-piece for ducks, as ſhe paid for chickens, which coſt together 1 l. 13 s. 4 d. the ſum of the ſquares of the number ſhe bought of each ſort was 326. What number of geeſe, ducks, and chickens did ſhe buy? And what was the price of each?

XV. QUESTION 309, *by* Hurlothrumbo.

What are the odds of battle, or the different probabilities of ſucceſs of two armies going to engage, the chances of each army for victory being reſpectively equal to the ſum raiſed to a power expreſſed by the difference, and the difference raiſed to a power expreſſed by the ſum, of thoſe chances?

XVI. QUESTION 310, *by Mr.* Aſh.

A ſpider, at one corner of a ſemi-circular pane of glaſs, gave uniform and direct chace to a fly, moving uniformly along the curve before him: The fly was 30° from the ſpider at their firſt ſetting out, and was taken by him at the oppoſite corner. What is the ratio of both their uniform motions?

PRIZE QUESTION *by Mr.* R. Heath.

On what days of the year does the city of London travel the greateſt and leaſt number of miles, by the diurnal and annual motion of the earth? And how many miles per day, and alſo per hour about noon does it travel, when the days are longeſt and ſhorteſt in that place?

Queſtions

1749.

Questions answered.

I. QUESTION 295 *answered by Mr.* John Turner *of* Heath, *near* Wakefield.

IT is evident, that with the number of pounds printed (by miftake) the queftion is impoffible; but fuppofing 351 l. intended (as the author informs me of) inftead of 341, it is thus anfwered.

Put $x =$ number of guineas, $y =$ number of moidores $= \frac{7020 - 21x}{27}$ by confequence, which muft be a whole number. Whence $\frac{21x}{27}$, or $\frac{x}{9}$ is a whole number; confequently the leaft value of $x = 9$; whence $y = 253$, whofe fum is 262 the leaft number of pieces; becaufe there are taken the moft moidores, except paying the whole with them. Let $\frac{x}{9} = m$, then $x = 9m$, and $y = 260 - 7m$ by fubftitution; by which the greateft value of $m = 37$, when $x = 333$ guin. and $y =$ 1 moid. the greateft number of pieces. The firft term of an arithmetical progreffion being 9 (a), the laft 333 (y), and the difference 9 (e) as it is evident, then the number of terms $= \frac{y + e - a}{e} = 37$, the ways to pay the fum of 351 l. which being 37 times taken is $= 12987$ l. due to Mr. Heath.

N. B. The intermediate numbers are found by continually adding 9 to the guineas, and fubtracting 7 from the moidores. There will be but 36 ways, if the payment with one moidore is not admitted. *Q. E. F.*

We defire all thofe who anfwered this queftion, or any other wrong, not to be difobliged at our omiffion of their folutions; being not againft their fhining in a proper place.

II. QUESTION 296 *anſwered by Mr.* Heath.

It is a maxim, *That whole numbers added to, ſubtracted from, or multiplied into whole numbers, ſhall produce whole numbers*: Upon which fundamentals theſe kind of queſtions are naturally reſolved; though I have not ſeen the method clearly explained. Let $x =$ the number ſought, then $\frac{x-5}{6}$, $\frac{x-4}{5}$, $\frac{x-3}{4}$, $\frac{x-2}{3}$, $\frac{x-1}{2}$ are all whole numbers by the queſt. Put $\frac{x-5}{6} = m$, then $x = 6m + 5$ a whole number, and by ſubſtitution in the ſecond whole number for the value of x, $\frac{6m+1}{5} = m + \frac{m+1}{5}$ a whole number; ∵ $\frac{m+1}{5} = n$ a whole number, and $m = 5n - 1$; which ſubſtituted for m, in the firſt value of x, is $x = 30n - 1$ for the ſecond value; ∵ $\frac{30n-4}{4}$ by ſubſtitution is the value of the 3d number, and $7n - 1 + \frac{2n}{4}$ is a whole number, and $n =$ a whole number; ∵ ſubſtituting again the 2d value of x in the 4th whole number, $\frac{30n-3}{3} = 10n - 1 =$ a whole number, ∵ n is a whole number (firſt ſuppoſed); and ſubſtituting again the 2d value of x in the 5th whole number, $\frac{30n-2}{2} = 15n - 1$ a whole number; ſo that $x = 30n - 1$ is the general value, after aſſuming $n = 2$, by which the leaſt value of $x = 59$.

Mr. *Richard Gibbons* of Plymouth has ſhewn an eaſy method of finding the leaſt number only, to anſwer theſe kind of queſtions. For he obſerves that $2 \times 3 \times 4 \times 5 \times 6 = 720$ is a number, which being divided reſpectively by 6, 5, 4, &c. will leave no remainder; and therefore $720 - 1 = 719$ will leave the required remainders, of 1 leſs than each factor. But 4 being a ſquare of 2, and 6 a multiple of 2 and 3, ∵ $2 \times 3 \times 2 \times 5 : - 1 = 59$ the leaſt number required.

Mr. *Flitcon* has made exactly the ſame obſervation; and alſo Mr. *James Terey* of Portſmouth.

Mr. *Farrer* gives $x = 60q - 1$, where q is a whole number, for the general value of x; which it ſeems is according to Mr. *Robinſon*'s method: but the next value of x to 59 is 89, which Mr. *Farrer*'s equation does not find, and therefore not

not true, as is proved above. This gentleman obſerves, that this queſtion is of like kind with one in another kind of place; to which we did not deſign any reſemblance, nor conſulted ſuch pattern, if we have done any honour. Mr. *Collingridge*, Mr. *Hare*, and ſome others, ſolved this queſtion.

III. QUESTION 297 *anſwered by Mr.* Nich. Farrer.

Let $x =$ s. time from ſun-riſe, then $3x - 4x^3 =$ s. time from noon. By queſt. $9 : 4 :: 3x - 4x^3 : x$; hence $x = \frac{\sqrt{3}}{4}$ $= \cdot 4330127 =$ s. $25^\circ\ 39'\ 32''$, and $3x - 4x^3 = \cdot 9742786 =$ s. of $76^\circ\ 58'\ 36'' = 5$h. $7'\ 54''$ time. Hence the ceremony begun at 6 h. $52'\ 6''$ in the morning. ☉ riſes 5 h. $9'\ 27''$, aſcenſional diff. $= 12^\circ\ 38'\ 8''$ lat. Rocheſter $51^\circ\ 28'$, from which the ſun's declination $= 9^\circ\ 53'$, anſwering to April 4th, or Auguſt 16th. *Q.E.F.*

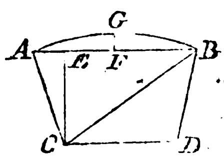
Mr. *Gibbons* has ſolved this queſtion by the method of trial-and-error; thereby proportioning the truth very exactly. He obſerves the advantages of trial above the uſe of ſeries, in many caſes. The ſame was ſolved by the Rev. Mr. *Baker*, and ſome others.

IV. QUESTION 298 *ſolved by Mr.* Collingridge.

Let $x =$ the number of ſervants; then, by Mr. Stone's Mathematical Dictionary, $\frac{x^{x+1} - x}{x - 1} = 960799$, all the poſſible variations of the ſervants; which equation ſolved, $x = 7$ ſervants exactly. *Q.E.F.*

Mr. *Turner* ſolved the ſame; referring to Wolfius' Elementa Matheſeos, p. 305, for the equation; which however is eaſy to be deduced from the principles of combination. Mr. *Farrer* ſolved it likewiſe.

V. QUESTION 299 *anſwered by Mr.* James Terey *of* Portſmouth.



Let $CB = 7 = d$, $AB = x$, then $CD = \frac{2}{3}x$, per queſt. alſo $BE = \frac{5}{6}x$. Then $\sqrt{dd - \frac{25}{36}xx} = CE$. Now $\frac{19}{9}xx \times \sqrt{dd - \frac{25}{36}xx} \times 43{\cdot}313864$ $=$ content in ale gallons, a maximum; or $x^4 \times 36dd - 25xx$, a

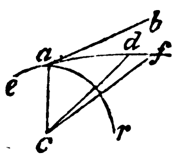
 maximum;

maximum; in fluxions and reduced, $x=\frac{d\sqrt{24}}{5}=6·858571279$ yards $= AB$; and $CD = 4·572380852$, $CE = 4·04145188$ yards. The content in ale gallons $= 17383·721448$ &c. But to hold 3 gallons more AGB must be a spherical segment, containing 3 gallons. Let $FG = z$, $AF = FB = b$. By a theorem, $·523598\,z$ &c. $\times \overline{3bb + z^2}$ = cont. spher. seg. in numbers $z^3 + 35·28\,z = ·0346309367$, where $z = ·00098160248$ yards. The diameter of which sphere $= 11980·410811$ &c. yards. Consequently, if the brim of the tub is placed 5990·205405 yards from the earth's center, it will hold 3 gallons more. *Q.E.F.*

Mr. *Bamfield* has found the distance from the earth's center exactly the same to a yard; but his figure was drawn so large, that we were obliged to omit inserting his answer: as we omit other answers to questions on account of the unfit size of the schemes drawn, which require too much time and trouble to alter. Mr. *Baker*'s scheme was very proper, and his solution very elegant to this question; but we cannot insert all good performances. Mr. *Farrer*'s figure was too large. Mr. *Hampson* and Mr. *Himbol* sent us their solutions. Mr. *Bulman* the proposer died near Rochester, in Kent, about Michaelmas 1747, who may have given a practical solution himself before this time; though he was (like most of the brother conjurers) a very honest, odd kind of person.

VI. QUESTION 300 *solved by the Excellent* J. Landen, *near* Peterborough.

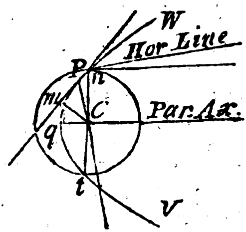
Let c be the earth's center, ear the surface, ab the projectile's direction, and adf its trajectory. Suppose cd, ef indefinitely near each other, and call ca, (the earth's radius $=$ 21000000 feet) a; cd, x; 32·2 feet, the velocity generated in a second at the earth's surface, b; v the velocity in d; V the required velocity. Then the centripetal force in d will be $\frac{a^2 b}{x^2}$ (being reciprocally as the square of the distance from the earth's center) and the force to retard the motion in the direction df, $\frac{a^2 b \dot{x}}{x^2 \times af}$; this retarding force drawn into the fluxion of the time, being equal to the fluxion of the velocity, $\frac{a^2 b \dot{x}}{v x^2}$ will be $=$ $-\dot{v}$;



$-\dot{v}$; therefore $v\dot{v} = -\frac{a^2 b\dot{x}}{xx}$, and the fluent $\frac{vv}{2} = \frac{a^2 b}{x}$: But in a, (v being $= V$, and $x = a$) the correct fluent gives $v = \sqrt{VV - 2ab + 2a^2 bx^{-1}}$. After an infinite time, x will be infinitely great, and $\sqrt{VV - 2ab + 2a^2 bx^{-1}}$ infinitely small, and therefore may be put $= 0$, in which equation a is nothing in respect of the value of x: and therefore $V = \sqrt{2ab}$. Hence, without regard to the angle of direction, if a body be projected from the earth's surface, in any direction whatever above the horizon, with such a velocity as will carry it above 7 miles per second, it will never return. *Q.E.F.*

Mr. John Turner *of* Heath, *sends us his Solution as follows.*

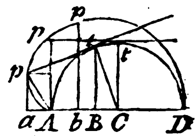
Let C represent the earth's center, and let a body be supposed to revolve in a circle at the superficies, the angle of the projectile's elevation being 40°, and a tangent to the parabolical curve qPW at P, the $\angle CPm$ will be 50°, (letting fall Cm perpendicular to Pm) whose complement to 180° = 130° = angle mPb. By prop. 17 and corol. of prop. 16, Newton's Principia, the axis of the parabolic trajectory will be parallel to Pb, passing through C; and likewise the latus rectum of the orbit = $2CP + 2Pn = 2{\cdot}3472$ semi-diameters of the earth (letting fall Cn perpendicular to Pb) and the focal distance from the vertex = $\cdot5868$ semi-diameters. Lastly, the velocity of a body moving in a circular orbit at the earth's superficies (by the prop. aforesaid) is such as would carry it through $4{\cdot}92$ miles uniformly, in each second; therefore, if x = velocity in the parabolic curve at P, we have (by prop. 15, 16, and 17 of the said Principia) As $24{\cdot}2064 : \cdot5868\,xx :: 2 : 2{\cdot}3472$; whence $x = 6{\cdot}958$ miles, or about 7 miles per second, the uniform velocity with which the body must be projected from P in the given angle of elevation, not to return. Or, since it is demonstrated (by the writers on physics) that the velocity of a body moving in a parabola is to the velocity of a body moving in a circle, at the same distance from the center of force, as $\sqrt{2}$ to 1, $\because 4{\cdot}92 \times 1{\cdot}4142 = 6{\cdot}958$ miles, the projectile's velocity per second, as before. *Q.E.F.*



VII. QUES-

VII. QUESTION 301 *answered by Mr.* Landen.

It is easily proved that the perpendicular Ap is always = its respective abscissa AB of the semicircle, and the $\angle pAe = \angle tCt$. Then, if Aep be supposed infinitely near Ap, and Ap or AB be called x, and AD, $2a$; it follows, as $a : \frac{a\dot{x}}{\sqrt{2ax-xx}} ::$



$x : \frac{x\dot{x}}{\sqrt{2ax-xx}}$ = the infinitely small arch pe, and the fluxion of the curve $= \sqrt{2a} \times \frac{\dot{x}}{\sqrt{2a-x}}$, the fluent of which corrected is $4a - 2\sqrt{4aa - 2ax}$ = length of the curve; which, when $x = 2a$, in the present case, is equal to twice the diameter AD, or 40 inches.

But $\frac{1}{2}x \times (pe) \frac{x\dot{x}}{\sqrt{2ax-xx}} = \frac{\frac{1}{2}x^{\frac{3}{2}}\dot{x}}{\sqrt{2a-x}}$, the fluxion of the area; whose fluent, by Mr. Cotes' 6th form, or Mr. Emerson's 10th and 11th forms, is $\frac{3}{2}aa \times$ arch of a circle whose radius = 1 and nat. sine $\sqrt{\frac{x}{2a}}$, $-\frac{3}{4}ax^{\frac{1}{2}}\sqrt{2a-x}$ $-\frac{1}{4}x^{\frac{3}{2}}\sqrt{2a-x}$ = area sought; in the present case $= \frac{3}{2} \times$ area of the semicircle, when $x = 2a$. Consequently the space $ApDtA$ = quadrant $AttCA$ = 78·54 inches.

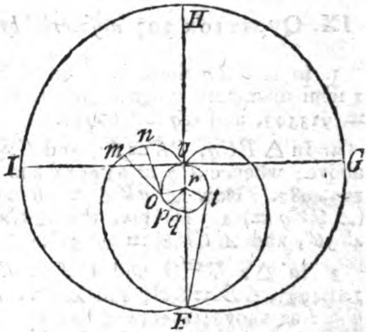
For the greatest ordinate, the proportion is $a : \sqrt{2ax-xx} :: x : \frac{\sqrt{2ax^3-x^4}}{a}$ = any ordinate. Its fluxion being made $= 0$, $x = \frac{3}{2}a = 15$ (though $= bD$), which being substituted for x in the expression of the ordinate above, then $\frac{3\sqrt{3}}{4} \times a$ will be the greatest ordinate in all cases; but in this case = 12·9903 inches $= bp$. *Q.E.F.*

We cannot help admiring this gentleman's art and sagacity, in performing what Sir Isaac Newton must have commended. And as we know him to be acquainted with most of Sir Isaac Newton's excellencies, great things are promised from his genius in the sciences. Mr. *Farrer* sent us a fluxionary solution to the same, with the true numbers; but as the general fluent to the fluxion was not found, we could not see the use of it.

VIII. QUES-

VIII. QUESTION 302 *answered by Mr.* Heath.

Let all the rope be wrapt round the rails of the pond *vopqtv*, and the horse begin to move, or unwrap his rope from *v* (where it is fixed) in the track of *vnm FGHIFv*; then the space *FGHIF*, deducting the pond's area, is the greatest he can feed. When his rope is completely unwrapt at *G*, he describes the semicircle *GHI*, with rad. *vG* = 160 yards, the area of which = 40212·48 yards square. Then

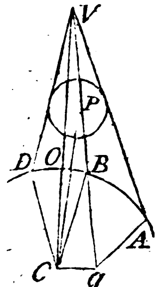


he begins to wind his rope round the rails the contrary way to the former winding; describing the track *IFv*, similar and equal to the track *vnmFG*, at the unwinding of his rope. *To find the area he describes, at unwinding, or winding?* Put r = pond's rad. = 35·464731 yards; z = any arch of the rails unwound, suppose *vo* = *on*, and $\dot{z}$ its fluxion = *op*. Now, little sector *rop* is the fluxion or next increment of circular space *rvo*, as little sector *pnm* (with radii of curvature *pn*, *pm*) is the next increment of the space *von*; both sectors, when infinitely little, being similar, say, r (*ro*) : $\dot{z}$ (*op*) :: z (*on*) : $\frac{z\dot{z}}{r}$ = *nm*, the fluxion of the involute arch *vn*, whose fluent is $\frac{z^2}{2r}$: but sector *pnm* = $\frac{z^2\dot{z}}{2r}$, the flux. space *von*, whose fluent is $\frac{z^3}{6r}$ = (when z = 160) to 26808·317 yards square, being area space *vnmFGvtqpov*, when the rope is quite unwound. Also *Ft* = *tqpov* = 114·42 yards, best found by trial and a table of logarithms (series and the reversion of series being less certain and expeditious); whence area *vnmFtqpov* = 9804·2 square yards; whence area *GvtFG* = 17004·117 by substitution, to which adding area *tqF* = 1018·58 gives 18022·69 = area *GvtqFG*, which doubled and added to semicircle *GHI* make 76257·86 square yards = 15 a. 2 r. 12 p. And length of the whole track = 1507·96 yards. N. B. *vnmFG* = *GHI* = 502·656. Q. F. F.

Mr.

Mr. *Landen* ſent us an exact ſolution by another method; which are all the ſolutions given.

IX. QUESTION 303 *anſwered by the Rev. Mr.* Baker.



1. In △ CBq there is given ∠ $CBq = 24°$, and $CB =$ 1 ſemi-diameter; whence (by trigon.) $Bq = \cdot913545$, and $Cq = \cdot406736$.

2. In △ PCq, $PC = 64$, and $CQ =$ as above; whence $Pq = 63\cdot9987$ and $Vq = 122\cdot4987$. Now ∠ $qVA = 16' \, 23'' \mp$ (∠ $CVq \pm$) $11' \, 27''$ give the ∠ $CVD = 4' \, 56''$, and ∠ $CVA = 27' \, 50''$.

3. In △s CVD and CVA, $CV = 122\cdot4993$, $CD = CB$, and ∠s CVD and CVA as above; thence ∠ $DCV = 9° \, 59'$ fere, and ∠ $VCA = 81° \, 12'$, which added $= 91° \, 11' =$ arch $Do +$ arch oA, the quantity of the earth's ſurface included in the penumbral ſhadow. *Q.E.F.*

Mr. *J. Turner* has likewiſe ſent a ſolution; being the only ones we received.

X. QUESTION 304 *anſwered by Mr.* Landen.

This gentleman puts zy for the 1ſt number, zx for the 2d, and zv for the 3d, p^2z for the ſum of the three; zs for the ſide of the 1ſt cube, zr for the ſide of the 2d, and zq the ſide of the 3d, and ſo proceeds to equations; from whence, in a curious manner, he determines the 3 cube numbers $\frac{341}{4913}$, $\frac{854}{4913}$, and $\frac{350}{4913}$. But we muſt reſerve his method for more room than we have at preſent.

Mr. *Farrer* informs us, that this queſtion was taken from Branker's Algeb. where the ſolution may be ſeen at large; and ſays that one ſet of numbers is $\frac{494424}{2352637}$, $\frac{242696}{2352637}$, $\frac{448000}{2352637}$.

Mr. *Hampſon*'s numbers are $\frac{854}{4913}$, $\frac{341}{4913}$, $\frac{250}{4913}$, beſides other of his numbers ſent.

XI. QUES-

XI. QUESTION 305 *answered by Mr.* Landen.

Put $4x$ for one of the amiable numbers, and $4yz$ for the other, x, y, and z being primes; we shall have $7+3x=4yz$, or $x=\frac{4yz-7}{3}$, and $4x=7+7y+7z+3yz=\frac{16yz-28}{3}$; hence $z=3+\frac{16}{y-3}$. Now taking $y=5$, z will be $=11$, and $x=71$, $4x=284$, and $4yz=220$, the first and least amiable pair.

This gentleman proceeds in a new method of substitution (shewing the defect of the present method) and finds 18416 and 17296 the next amiable pair, and 9437056 and 9363584 for a third amiable pair. And then gives this general rule: If 2^n (n being an affirmative integer) be taken such, that $3\times2^n-1$, $6\times2^n-1$, and $18\times2^{2n}-1$ be primes, then $2^{n+1}\times18\times2^{2n}-1$ shall be an amiable number, and $2^{n+1}\times3\times2^n-1\times6\times2^n-1$ its amiable correspondent or partner.

N. B. Mr. Stone's theorem, in his Mathematical Dictionary, for finding amiable numbers, is erroneous. The above true rule is said by Mr. Landen to be first shewn by the famous Descartes.

The same ingenious gentleman proceeds to the solution of the 2d part of the question, putting $4x^n=$ the numb. sought, x being a prime, and n an affirmative integer.

$7+7x+7x^2+\ldots.7x^{n-1}+3x^n$ is the sum of its aliquot parts, which must be $=4x^n+7$. Therefore $x^n-7x-7x^2-\ldots7x^{n-1}=\frac{x^{n+1}-8x^n+7x}{x-1}=0$. Whence $x^n-8x^{n-1}+7=0$. Herein, if $n=2$, x will be $=7$. Whence $4\times7^2=196$ the least whole number, whose aliquot parts summed up shall exceed it by 7.

XII. QUESTION 306 *answered by Mr.* Heath *only.*

Instead of each proportional number raised to the power expressed by the natural logarithm of the other, it should have been expressed *root extracted*, or each proportional number raised to a power expressed also by the logarithm root

root of the other. Then putting x and z for the numbers, $x^{\frac{1}{l.z}} = x+z$, and $z^{\frac{1}{l.x}} = x-z$; which reduce to $x = \overline{x+z}|^{l.z}$ and $z = \overline{x-z}|^{l.x}$; and if $b = 2{\cdot}3025$ &c. then $x = \overline{x+z}|^{l.z \times b}$, and $z = \overline{x-z}|^{l.x \times b}$; whence, by the table of common logarithms (performing beyond the art of series) $x = 3{\cdot}8761$, and $z = 2{\cdot}1292$ fere. And hence the odds of proportion of payment are as $1{\cdot}8204$ to 1. As the question was expressed $x^{l.z} = x+z$, and $z^{l.x} = x-z$, where, if $z = 0$, the Gloucester's share of prize money, in the first equation, then x, in the same equation has an impossible value for the Centurion's lot. *Q. E. F.*

We thank Mr. *Farrer* for his observation on this question; truth being always welcome to us.

XIII. QUEST. 307 *answered by Mr.* Landen *the Proposer.*

When the velocity of the ball, in a direction parallel to the horizon, is the greatest it can acquire by such a descent, the ball will fly off, and describe a portion of a parabola, to which the plane, at that instant of time, will be a tangent. *To find the said Velocity.* Put a = angular velocity of the plane per second, measured by the arch of a circle, whose rad. = 1; $A = 32\frac{1}{6}$ feet, the absolute force of gravity, computed by the velocity generated in a second; x = sine of angle of inclination; rad. being = 1. The force of gravity to accelerate the motion of the ball along the plane = Ax; the absolute gravity being to the relative gravity of the descending ball, as rad. to sine of the angle of inclination. The fluxion of the velocity along the plane $= \frac{A}{a} + \frac{x\dot{x}}{\sqrt{1-xx}}$, i. e. the accelerating force drawn into the fluxion of the time. The fluent of which corrected is $\frac{A}{a} \times \overline{1 - \sqrt{1-xx}} = \frac{A}{a} \times$ versed sine of the plane's inclination, for the velocity itself: But this velocity is to the velocity in a horizontal direction as rad. to cos. angle of inclination. Therefore $1 : \sqrt{1-xx} :: \frac{A}{a} \times \overline{1-\sqrt{1-xx}} : \frac{A}{a} \times \overline{\sqrt{1-xx} - 1 - xx}$, the velocity of the ball in the said direction; the fluxion of which being

being made = o, and reduced $x = \sqrt{\frac{3}{4}}$, the sine of the angle of inclination, when the ball quits the plane. *To find the length then descended by the ball along the plane.* Multiply the velocity $\frac{A}{a} \times \overline{1 - \sqrt{1 - xx}}$ by $\frac{\dot{x}}{a\sqrt{1 - xx}}$ the fluxion of time, and we have $\frac{A}{aa} \times \overline{1 - xx}|^{-\frac{1}{2}} - 1 \times \dot{x}$ the fluxion of the said length, whose fluent is $\frac{A}{aa} \times$ excess of the arch of a circle (rad. being unity) above its sine x; which when $x = \sqrt{\frac{3}{4}}$, will be the length required.

COROLLARY 1. The velocity of the ball along the plane, at its becoming inclined in any given angle, is directly as the time it has been in motion, or reciprocally as the angular velocity of the plane.

COROLLARY 2. The length descended by the ball along the plane, at its becoming inclined in any given angle, will be directly as the square of the time it has been in motion, or reciprocally as the square of the angular velocity of the plane.

COROLLARY 3. The ball will quit the plane when its angle of inclination becomes equal to 60°, let the uniform angular velocity and the force of gravity be what they will.*

XIV. QUESTION 308 *answered by Mr.* James Terey.

All the numbers whose squares equal 326, are easily determined:

18	1	1
17	6	1
15	10	1
14	11	3
14	9	7
13	11	6

Putting x, y, and z for the number of geese, ducks, and chicken, and v the pence paid for the chicken; then, among some of those 3 numbers (varied for geese, ducks, and chicken) $4vx + 2vy + vz = 400$; whence $v = \frac{400}{4x + 2y + z}$ a whole number; which only admits

* Mr. *Landen* remarks that this question is not right, but that a true solution may be made out from *Simpson*'s Fluxions.——He farther observes that his solution to the prize question Diary 1747 is not entirely right. The fluent for the time of descending along the mountain's side would be infinite if properly corrected. The stone should be laid at some distance from the top of the mountain, otherwise it will not move.

mits of $\left\{\begin{array}{l}\text{13 geese}\\ \text{11 ducks}\\ \text{6 chicken}\end{array}\right\}$ price of each $\left\{\begin{array}{c}\text{1s. 8d.}\\ \text{10}\\ \text{5}\end{array}\right\}$. And $\left.\begin{array}{l}\text{9 geese}\\ \text{7 ducks}\\ \text{14 chicken}\end{array}\right\}$ price of each $\left\{\begin{array}{c}\text{2s. 1d.}\\ 1\ \ 0\frac{1}{2}\\ 6\frac{1}{4}\end{array}\right\}$. Amounting each way to 1l. 13s. 4d.

Mr. *J. Turner*, Mr. *Collingridge*, Mr. *Sam. Atkin*, Mr. *Flitcon*, and others, solved the same; but only Mr. *Terey* and Mr. *Hampson* in the variety.

XV. QUESTION 309 *answered by the Proposer*.

Raised to a power, &c. should have been expressed *root extracted, &c.* Then, if v and y represent the respective chances of each army for victory, $\overline{v+y}|^{\frac{1}{v-y}} = v$, and $\overline{v-y}|^{\frac{1}{1+z}} = y$; whence $v+y = v^{v-y}$, and $v-y = y^{v+y}$; where $v = \cdot5806$ and $y = \cdot2590$ very correctly, as may be proved by a table of logarithms; which numbers were not a little curious to determine. Hence the odds of battle are as 2·2417 to 1.

As most persons of science are but little conversant with these sort of equations, it may not be improper here to unfold the mystery of raising powers of all sorts. And 1. A decimal raised to a decimal power produces a greater value than the root. 2. A decimal raised to an integral power produces a less value than the root. 3. An integer raised to a decimal power produces a less value than the root.

$x^0 = 0^0 = 1$; all very small powers of quantities approaching the value of unity. x^x is least $= \cdot6922$ correctly, when $x = \cdot3678798$ &c. $= \frac{1}{2\cdot30258509\text{ \&c.}}$. The logarithm of a decimal is negative, or so much less than nothing.

To determine the value of the unknown quantities in all exponential equations, it is convenient to suppose the least quantity unknown $= 0$, and thence to find the value or values of the next greater, noting the error in the next equations. Again, suppose the value of the least unknown quantity $= 1$, and thence find the value or values of the next greater; noting the quality of the error, in the next equations, as before; and so on to 10, 20, &c. for the least value, if need

need requires. Or supposing the value of the greatest unknown quantity to be 1, 10, 100, &c. determining the value of the next less unknown quantity to each supposition; at the same time always denoting the quality of errors, by which the true values of the unknown quantities are determined by a table of logarithms very exactly.

N. B. It is often very easy and convenient to suppose the value of one unknown quantity in two equations, in such a manner as that by it the other may be determined in a whole number. *Hurlothundro.*

XVI. QUESTION 310 *answered by* Nobody.

Mr. *Farrer* sent us a solution which was not true. Mr. *Landen* sent us a true method; but the calculus being so operose, it was not wrought out. And no method appearing to us yet elegant enough for a place, it will be next year before we shall have time to catch the solution to this famous spider and fly question. The ratio of the motion of the insects are little different from an equality; though a certain gentleman makes the motion of the chasing insect the slowest, to overtake the fly.*

PRIZE QUESTION *answered by Mr.* Heath *the Proposer.*

Anno 1748, June 18d. 10h. 17m. and Dec. 18d. 1h. 22m. by authentic tables the earth was in the aphelion and perihelion points of her orbit, or at the greatest and least dist. à ☉; her motion (by describing equal areas in equal times round him) being at those times of aphelion and perihelion, (later by 9 hours than according to Street's Tables) the slowest and quickest respectively.

By the equation table of the earth's orbit, her true diurnal motion round ☉ at coming to aphelion is 57′ 12″ (earth retarding variously from aph. to perih.) and true diurnal motion coming to perihelion 61′ 10″ (earth accelerating variously from perih. to aph.) Now allowing 10″ one-half ☉'s mean parallax, or angle at ☉, subtended by earth's semi-diameter, then by trigon. 19644 earth's semi-diameters is her mean dist. à ☉; being then at the conjugate end of her orbit; which dist. à ☉ = length of the semi-transverse.

Mr. Flamsteed's eccentricity of the earth's orbit, or ☉'s focal dist. à center of her orbit is 1692 such parts as semi-

* A solution to this question may be seen in the *Mathematician.*

transverse or mean dist. is 100000, consequently 100000 : 1692 : : 19644 : 332·37 earth's semi-diameters, the ☉'s true dist. à center earth's orbit: whence 19976·37 and 19311·63 semi diameters are the earth's greatest and least dist. à ☉. And by her describing equal elliptical sectors round him each day, with the respective angles 57′ 12″ and 61′ 10″, as before observed, the correspondent elliptical arches, which may be considered as circular for a day, will be 332·28 and 343·6 &c. semi-diameters which the earth's center goes over in 24 hours, when her motion is slowest and fastest. Whence, by allowing the earth's semi-diam. = 3967 miles, her center goes over at fastest 56794 miles per hour, 946 per minute, and almost 16 miles per second! an amazing swiftness! Also at her slowest rate, in aphelion, 54923 miles per hour, 915 per minute, and 15 per second.

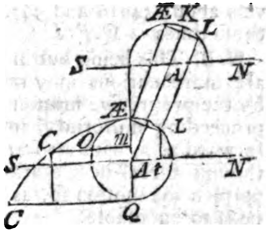
Anno 1748, June 9d. 16h. 34m. 46s. and Dec. 9d. 21h. 8m. equal time, ☉ enters Cancer and Capricorn, the respective mean anomalies being then 11 S. 21° 23′ 15″ and 5 S. 21° 56′ 28″ (by Halley's and Flamsteed's numbers); whence the proportional distances of earth à ☉ are 101675 and 98322 respectively, and thence the true distances 19973 and 19314 earth's semi-diameters (by multiplying 19644, a constant multiplier, and cutting off 5 fig.); and the earth's diurnal angular motion round ☉ being 57′ 11″ and 61′ 9″ respectively (by tables of the earth's orbit) to the said radii, consequently the arches respectively moved over by the earth's center, when the days are longest and shortest, are 332·230 and 343·554 &c. semi diameters of the earth; nearly equal to her slowest and fastest motions as above; she being at those times but a few days from the aphelion and perihelion points.

N. B. Anno 1748, March 19d. 2h. 41m. and Sept. 17d. 17h. 51m. the mean anomalies of the earth are respectively 9 and 3 degrees when her mean and true places differ the most, viz. 1° 56′ 20″; about which times she neither accelerates nor retards for some days.

To find the distance travelled over by a spot on the earth's surface for a day.

The earth's radius bearing so small a proportion to her dist. à ☉, she may be considered as revolving forward in the direction of a streight line, for a small interval of time, with a progressive motion as p, to 1 rotatory, or over 332·23 and 343·554 semi-diameters respectively, on the days about the solstices of Cancer and Capricorn; to each of which distances gone forward she makes but one revolution.

Put



Put $a = ÆQ$, the diameter of a circle generating a cycloidal curve, with progressive motion p, and rotatory 1; $x = Æm$; then $om = \sqrt{ax - xx}$ per circle. And fluxion arch $Æo = \frac{a\dot{x}}{2\sqrt{ax - xx}}$; whence $1 : p :: \frac{a\dot{x}}{2\sqrt{ax - xx}} : \frac{pa\dot{x}}{2\sqrt{ax - xx}}$ = flux. Co; and flux. $om = \frac{a - 2x}{2\sqrt{ax - xx}} \times \dot{x}$, whose sum $\frac{ap + a - 2x}{2\sqrt{ax - xx}} \times \dot{x}$ = fluxion Cm. But $\sqrt{\overline{\text{flux. } Cm}|^2 + \overline{\text{flux. } Æm}|^2}$ = fluxion arch $CÆ = \frac{\dot{x}}{4x} \times \frac{\sqrt{\overline{p+1}|^2 \times a^2 - 4pax}}{a - x}$; whose fluent is $\sqrt{ax} \times : p + 1 + \frac{\overline{p+1}|^2 - 4p}{2 \cdot 3 \cdot \overline{p+1}} \times \frac{x}{a} + \frac{\overline{p+1}|^2 - 4p}{2 \cdot 5 \cdot \overline{p+1}} - \frac{\overline{\overline{p+1}|^2 - 4p}|^2}{2 \cdot 4 \cdot 5 \cdot \overline{p+1}|^3} \times \frac{x^2}{a^2} + \frac{\overline{p+1}|^2 - 4p}{2 \cdot 7 \cdot \overline{p+1}} - \frac{\overline{\overline{p+1}|^2 - 4p}|^2}{2 \cdot 2 \cdot 7 \cdot \overline{p+1}|^3} + \frac{\overline{p+1}|^2 - 4p}{2 \cdot 2 \cdot 4 \cdot 7 \cdot \overline{p+1}|^5} \times \frac{x^3}{a^3}$ &c.

The poles of the earth S, N, moving nearly parallel, for a short time, the earth's center A is carried with an oblique direction AK, in an angle of 23° 30′ with the poles of the equator $ÆQ$; and the city of London, on the surface, at L, in the parallel rotatory direction to $ÆQ$, on the surface; the distance betwixt the finishing of each revolution being $LL = AA = ÆÆ = kK$; each town on the earth's surface describing cycloidal curves whose bases are equal. Lt the dist. of the city of London from the earth's axis = ·622059 semi-diam. being nat. s. co-lat. London 88° 28′ (vid. curve of the tack, D. 1747). Now, when $x = 2$, $Lt = 1·244118$ earth's semi-diam. $= a$, $p = 85$, and $p = 87·9$ fere (dividing 332·230 and 343·554 by 3·90852 = lesser circle's circumference whose diam. $= a = 2Lt$) the semi-cycloids = 166·12 and 171·78 fere; whence 332·24 and 343·56 earth's semi-diam. = distances travelled by the city of London in 24 hours, when the days are longest and shortest: consequently it tra-

 vels

vels about 54916 and 56787 miles per hour, respectively, at those times. *Q. E. I.*

N. B. This being but in consequence of theory, those who are more curious may rectify the cycloidal curve described by the progressive motion of a point on a revolving globe, proceeding uniformly, in a circular or elliptical direction, forward as p, with a rotatory motion as 1, at the same time: though a real acceleration or retardation in either, would perplex the motion so, as to render the solution of the track next to impossible.

The Prize of 10 *Diaries was won by Mr.* John Turner.

The Eclipses calculated for 1749, *by* Mr. John Smith.

There will happen five eclipses, three of the sun, and two of the moon; but only one of each luminary will be visible to the inhabitants of Great Britain.

1. The sun eclipsed on saturday January 7th, past 7 at night, but invisible, because of the sun being set.

2. The moon eclipsed on monday June 19th, at 9h. 2m. in the morning.

3. The sun eclipsed on monday the 3d of July, at 3 h. afternoon.

4. The moon eclipsed on tuesday the 12th of December. Beginning 6h. 48m. Middle 7h. 59m. End 10h. 22m. Duration 2h. 23m. Digits eclipsed 5° 3'.

Calculated by		Beg. h.m.	Mid. h.m.	End h. m.	Dur. h.m.	Dig. ° '
Mr. T. Cowper,	Wellingborough	6 55	8 4	9 13		4 42
Mr. J. Brown	London	6 58	8 8	9 18	2 20	
	Witton-le-wear	6 52	8 2	9 12		4 45
Mr. W. Caile	Great Musgrave	6 51	8 1	9 11	2 20	4 35
	Etherly	6 52	8 2	9 12		
Mr. A. Man	Rome	7 48	9 0	10 12		
	Lisbon	6 19	7 31	8 43	2 24	5 6
	Paris	7 6	8 18	9 30		5 6
Mr. Hawkins	London	7 0	8 7	9 14	2 14	4 36
	Hanover	7 40	8 47	9 52		
	Oxford	6 55	8 2	9 9		
	Rome	7 52	8 59	10 46		
Mr. J. Hampson,	Leigh, Lancash.	6 39	7 50	9 1	2 22	4 50
Mr. Jos. Walker,	Kettering	6 52	8 1	9 10	2 18	4 42
Mr. Farrer,	Sunderland	6 41	7 53	9 4	2 23	5 10

5. The

5. The sun eclipsed on thursday Dec. 28th, 9h. 11 m. morning. Beginning 8 h. 4 m. Middle 9 h. 11 m. End 10 h. 22 m. Duration 2 h. 18 m. Digits eclipsed 7° 8′.

Calculated by		Beg. h.	m.	Mid. h.	m.	End h.	m.	Dur. h.	m.	Dig. °	′
Mr. T. Cowper,	Wellingborough	8	12	9	8	10	18	2	16	6	49
Mr. J. Brown,	Witton-le-wear	8	4	9	10	10	16	2	12	7	9
Mr. W. Caile,	Great Musgrave	8	3	9	9	10	15				
	Etherly	8	4	9	10	10	16	2	12	7	9
Mr. A. Man,	Rome			10	9	11	29	2	20	7	39
	Lisbon	8	5								
Mr. A. Hawkins,	London	8	5	9	12	10	22	2	17	7	9
Mr. J. Hampson,	Leigh, Lancash.	8	1	9	6	10	16	2	5	7	1
Mr. Jos. Walker,	Kettering	8	2	9	8	10	18	2	16	6	47
Mr. Farrer,	Sunderland	8	1	9	5	10	7	2	6	6	25

Mr. *Abraham Clark* sent us the first three eclipses right.

We cannot but be pleased with the ingenuity and pains of Mr. *Aaron Hawkins*, who has calculated the appearances of the solar eclipses on Jan. 7th, in New Spain, Terra Firma, Jamaica, Cuba, and places adjacent; all the penumbra falling within the earth's disk. And this gentleman has given the times of the appearances with respect to the meridian of London. He has also shewn that the solar eclipse on July 3d, 27′ P. M. invisible in England, will be seen in the more southern parts of the globe, where all the penumbra will fall within the earth's disk, and be central and annular. And at long.

The Eclipse of the moon on the 12th of December was observed (as below) at Mr. *Grahm's* in *Fleetstreet*, by *John Bevis*, M. D. and Mr. *James Short*, F. R. S.

App. time			
6 h.	46 m.	36 s.	A sensible penumbra
6	50	56	Eclipse began
9	12	38	Eclipse ended
9	17	3	Penumbra gone.

The solar Eclipse of the 28th of December was thus observed:

	Beginning			End			
At *Rome*, by Mr. *Chr. Maire*	8 h.	34 m.	35 s.	11 h.	11 m.	32 s.	
At the Observatory at *Berlin*, by M. *Grischow*, jun. and M. *Kies*	8	59	19½	11	20	5¼	tr. t.
At *Berlin*, near the Observatory, by M. *Euler*	8	58	30	11	19	50	tr. t.

the middle of the eclipse the sun will be vertical at St. Anthony's river, on the western coast of Barbary, lat. 21° 42′ north; will be seen very great at guinea; and set at St. Helena, and places adjacent. Also the solar eclipse on Dec. 28, visible, will appear eclipsed at his rising in 14° 3′ S. lat. and long. 23° 9′ west, at Ethiopia. Centrally eclipsed as he rises lat. 32° 10′ S. long. 50° 49″ west, at St Domingo. Centrally eclipsed in nonagesimal lat. 26° N. long. 14° 53′ E. at Barbary. Centrally eclipsed in the meridian lat. 28° 4′ N. long. 32° E. at the Red Sea. Sets centrally eclipsed lat. 54° 37′ S. long. 144° 25′ E. at the South Sea. Eclipse ends at sun-setting lat. 38° 10′ S. long. 109° 23′ E. at Holland Nova.

Our worthy correspondent Mr. *J. Turner*, of Heath, near Wakefield, writes as follows: 'On monday, April 18th at 'midnight, as I was looking towards the north part of the 'heavens, I accidentally cast my eye upon a comet, near 'the chain of Andromeda. Its splendour is not very great at 'present, yet the tail is perfectly distinct, stretching towards 'Lyra. The motion of it is very swift, amounting to near '4 degrees of a great circle in a day, and tending almost 'towards the north pole. It comes to the north part of the 'meridian about 9 at night, being then about 10° high. On 'thursday the 21st, half an hour past 10 at night, the comet 'was in a right line with ε and β in Cassiopeia, and with the 'pole star and γ, in Cepheus's foot, or rather the line passed 'between γ and τ. Also with the bright star of the swan's 'tail, and Cassiopeia's head; by which its place may be 'exactly determined. Its ascending node is in about 25° of 'Pisces: and the inclination of its orbit to the ecliptic about '52° 0′.' This comet we also had the pleasure to observe; and should be glad to discover the certain paths and periods of comets; the substances, quantities, and qualities of which they are composed; as well as their proper designed uses, and laws of continuation and support: And the like of all the other celestials wandering in infinite space.

Mr. *T. Cowper*, of Wellingborough, has communicated the following occultation of the scorpion's heart, by the moon, 1749.

		d.	h.	m.	s.	
Apparent time at Wellingborough	Immersion, March	27	12	50	9	P.M.
	Middle of Occulation	28	1	24	32	Mane
	Visible conjunction	—	1	25	10	
	Emersion — — —	—	1	59	4	
	Duration — — —	—	1	9	31	

New Questions.

I. QUESTION 311, *by Mr.* Landen, *near* Peterborough.

To find three such numbers, that the sum or difference of any two of them shall be a square number?

II. QUESTION 312, *by the Rev. Mr.* Baker, *at* Stickney, Lincolnshire.

A bowl, by its byas, describes a spiral, expressed by z, whose equation is $\frac{3}{5}y^{\frac{5}{3}} - \frac{3}{2}z^{\frac{3}{2}} + y^{\frac{2}{3}}z^{\frac{3}{5}} = 0$. In what direction must the bowl set out to fall upon the jack, when the length of the cast is 47 yards?

III. QUESTION 313, *by Mr.* Landen.

There are four remarkable high trees growing in a strait hedge-row, the distance of the 1st and 2d is 60 yards, of the 2d and 3d 40 yards, and of the 3d and 4th 20 yards. Where must I stand to observe them, so that the three intervals may appear equal?

IV. QUESTION 314, *by the Rev. Mr.* Baker.

A hare sets out 50 yards before a grey hound, at the rate of 31 yards per second, and continues a strait course in the subquadruplicate inverse ratio of the time taken up in running: The dog sets forward only at the rate of 26 yards per second, and maintained his pace in the subquintuplicate inverse ratio of his time spent in running: How far had the dog run when his speed was equal to that of the hare's? Also when he was again as near to the hare as at first? And lastly, when he killed her?

V. QUESTION 315, *by Mr.* Philip Stevens, *of* Bristol.

If the diurnal rotation of the earth was stopt from the 10th of December, 1748, at midnight, to the 10th of December, 1749, what time of the year would it be day-break, sun-rise, and mid-day, at London?

VI. QUESTION 316, *by Mr.* John Hampson, *of* Leigh, Lancashire.

At Bedford mill, near old Leigh town, is found,
In form triangular, a piece of ground,
Whose sides and area none can yet explain,
Tho' these subsequent hints may them obtain.

One

One angle makes degrees just seventy-nine,
Which being, as three to ten, cut by a line,
Of chains eleven, drawn to its side oppos'd,
The area is the least can be inclos'd.
The miller thus—' Who best explains the truth,
' Wins for reward our buxom daughter Ruth.'

VII. QUESTION 317, by Mr. James Collingridge.

A marble table of six equal hexagon sides, each 1¼ feet length, and the table 1½ inch thickness, is levelly suspended by a point on the under surface, and there hangs 7, 11, 15, 19, 23, and 27 pound weights, in a successive order, from each corner of the table. Quere the point of suspension?

VIII. QUESTION 318, by Mr. James Terey.

Two roads, perpendicular to each other, issue from the extremity of a semi-elliptical enclosure, next to a gentleman's seat; one road runs along the transverse close by the side of the strait paling, some distance beyond the other end of the enclosure, and the other road proceeds forward next the crooked paling: A strait diagonal visto of 20 yards breadth is to be made, so as to communicate between the two roads, with one of its fences touching the curved fence of the elliptic enclosure; how must the visto fences be drawn, and of what dimensions, so that the visto may take up the least quantity of ground possible; the transverse axis of the elliptic enclosure being 100, and its semi-conjugate 40 poles?

IX. QUESTION 319, by Harmonicus.

There are 13 musical cords of equal thickness and tension, each an inch longer than the other, from the shortest of 12, to the longest of 24 inches; the tone of the longest cord to the shortest is as 2 to 1. Quere the proportion of the tones of all the intermediate cords?

X. QUESTION 320, by Mr. Heath.

At what time (next ensuing) will Mars and Venus, Sol and Terra, be conjunctly in a right line?

XI. QUESTION 321, by Mr. Farrer.

A musket ball being shot 3½ furlongs perpendicularly upwards, at what distance in its return will the force of the ball be equal to the weight of 9 pounds? And what will be its force at coming to the earth's surface, supposing the ball to weigh an ounce when at rest?

XII. QUESTION 322, *by Mr.* Landen.

A cylindric pillar of ſtone, of 2 yards in circumference, being drawn up 20 yards above ground at a building, a rope being at the ſame time wrapt ten times round its convexity, with its lower end fixed to a hole upon the middle part where it begun to wrap; but before the pillar could be lodged upon the ſcaffolding, as it was drawn up with the rope's upper part paſſing through pulleys above, the rope unfixed its ſecurity at the tenth round, and the ſuſpended pillar by its weight then unwrapped itſelf of the rope, and deſcended to the ground. How long was the time of its deſcent?

XIII. QUESTION 323, *by Mr.* John Corbet, *Surveyor.*

How many acres of the moon's ſurface are ſeen enlightened 10 days after her conjunction with the ſun? And how many acres are contained on the convex ſuperficies of a lunar mountain (part of a gentleman's eſtate) its height being 3 furlongs, and its ſuperficies equal to that by the rotation of the ſemi-cycloid of that height about its axis?

PRIZE QUESTION, *by the Excellent Mr.* J. Landen.

An eagle (200 yards above) ſtooped to a kite, then taking flight from the ground with a chicken, at an angle of 60°, the eagle ſoaring directly towards the kite, then flying from her, at an uniform rate of ſwiftneſs of 3 to 1, the ratio of both their uniform motions; when, after ſome time flying, the kite finding herſelf cloſely purſued, quits her little captive, which fell to the ground at the inſtant the bird of Jove ſeized her priſoner, who was then juſt as high from the ground as at her firſt ſtooping. The eagle's diſtance from the kite, at firſt ſetting out? Her neareſt approach to the ground during the purſuit? Her height above it, and diſtance from the kite, when the chicken begun to fall? And alſo the time of flight, are from hence required?

A PARADOX, *by Mr.* Landen.

Two ivory balls of five inches diameter, each being placed at the diſtance of two inches from one another, and both ſtruck by another ivory ball of the ſame ſize, in a perpendicular direction to their line of diſtance, with any given velocity, they will move ſwifter after the ſtroke, than if they had been placed cloſe together, or at any other diſtance.

Queſtions

1750.

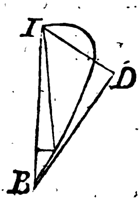
Questions answered.

I. QUESTION 311 *answered by Mr.* C. Bumpkin.

BY a method of substitution, too tedious to insert, he finds 1873432, 2399057, 2288168, the three numbers, answering the conditions of the question.

II. QUEST. 312 *answered by Mr.* W. Jepson *of* Lincoln.

In the equation $\frac{2}{3}y^{\frac{5}{3}} - \frac{3}{2}z^{\frac{3}{2}} + y^{\frac{4}{3}}z^{\frac{3}{5}} = 0$, if we write v^5 and v^2 for $z^{\frac{3}{2}}$ and $z^{\frac{3}{5}}$, it will become $v^5 - 8{\cdot}68245v^2 = 272{\cdot}05$ when $y = 47$; whence, by converging series, $v = 3{\cdot}252325$, and thence $z = 50{\cdot}9695$.



To the tangent BD, which is the line of direction, draw the perpendicular ID. Then,

$\dot{z} : \dot{y} :: y : \frac{y\dot{y}}{\dot{z}} = BD$. And the above equation in fluxions is $\frac{10}{9}y^{\frac{2}{3}}\dot{y} - \frac{9}{4}z^{\frac{1}{2}}\dot{z} + \frac{2}{3}z^{\frac{3}{5}}y^{-\frac{1}{3}}\dot{y} + \frac{3}{5}y^{\frac{2}{3}}z^{-\frac{2}{5}}\dot{z} = 0$, which multiplied by $y^{\frac{1}{3}}z^{\frac{2}{5}}$ gives $\dot{y} = \frac{\frac{9}{4}y^{\frac{1}{3}}z^{\frac{9}{10}} - \frac{3}{5}y}{\frac{10}{9}yz^{\frac{2}{5}} + \frac{2}{3}z} \times \dot{z}$, whence $BD = \frac{\frac{9}{4}y^{\frac{1}{3}}z^{\frac{9}{10}} - \frac{3}{5}y}{\frac{10}{9}yz^{\frac{2}{5}} + \frac{2}{3}z} \times y$. Now $BI\ (=y) : 1 :: BD : \frac{\frac{9}{4}y^{\frac{1}{3}}z^{\frac{9}{10}} - \frac{3}{5}y}{\frac{10}{9}yz^{\frac{2}{5}} + \frac{2}{3}z} = {\cdot}379268$ &c. the cosine of 28° 27′ (nearly) the angle of direction required.

The same answered by Mr. J. Powle.

It is evident that the line of direction will be a tangent to the curve where the bowl is delivered, the position whereof, with

with a line drawn from thence to the jack is what is required? The equation of the curve in fluxions is,

$\frac{10}{9}\dot{y}y^{\frac{2}{3}} - \frac{9}{4}\dot{z}z^{\frac{1}{2}} + \frac{2}{3}z^{\frac{3}{5}}y^{-\frac{1}{3}}\dot{y} + \frac{3}{5}y^{\frac{2}{3}}z^{-\frac{2}{5}}\dot{z} = 0$, therefore $\frac{\dot{y}}{\dot{z}} = \frac{\frac{9}{4}z^{\frac{1}{2}} - \frac{3}{5}z^{-\frac{2}{5}}y^{\frac{2}{3}}}{\frac{10}{9}y^{\frac{2}{3}} + \frac{2}{3}z^{\frac{3}{5}}y^{-\frac{1}{3}}}$, consequently $\frac{y\dot{y}}{\dot{z}}$ the subtangent

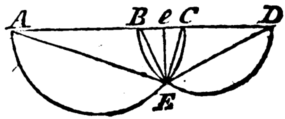
$$BD = \frac{\frac{9}{4}z^{\frac{1}{2}} - \frac{3}{5}z^{-\frac{2}{5}}y^{\frac{2}{3}}}{\frac{10}{9}y^{-\frac{1}{3}} + \frac{2}{3}z^{\frac{3}{5}}y^{-\frac{4}{3}}}.$$

But y being given, z is known from the equation of the curve. Therefore $BD = 41{\cdot}47$ nearly. Then, by trigonometry, as $BI : BD :: \text{rad.} :$ cosine angle of direction, $IBD = 28^\circ\ 8'$ nearly.*

These are the only solutions that we received to this curious question, except one from the proposer.

III. QUEST. 313 *answered by Mr.* Terey *of* Portsmouth.

Let A, B, C, D, represent the four trees, and E the station sought of the observer.



$AB = 60$ yards $= a$, $BC = 20 = b$, $CD = 40 = c$. *Geometrically.* In this particular case, draw two semicircles AEC and BED, on the diameters AC and BD, and the point of intersection E will be the observer's station: For then $AB : BC :: AD : CD$; agreeing with the data; and $AB : BC :: AE : EC$; also $CD : BC :: ED : EB$, 3 Euc. 6. (vid. Universal Arith.) Letting fall the perpendicular $Ee = y$, and putting $x = eC$, per property of the circle $\frac{2cb}{c-b}x - xx = \frac{2ab}{a-b} \times \overline{b-x} - \overline{b-x}^{2}$, whence

$x =$

* *Mr.* LANDEN (in the Diary for the year following) says, that Mr. *Jepson* and Mr. *Powle* have both absurdly considered the bowl running backwards to the point where the spiral begins, and calls their solutions erroneous; though the proposer, Mr. *Baker*, meant that it should do so, and solved the question himself the same way. Mr. *Landen* thinks it too easy a question, in the case of drawing a tangent to a curve, whose equation is given; and therefore, correcting the fault, proposes it should be solved by drawing a tangent to the spiral, at the point where it begins, which will make it the more hard: and says it is a case not taken notice of by authors. He has gone through part of the solution this way himself.

$x = \frac{1}{2}b \pm \frac{c-a}{2ac-bb} \times bb = 12$ or 8 yards; whence $y =$ 24 yards, and each of the angles of interval 45°.

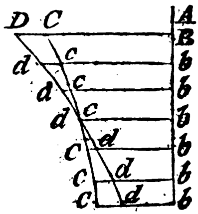
G. Bumpkin's solution to this question was of the like nature.

General Solution by Mr. Ch. Smith.

Let E be the place of the observer; A, B, C, D, the places of the trees; put $y =$ the perpendicular Ee, $a = 20 = BC$, $x = eC$, then $4a - x = Ae$, $a - x = Be$, and $2a + x = eD$; whence the tangents of the angles AEB, BEC, and CED are $\frac{3ay}{yy+4aa-5ax+xx}$, $\frac{ay}{yy-ax+xx}$, and $\frac{2ay}{yy+2ax+xx}$, which must be all equal by the question. Whence $yy = 4ax - xx = 2aa - ax - xx$, $x = \frac{2}{5}a = Ce = 8$ yards, $Ee = 24$, and each angle $= 45°$. *Q.E.F.*

Mr. *John Turner* solved this question in an elegant and general manner; so did Mr. *Garrard*, the Rev. Mr. *Baker*, Mr. *William Spicer*, Mr. *Enefer*, and some others.

IV. Question 314 *answered by the Rev. Mr.* Baker, *proposer.*

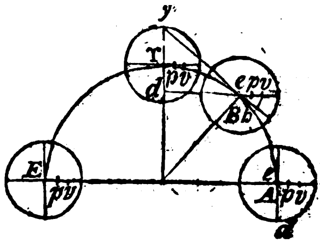


Let Bb, bb &c. represent equal portions of time, indefinitely small; BC, bc &c. the celerities of the dog; BD, bd &c. the celerities of the hare, at each of those times. Then, by mechanics, the areas $BCcb$ and $BDdb$, will denote the respective distances run by the dog and hare, in any given time. Putting $d = 50$, $AB = 1$, $BC = 26$ yards $= a$, $BD = 31 = b$, $Bb = x$, $bc = y$, and $bd = v$, we have, by the question, $\overline{1+x}|^{\frac{1}{5}} : 1^{\frac{1}{5}} :: a : bc = y = \frac{a}{\overline{1+x}|^{\frac{1}{5}}}$, ∵ $\dot{x}y = a\dot{x} \times \overline{1+x}|^{-\frac{1}{5}} =$ fluxion of the space $BCcb$, whose corrected fluent is $\frac{5a}{4} \times \overline{1+x}|^{\frac{4}{5}} - \frac{5a}{4}$ = dist. run by the greyhound in the time x. After the same manner we get $v = \frac{b}{\overline{1+x}|^{\frac{1}{4}}}$, and $\frac{4b}{3} \times \overline{1+x}|^{\frac{3}{4}} - \frac{4b}{3} =$ the space $BDdb$, run by the hare in the same time. Hence, by by making $\frac{a}{\overline{1+x}|^{\frac{1}{5}}} = \frac{b}{\overline{1+x}|^{\frac{1}{4}}}$, we have $x = \frac{b}{a}\Big]^{20} - 1 =$ 32·7107 seconds, and 509·623 yards, the diſtance run by the dog, when his pace equalled the hare's. Again, making $\frac{5a}{4} \times \overline{1+x}|^{\frac{4}{5}} - \frac{5a}{4} = \frac{4b}{3} \times \overline{1+x}|^{\frac{3}{4}} - \frac{4b}{3}$, we have $x =$ 106·8167 ſeconds, and 1341·577 yards run by the greyhound, when he had regained his loſt ground. Laſtly, making $\frac{5a}{4} \times \overline{1+x}|^{\frac{4}{5}} - \frac{5a}{4} = \frac{4b}{3} \times \overline{1+x}|^{\frac{3}{4}} - \frac{4b}{3} + d$, we have $x =$ 181·3635 ſeconds, and conſequently 2059·846 yards = diſtance run by the dog when the hare died.

We have received no other anſwer to this queſtion, nor was the application of the inverſe ratio, as expreſſed, clearly underſtood by any. One of the greateſt mathematicians of the age, and a fluxioniſt, has aſſerted it unintelligible, as he does of the exhalation queſtion, this year inſerted.*

V. QUESTION 315 *anſwered by Mr.* W. Sutton.

Let e, e, ♈, E repreſent the pole of the earth's ecliptic, in the reſpective poſitions of the earth, when the required appearances happen, as the earth moves along in the plane of the ecliptic, from A to B, ♈, and E about ☉, the ſun's center. Let A be the place of the earth on Dec. 10, 1748, at midnight, p the pole of the equator, and v the vertex of London, in the obſcure hemiſphere of the diſk, at which time the vertex of London is diſtant 61° 57′ from the horizon of the diſk (noted b, e, d) = ſum of the co-latitude and ſun's greateſt declination.



2. Con-

---

* *Mr.* LANDEN corrects this queſtion (in the year following) by adding *ſpeed*, for rate of going forward; which ſpeed, at the end of the firſt ſecond, according to his Commentary, was 31 yards, &c. ſolving this queſtion throughout, except giving the numbers; and ſays, the propoſer's ſolution would have agreed exactly the ſame with his, had he brought the ſpaces paſſed over in the firſt ſecond

2. Conceive the earth with its axis, still keeping its parallelism, carried to *B*, where the distance of the vertex and horizon is 18° = *vh*, which sides *vh*, *ev*, and *eh*, constitute a right-angled spherical triangle, right-angled at *h*, in which is given *vh* = 18° 0′, and *ev* = 61° 57′, to find the ∠*veh*, = *yed* = *d*☉*e*, the cos. of the diff. of long. from Dec. 10, = 69° 31′, which added to the sun's longitude on Dec. 10, gives the longitude for the day 11 s. 10° 8′ when 'tis day-break first at London, according to the question, answering to Feb. 17.

3. 'Tis evident, that when the earth comes to the right angle in the ecliptic, *A*☉♈ or 90°, from its place Dec 10, at ♈, the vertex of London at *v* will first arrive in the horizon of the disk, where the sun will first appear to rise: Therefore the longitude answering that appearance 0 s. 0° 37′ answering with March 10.

4. When the sun comes to the opposite point of the ecliptic, at *E*, or 180° from its long. Dec. 10, the vertex of London will transit the meridian at *v*, whence the sun's longitude then is 3 s. 0° 37′, to which agrees June 11, 1749.

Mr. *J. Powle*, drawing a scheme, says, that since on Dec. 10. the sun enters Capricorn, and that sign being on the meridian at midnight, it is evident, on the earth's rotation being stopt, that when the sun is depressed below the horizon 18° in his progress through the ecliptic, day will break.

In Aries he will rise; in Cancer it will be mid-day, *i. e.* sun-rising and Mid-day are on the 10th of March and 10th of June respectively.

To find day-break. Say, sine sum complement lat. and declin. 61° 58′ : sine sun's depression 18° :: radius : sine sun's distance from vern. equinox 20° 29′, answering to the 20th of April, the time of day-break required.

These being the only answers received, we thought fit to insert both, that each gentleman may be convinced of the truth.*

VI. QUES-

---

second into consideration. He says, it will be as $1 : 31 :: \frac{1}{x^{\frac{1}{4}}} : \frac{31}{x^{\frac{1}{4}}}$ the hare's speed; and $1 : 26 :: \frac{1}{x^{\frac{1}{5}}} : \frac{26}{x^{\frac{1}{5}}}$ the dog's speed. And the speed into $\dot{x}$, the fluxion of the time, will be equal to the fluxion of the distance run, &c.

* *Mr.* LANDEN (in the year following) says, Mr. *Powle*, in his solution to this question, is right, by reckoning 20° 29′ from Aries into Pisces; but, by mistake, makes day-break to follow sun-rise; otherwise his solution had been like Mr. *Sutton*'s.

*** ☉ is wanting at the center of the last figure.

VI. QUESTION 316 *answered by Mr.* W. Jepson.

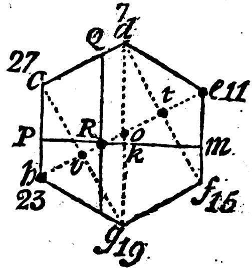
Let s. $\angle APC = s$, s. $\angle APB = n$, [*See the* $\triangle$ *p.* 77] s. $\angle BPC = m$, $BP = a$, $AP = x$, $DC = y$; then, by a well-known theorem, $sxy = nax + may$, $\because sxy - may = nax$, and $y = \frac{nax}{sx - ma}$; whence $\frac{snaxx}{sx - ma}$ or $\frac{xx}{sx - ma}$ is a minimum by the question; in fluxions $2x\dot{x} \times \overline{sx - ma} - s\dot{x}xx = 0$; $\because 2sx - 2ma - sx = 0$, or $sx = 2ma$, and $x = \frac{2ma}{s} = 19.558$ &c. Hence $y = \frac{2na}{s} = 7.012$ &c. and the area $= \frac{2mnaa}{s} = 6.731$ acres $= 6$ a. 2 r. 36.96 p.

COROLLARY. The area $APB =$ area $BPC = \frac{nmaa}{s}$, and $\because AB = BC$.

The Rev. Mr. *Baker*'s solution agrees with the above; as likewise does Mr. *John Turner*'s, Mr. *Charles Smith*'s, Mr. *Terey*'s, Mr. *Enser*'s, Mr. *Spicer*'s, Mr. *Gibbon*'s, Mr. *Tho. Hare*'s, and others, which we omit inserting to make room for other variety of subjects.

VII. QUESTION 317 *answered by Mr.* Turner, *of* Brumpton, Kent.

The solidity of the hexagon $= 631.33$ inches $= 26.5$ pounds $= w$; and let the weights be represented by 27, c; 7, d; 11 e; 15, f; 19, g; 23, h; whose sum, with w, $= 128\frac{1}{2}$ pounds $= s$: Let R be the center of gravity, draw $QR \parallel ch$, and $Rm \perp QR$; put $2p =$ side hexagon $= ch$, $Pk = a = 15.5884$, $PR = x$, $gk = y$. Then, by mechanics, $sx = \overline{d + g + w} \times a + \overline{e + f} \times 2a$, and $sy = \overline{h + f} \times p + w \times 2p + \overline{c + e} \times 3p + d \times 4p$. These equations, reduced, are



$x =$

$$x = \frac{d+g+w+2e\,2f}{s}a = 12\text{'}677 \text{ inches,}$$

$$y = \frac{h+f+2w+3c+3e+4d}{s}p = 16\text{'}319 \text{ inches.}$$

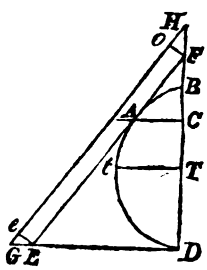
Whence the point R is determined by a general method, let the weights be what they will.

Anſwered by Mr. Heath.

With the given weights, the center of gravity R will fall on the axis eh at Ro diſt. from the table's center $= 3\text{'}31618$ inches (which would be $4\text{'}23529$, as anſwered by Mr. *Baker*, the weight of the table $26\frac{1}{2}$ lb. not being conſidered) for as the ſum of the weights ſuſpended on each ſide of the axis eh are equal, the center of gravity muſt needs fall on that line. The oppoſite weights 27 and 19 may be conſidered in one ſum $=46$ pounds, ſuſpended from the point v; and the weights 7 and $15 = 22$ ſuſpended from t, and placing the weight of the table $= 26\frac{1}{2}$, in the middle at the point o, the queſtion will be reduced to find the center of gravity to five weights 23, 46, $26\frac{1}{2}$, 22, and 11, placed at the equal diſt. of 9 inches on the line $he = 36$. Then ſubſtituting for the diſtance of the center of gravity from o, and multiplying the reſpective diſtances therefrom into the reſpective weights ſuſpended, making the products on each ſide the center of gravity, R, equal, the equation will give the value of oR as before.

VIII. QUESTION 318 *anſwered by the Rev. Mr.* Baker.

Seeing that on every variation of the viſto fence EF, the $\square\, eoFE$, increaſes or decreaſes, in magnitude, much faſter than the ſim. $\triangle$s GeE and oHF, therefore when the viſto $GHFE$ is a minimum, the fence EF will be alſo a minimum. Put $b = Tb = 50$, $c = Tt = 40$, $x = BC$; then $CA = \frac{c}{b}\sqrt{2ax - xx}$, $FC = \frac{2bx - xx}{b - x}$, $FD = \frac{2bb - bx}{b - x}$: Now by ſim. $\triangle$s $FC : CA :: FD : DE = \frac{c}{x}\sqrt{2bx - xx}$, and by 47 Euç. 1,



$\frac{cc}{x}$

$\frac{cc}{x} \times \overline{2b-x} + bb \times \frac{\overline{2b-x}|^2}{\overline{b-x}|^2} = EF^2$, a minimum; or, by reduction, $\frac{2cc}{bx} + \frac{\overline{2b-x}|^2}{\overline{b-x}|^2}$ is a minimum; whose fluxion made $= 0$, and reduced, gives $x^3 + \frac{3cc - 2aa}{aa - cc} axx - \frac{3aacc}{aa-cc} x = \frac{-a^3cc}{aa-cc}$, where $x = 16.785$ fere, whence $EF = 153.702$ required.

Mr. *John Turner* solved this question in the same elegant manner, as did Mr. *Terey* the proposer: And therefore a certain gentleman is out in his calculation, who undertook to demonstrate that this was not a question *de maximis & minimis*, as he may perceive his mistake by reviewing the breach of connexion in his fluxionary process.

IX. QUESTION 319 *answered by Mr.* Turner, *of* Brumpton, *near* Rochester.

The longest cord to the shortest being as 2 to 1, which is as 24 to 12, consequently the second cord will be as 24 to 13, the third as 24 to 14 or 12 to 7, the fourth as 24 to 15 or 8 to 5, the fifth as 24 to 16 or 3 to 2, the sixth as 24 to 17, the seventh as 24 to 18 or 4 to 3, the eighth as 24 to 19, the ninth as 24 to 20 or 6 to 5, the tenth as 24 to 21 or 8 to 7, the eleventh as 24 to 22 or 12 to 11, the 12th as 24 to 23, the thirteenth as 24 to 24 or 1 to 1. *Q. E. F.*

N. B. This question, intending to shew the nature of harmonical proportion, was mistakenly proposed.

X. QUESTION 320 *answered by Mr.* T. Cowper.

The last mean opposition of the Sun, Mars, and Venus (in the superior part of her orbit) was January 20, 1695, and I cannot find (by the proportion of the one conjunction of the sun and another with those planets) that they will be so conjoined again till the 11th of January 2942, though they happen very nearly in a right line about the 20th of October, anno 2006.

N. B. Those who are more curious may calculate the time of their being conjoined, according to the true motions, as we expect to see performed by Mr. Gael Morris, whose numbers

numbers given us from the Royal Observatory, correct the places in Dr. Halley's tables, lately published by Mr. Innys, by exactly 7 min. less of sun's anomaly, &c.

XI. QUESTION 321 *answered by Mr.* John Turner, *of* Heath, Yorkshire.

Putting $b = 2310$ feet in $3\frac{1}{2}$ furlongs, $c = 16\frac{1}{12}$ feet, then $2\sqrt{bc}$ = the celerity of the musquet ball when it falls to the ground, which being multiplied by its weight (viz. 1 ounce) produces 358 ounces, or 24 pounds, equal to its absolute force.

2. Let x = the time of its fall in seconds, when its absolute force = 9 pounds, or 144 ounces. Say, $1'' \times 1'' : c :: x \times x : cxx$ the space descended; whence $2cx$ = the celerity at that time, which multiplied by 1 ounce, is $144 = 2cx$; whence $x = \frac{144}{2c} = 4\frac{1}{2}$ seconds nearly, and the space descended, when the ball's force = 9 pounds, is 322·321 feet.

The Rev. Mr. Baker's *Solution.**

Putting $r = 21000000$ feet = the earth's rad. $d = 2310$ feet $= 3\frac{1}{2}$ furlongs, $b = 9$ pounds, $c = \frac{1}{16}$ of a pound, the ball's weight at rest, $s = 16\frac{1}{12}$ feet, x = space descended. The velocities of falling bodies being in the subduplicate ratio of the spaces descended through, we have $\sqrt{s} : 2s :: \sqrt{x} : 2\sqrt{sx}$ = the ball's velocity at x distance descended. By mechanics, and the condition of the question, $2c\sqrt{sx} = b$, $\because$ $x = \frac{bb}{4ccs} = 322{\cdot}321$ feet, exactly agreeing with the foregoing number by Mr. Turner. And when $x = d$, then $2c\sqrt{sd} = 24{\cdot}0937$ pounds, the ball's force coming to the earth's surface.

* *Mr.* LANDEN (in the year following) says, Mr. *Turner* and Mr. *Baker* are both wrong in their solutions to this question. This gentleman solves it, by assuming gratis, that 1100 grains, descending one-fourth of a foot, acquires a force = 4660 grains weight; and the rest upon the same principles with others, supposing the force to be as the velocity and quantity of matter; and so it may be solved by as many different suppositions as any one pleases. But *Newtoniensis* says there can be no proper solution to this question, for want of proper data, like the exhalation question.

face. But accurately thus. From p. 369 of Mr. Emerſon's Flux. we get $2r\sqrt{\frac{sx}{aa-ax}}$ = the velocity, whence $2cr\sqrt{\frac{sx}{aa-ax}}=b$, and $x=\frac{aabb}{4sccrr+abb}=322{\cdot}3873$ feet (where $a=d+r$). And when $x=d$, then $2c\sqrt{\frac{sdr}{a}}$ $=24{\cdot}0924$ pounds &c. *Q.E.F.*

Mr. *Powle* ſent his ſolution.

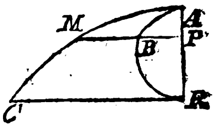
XII. QUESTION 322 *anſwered by Mr.* C. Bumpkin *only*.

If the ſtone were ſuſpended by two ropes hanging perpendicular, with one fixed at a point in the middle of its convex ſurface, and the other, at the center of percuſſion of that circular ſection in which the aforeſaid point is ſituated (which point in computing the place of the center of percuſſion is to be conſidered as the point of ſuſpenſion) and likewiſe the diam. in which the center of percuſſion is found, were parallel to the horizon, the tenſion of the rope fixed at the center of percuſſion would then be equal to the motive force bringing the ſtone down, if that rope was unfixed, and the ſtone left to deſcend in the manner deſcribed in the queſtion. Conſequently, putting W for the weight of the ſtone, a for the rad. thereof, then $\frac{3a}{2}$ will be the diſt. of the center of percuſſion from the center of ſuſpenſion; and $\frac{2W}{3}$ that motive force which would act continually upon the ſtone. Moreover, $W : 32{\cdot}2\ (=s) :: \frac{2W}{3} : \frac{2s}{3}$, the accelerating force, or velocity generated per ſecond, by the deſcending ſtone. Putting v for the velocity of the ſtone's center of gravity, and z the ſpace deſcended, then $\frac{\dot{z}}{v}$ being the fluxion of the time $\frac{2s}{3}\times\frac{\dot{z}}{v}$ will be $=\dot{v}$, or $\frac{2s}{3}\times\dot{z}=v\dot{v}$, whence $v=2\sqrt{\frac{sz}{3}}$, and applying $\dot{z}$, we have $\frac{\dot{z}}{2}\sqrt{\frac{3}{sz}}$ = fluxion of the time, whoſe fluent $\sqrt{\frac{3z}{s}}$, when $z=60$, is $=2''\ 21'''\ 51''''$. *Q.E.F.*

XIII. QUES-

XIII. QUESTION 323 *answered by Mr.* Heath.

Say 14d. 18h. 22′ 1½″ (the time of half a mean lunation) : ½ (the moon's surface at most seen enlightened) : : 10 days (the time after conjunction with the sun) : $\frac{10}{14\text{·}765299} =$ ·3386319 &c. parts of the moon's surface then seen enlightened = 3245609437 acres; the moon's whole surface being 9584476353 acres.



Put a = lunar mountain's height = AR = 3 furlongs, $x = AP$, any indefinite part of the cycloid's axis, then $BP = \sqrt{ax - xx}$, whose fluxion is $\frac{a - 2x}{2\sqrt{ax - xx}} \times \dot{x}$ and fluxion arch AB = flux. $MB = \frac{a\dot{x}}{2\sqrt{ax - xx}}$, whence flux. $\overline{MB + BP} =$ flux. $MP = \dot{y} = \dot{x}\sqrt{\frac{a-x}{x}}$, whose fluent $y = \sqrt{ax} \times :$ $2 - \frac{x}{3a} - \frac{x^2}{20a^2} - \frac{x^3}{56a^3} - \frac{5x^4}{576a^4}$ — &c. by series; put $c = 2 \times 3\text{·}1416$, then it being known that $\dot{x}\sqrt{\frac{a}{x}}$ is the flux. of the cycloid's arch, $cy\dot{x}\sqrt{\frac{a}{x}}$ will be the fluxion of a curved surface, by the rotation of the space AMP about AP, whose fluent found is $2ax - \frac{x^2}{6} - \frac{x^3}{60a} - \frac{x^4}{224a^2} - \frac{5x^5}{2880a^3}$ — &c. × c = the curved superficies; which when $x = a$, becomes $2aa - \frac{aa}{6} - \frac{aa}{60} - \frac{aa}{224} - \frac{5aa}{2880}$ — &c. × $6\text{·}2832 = aa \times : 2 - \frac{1}{6} - \frac{1}{60} - \frac{1}{224} - \frac{5}{2880}$ — &c. × 6·2832 $= aa \times 11\text{·}3617 = 1022\text{·}553$ acres, or 1022 a. 2 r. 8·48 p. the quantity of the lunar mountain's surface required.

Solution by Curiosus.

Let s = surface, $AP = x$, $PM = y$, $AM = z$, $AB = v$, $c = 2 \times 3{\cdot}1416$; then will $\dot{z} = \dot{x}\sqrt{\frac{a}{x}}$, and $\dot{s} = c \times PM \times \dot{x}\sqrt{\frac{a}{x}}$; but $PM = \sqrt{ax - xx} + v$, whence $\dot{s} = c\dot{x}\sqrt{aa - ax} + cv\dot{x}\sqrt{\frac{a}{x}}$. And $s = -\frac{2c\sqrt{a}}{3} \times \overline{a-x}|^{\frac{3}{2}}$ + flu. $cv\dot{x}\sqrt{\frac{a}{x}}$. But (by rule 8 p. 50 Emerson's Fluxions) this fluent is $= 2c\sqrt{ax} + 2ca\sqrt{aa - ax}$, because $\dot{v} = \frac{a\dot{x}}{\sqrt{aa - ax}}$. Whence $s = -\frac{2c\sqrt{a}}{3} \times \overline{a-x}|^{\frac{3}{2}} + 2cv\sqrt{ax} + 2ca\sqrt{aa - ax}$, and corrected, $s = \frac{4ca + 2cx}{3}\sqrt{aa - ax} + 2cv\sqrt{ax} - \frac{4caa}{3}$; and when $x = a$, the whole surface $= \frac{3c - 8}{6} caa = 11{\cdot}36169aa$.

C. Bumpkin's solution, agreeing with the above, came too late to be inserted; which are the only solutions that we received to this question, except the proposer's.

The PRIZE QUESTION *answered by his Excelleny Sir* Stately Stiff.

The equation of the curve described by the eagle, is $2x = \frac{a^n y^{1-n}}{1-n} - \frac{a^{-n} y^{n+1}}{1+n}$, computing x from the point where the pursuit ended, y being an ordinate at right angles, $n = \frac{1}{3}$, and a an invariable quantity to be determined. Moreover, if the variable distance of the eagle and kite be $= d$, and the distance of the kite from the point where x begins $= z$, and s and c be the sine and cosine of 60°, we have, by the nature of the curve, $\frac{z}{n} - nx = d$, $xx - 2zx + zz + yy = dd$; and at the beginning of flight, when $z = 230{\cdot}9$ yards, $cy = sx$; at which time, as it is proved from these three last equations, $d = 582{\cdot}3$ yards, the eagle's distance from the kite at first setting out, $x = 331{\cdot}2$, $y = 573{\cdot}6$. Now by putting these values of x and y in the first equation, a is determined $= 967{\cdot}1$.

At

At the lowest point of the curve, $s : y :: 1 : \frac{y}{s} = d$, and $s : y :: c : \frac{cy}{s} = x - z$; therefore $z = \frac{sx - cy}{s}$, by means of which, and the former equations, x and y are found corresponding to that point; and then, by a short computation, the eagle's nearest approach to the ground = 117·8. Her height above it when the chicken began to fall, is found = 119·7 (by means of the former equations, trigonometry, and solving a cubit equation) and the eagle's distance from the kite, at the same instant is found = 251·7 yards; and lastly, the whole time of flight = 9·44 seconds.

N. B. The data should be corrected by writing, *when after 5 seconds flying*, instead of, *when after some time flying.* The whole operation requires too much room to be inserted at length. *Q. E. F.**

We wonder at some persons for sending criticisms on the impropriety of this question, who did not understand one step of the process in giving a solution: but like the author of the sham doctrine of ultimators, and the Irish conjurer who raised the ghosts of departed quantities, prove to be mere cyphers of mathematicians, whatever they may be in their own element.

The Prize of 12 Diaries was won by the above Answerer, and that of 8 by Mr. Baker.

The PARADOX *answered by* C. Bumpkin *only.*

Put m = mass of each ball, a the velocity of the striking ball before the stroke, w its velocity after the stroke, v the velocity of the balls impelled, and c = cos. angle made by the path of the striking ball with that of either of the impelled balls; then $mw + 2cmv$ will be the quantity of motion after impulse $= ma$, the quantity of motion before the stroke given. Moreover, $maa = mww + 2mvv$, as is proved by Mr. Mac Laurin, in his Treatise of Fluxions, and also by others. By which equations (expunging w) we get $v = \frac{2ac}{1 + 2cc}$, and is a maximum when $c = \sqrt{\frac{1}{2}}$. From whence it appears that the balls must be laid about 2 inches asunder, for the velocity, after the stroke of a third ball, to be the greatest possible.

The

* The above solution will be evident by reading prob. 15 *Simp. Flux.* p. 516.

The Eclipses calculated for 1750, by Mr. John Smith.

There will happen five eclipses, three of the sun, and two of the moon, in the following order: The times and appearances according to the meridian of London.

1. Of the moon, on friday June 8th, at 3 minutes past 6 at night, visible and total.*

2. Of the sun, on friday June 22d, at 45 min. past 6 at night, invisible.

3. Of the sun, on sunday the 18th of November, 56 min. in the morning, invisible.

4. Of the moon, on sunday Dec. 2d, at 31 min. past 6 in the morning, visible and total.†

5. Of the sun, on monday the 17th of December, at 17 min. past 6 at night, invisible to any part of Europe.

The quantities of the visible eclipses are given by the above ingenious young artist as follows; which are very correct, as appears by the calculations sent us by others.

A

---

* The lunar Eclipse of the 8th of June was observed in *Surry Street* in the *Strand*, *London*, by Mr. *John Catlin*, and Mr. *James Short*, F. R. S.

Emersion, or end of total darkness	9 h.	45 m.	0 s.
End of the eclipse — —	10	51	30

† The total Eclipse of the moon on the 2d of December, was observed in the *Strand*, *London*, (about 5″ of time W. of St. Paul's, and 27″ W. of *Greenwich* Observatory) by Dr. *Bevis* and Mr. *James Short*,

A sensible penumbra — —	4 h.	32 m.	0 s.
The eclipse judged to begin —	4	36	50
Total immersion — — —	5	36	5
Emersion — — — —	7	14	33

The end not observed.

A TABLE of the Moon eclipsed June 8th, at night, total.									Moon eclipsed Dec. 2d, morn. total.						
Calculated by		Beg.	Beg. to.d.	Mid.	End to.d.	End	Dur.	Dig.	Beg.	Beg. to.d.	Mid.	End to.d.	End	Dur.	Dig.
		h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	° ′	h. m	h. m.	h. m.	h. m.	h. m.	h. m.	° ′
Mr. J. Smith,	London	7 16	8 25	9 3	9 41	10 49	3 33	15 38	4 46	5 39	6 31	7 24	8 16	3 30	21
	Coventry	7 10	8 19	8 57	9 35	10 43			4 40	5 33	6 25	7 18	8 10		
Mr. W. Sutton,	Warwick	7 0	8 6	8 48	9 30	10 36	3 36	16 3	4 36	5 34	6 22	7 11	8 9	3 33	20 56
Moon rises totally eclipsed 8h. 16m.															
Mr. Cowper,	Wellingborough					10 41			4 33	5 32	6 20	7 8	8 7		20
	London								4 30	5 29	6 17	7 5	8 4		
Emersion 9h. 34m.															
Mr. Ralph Hulse,	Ellworth-hall, Ch.	7 25		9 9	9 34	10 43	3 26	16 20	5 18		6 50		8 20	3 2	21 12
Mr. Al. Man,	London	7 4	8 10	8 52	9 54	10 40	3 36	16 4	4 34	5 36	6 26	7 15	8 17	3 41	21
	Carbonear, Newf.								0 51	1 53	2 43	3 32	4 34		
	Boston, New Eng.								11 49	0 51	1 41	2 30	3 32		
	Paris	7 14	8 20	9 2		10 50									
	Rome	7 56	9 2	9 14		11 32									
Mr. Jos. Walker,	Kettering	7 4	8 11	8 52	9 33	10 40	3 35	16	4 41	5 42	6 31	7 20	8 22	3 40	20 31
Mr. J. Hampson,	Leigh, Lancashire	7 11	8 18	9 0	9 42	10 49	3 38	16 21	5 9	6 6	6 55	7 44	8 41	3 34	21 15

With several other calculations sent, of which these are the most correct.

New Questions.

I. QUEST. 325, *by Mr.* T. Cowper *of* Wellingborough.

Near Albion's center, in Northamptonshire,
Where bleating flocks on every side appear,
Stands * Wellingborough, † known to ancient kings,
For mineral waters, and salubrious springs;
Here wholesome air, and a rich soil is found,
With crops luxuriant here the fields abound.
The south-east side the river Nen glides through,
And spires beyond admiring travellers view:
Three neighbouring ones, which I shall ‡ here disclose,
Shaded my cottage, as bright Phœbus rose.
Strait from my house, next Higham, o'er the plains,
I measured twenty-four of Gunter's chains;
Where at sun-rise, on the solstitial days,
Irchester spire obscures the solar rays.
On February's 'leventh, the rising sun display'd,
On the same spot, a view of Rushden's shade.
If lines from Rushden, and from hence be drawn,
They will at Higham a right angle form.
Each spire's true distance from my house explore,
Counting refraction minutes thirty-four.

* Lat. 52° 20'.

† For the red wells, whose famous senative waters it's said were conducive to the conception of king Charles II. when king Charles I. and his royal consort the queen visited there; when there was great resort of nobility to drink the waters, as there is of the country inhabitants at this day.

‡ With Irchester, the nearest, on Jan. the 6th, Rushden, the remotest, on Feb. the 14th, and with Higham Ferrars on March the 13th, the center of the sun, at rising, appeared in a right line at Wellingborough.

II. QUESTION 326, *by Mr.* Christ. Mason, *Surveyor to the Right Hon. the Earl of* Northampton.

Suppose the radius of our earthly sphere
To be four thousand miles, or very near;
Then fit materials let us next prepare,
To build a pendant castle in the air;
Rais'd to such height, a ball let go from thence,
In falling takes the time found flies from hence,

Ingenious artists, tell me what degree
The ball's velocity at ground will be?
The different gravities next make appear,
Betwixt the ball below, and in the air?
If at the castle we suppose an eye,
How far can that the distant surface spy?

III. QUESTION 327, *by Mr.* W. Jepson *of* Lincoln.

Required two general theorems, with their investigation, to determine the least triangle, and least cone, that will circumscribe any segment of an ellipsis, and frustum of a spheroid, when the dividing ordinate is parallel, and in any given ratio to the conjugate axis?

IV. QUESTION 328, *by the Rev. Mr.* Baker *of* Stickney, Lincolnshire.

To what height will an exhalation ascend, whose specific gravity is, at the earth's * surface, equal to half that of common air, but decreases in the subtriplicate ratio of the spaces ascended?

* *Fluxoniensis* says it must be a mile, or some distance from the surface, to make it consistent.

V. QUESTION 329, *by Mr.* J Powle *of* Salop.

Three spheres of brass in contact, whose diameters are 8, 9, and 10 inches respectively, support a fourth sphere, weighing 12 pounds; what quantity of weight does each supporting sphere sustain?

VI, QUESTION 330, *by* Upnorensis.

To determine the path which a philosopher must describe, passing between two fires, at *d* distance from each other, and one fire *n* times as big as the other, so as to feel the least heat possible?

VII. QUESTION 331, *by Mr.* Christ. Mason.

There are two bridges over two different channels, having flood-gates underneath them; one has four gates, each 4 feet 2 inches wide; the other has two, each 3 feet 9 inches wide; there is 100l. a year paid as water-scot by lands which these channels help to drain: A mean depth of 45 inches was taken at the greater bridge, and 24 inches at the lesser; the beds of both channels are supposed to incline alike in their level, or or declivity; what part of the 100l. must be allotted to each channel, according to the proportion of water which they respectively discharge at the aforesaid depths?

VIII. QUESTION 332, *by Mr.* Powle.

An equation of a curve is expressed by $y = \frac{Xx}{\sqrt{rr+xx}}$ (where X is the hyper. log. of x): Required an expression of its area in finite terms?

IX. QUESTION 333, *by the Rev. Mr.* Baker.

What is the content of a cask, whose head and bung diameters are 30 and 40 inches respectively, supposed to be formed by the cassinian ellipse revolving on its principal axe, which is just four-thirds of the cask's length?

X. QUESTION 334, *by* Dictator Roffensis.

Three Irish evidences, namely, a pedant, a priest, and an alderman, offer their attendance to the plaintiff's attorney, on a trial at Westminster-hall, for the reward of half a hogshead of wine; the pedant can drink it out by himself in 12 days, the priest in 10, and the alderman in 15, when the days are 12 hours long: Quere, in what time can the pedant, priest, and alderman drink out the whole, drinking together, when the days are 10 hours long? And what will be each evidence's share?

XI. QUESTION 335, *by Master* Dickey.

If 16 packs of cards and 3 packs of knaves are of equal value with 9 packs of knaves and 4 packs of cards, what will be the value of one pack of knaves?

The PRIZE QUESTION, *by Mr.* Turner *of* Brumpton, *near* Rochester.

Three towns, *A*, *B*, *C*, at which make no wonder,
Seven, eight, and ten miles are exactly asunder;
A thousand good people in *A* live alert,
In *B* and *C* two and three thousand expert;
Religiously bent, must on Whitfield attend,
And wou'd chuse him a place, a la mode, for that end;
Where must he hold forth, that, in preaching to those,
All walking to hear him shall wear out least shoes?

1751.

Questions answered.

I. QUESTION 325 *answered by the Proposer.*

LET I, R, and H represent the situation of the spires, Irchester, Rushden, and Higham; and W, B, the places of observation at Wellingborough, and next it and Higham. The declination of ☉ on 6th of Jan. last at rising was 20° 42′ S. with which the complement of lat. of Wellingborough 37° 40′, and ☉'s zenith distance 90° 34′, I find (per spherics) the opposite angle, or ☉'s azimuth from $N.$ when his center appears in the horizon = 124° 27′. In like manner, the ☉'s apparent azimuth from N. at rising, Feb. 14th = 104° 29′, the difference of these 19° 58′ = $\angle IWR$. The ☉'s apparent amplitude March 13th was 3° 7′ N. Therefore the $\angle RWH$ = 17° 36′. Sun's apparent amplitude on the winter solstice is 39° 44′, and on the 11th of Feb. 15° 53′ S. Consequently, the $\angle IBH$ = 42° 51′ and $\angle RBH$ = 19° 0′; from whence, with the measured distance BW (by plain trigonometry) WI is found = 2 miles, 1 furl. 29 pol. WR = 3 m. 7 f. 39 p. and WH = 3 m. 6 f. 19 p. required.

Mr. William Sutton's *Answer to the same.*

The visible amplitudes of the sun, at rising, viz.

Dec. 10th	39° 45′	south	
Jan. 6th	34 25	south	
Feb. 11th	15 48	south	
Feb. 14th	13 57	south	
Mar. 13th	3 18	north.	

From whence the diff. of amplitudes from Jan. 6th to Feb. 14th = 20° 28′ = $\angle RWI$. And the sum of amplitudes from Feb. 14th to March 13th = $\angle RWH$ = 17° 15′.

Also diff. amplitudes in the right line from Wellingborough to Higham, between Dec. 10th and Feb. 11th = 23° 57′ = $\angle IBR$.

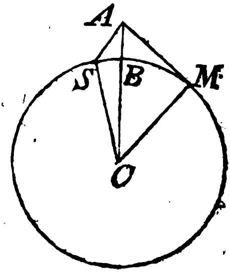
The diff. between Feb. 11th and 14th is $\angle BRW$ = 1° 51′, from whence the $\angle HBR$ = 19° 6′, $\angle BIW$ = 5° 20′, and consequently WI = 2 m. 1 f. 25 p. WR = 3 m. 0 f. 13 p. WH = 2 m. 7 f. 9 p.

The same was curiously answered by Mr. *W. Bevil*, Mr. *Roger Widger*, and others.

II. QUES-

II. QUESTION 326 *answered by Mr.* Chr. Turner, *the proposer.*

To find the height of the castle. Let x = seconds required, either in the acceleration of a body or velocity of sound propagated; $b = 16\frac{1}{12}$ feet = acceleration of a body the first second, $c = 1142$ feet the distance moved over by sound in a second.



Then, $bxx = cx$ = castle's height, per question. Hence $x = \frac{c}{b} = 71$ seconds, and the degree of velocity 141.

To find the proportion of gravity. Let $CB = r = 4000$ miles = radius given, $CA = s$, then $rr : 1 :: ss : \frac{rr}{ss}$, being in a reciprocal proportion. If the weight upon the earth be unity, the same weight at the castle will be ·99 or as 100 to 99 nearly.

Lastly, to find the visual, or tangent line AM. By 36 E. 3, and 47 E. 1, $AC^2 - CM^2 = AM^2$. Hence $AM = 35$• miles and 1745 yards.

Mr. *T. Cowper* elegantly solved the same.

Answered by Mr. Roger Widger *of* Plymouth.

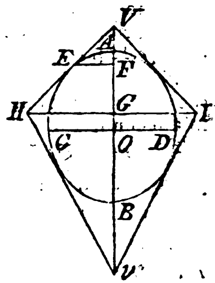
Put $a = 1142$ feet sound moves per second, $b = 16\frac{1}{12}$ feet descended by a heavy body in a second, x = dist. of the castle from the earth's surface. Then, $b : 1 :: x : \frac{x}{b}$ = square time of the falling body. As $a : 1 :: x : \frac{x}{a}$ = time of sound returning, whence $\sqrt{\frac{x}{b}} = \frac{x}{a}$ per quest. Whence $x = \frac{aa}{b} = 81087{\cdot}917$ &c. feet; the rest following as in the above answer.

Mr. *William Bevil* of *Harpswell* has calculated the same in the like manner.

III. QUES-

III. QUESTION 327 *answered by the proposer Mr.* Jepson.

Let $AB=2a$, $CD=2b$, GO (which by the data will always be a known quantity) $=c$, $HI=2y$, GV or $Gv=x$, OV or $Ov=x\pm c$. Then, by the properties of the figure, $VO:AO::AO:FO$, viz. $x\pm c:a::a:\frac{aa}{x\pm c}=FO$; ∴ $BF=a+\frac{aa}{x\pm c}$, $AF=a-\frac{aa}{x\pm c}$; and, by the properties of an ellipsis, $aa:bb::\overline{a+\frac{aa}{x\pm c}}\times\overline{a-\frac{aa}{x\pm c}}:bb-\frac{aabb}{\overline{x+c}^{\,2}}=EF^2$, therefore EF



$=b\sqrt{1-\frac{aa}{\overline{x\pm c}^{\,2}}}=b\sqrt{\frac{\overline{x\pm c}^{\,2}-aa}{\overline{x\pm c}^{\,2}}}$, and $VF=x\pm c-\frac{aa}{x\pm c}=\frac{\overline{x\pm c}^{\,2}-aa}{x\pm c}$: Now, by sim. Δs, $\frac{\overline{x\pm c}^{\,2}-aa}{x\pm c}:x::b\sqrt{\frac{\overline{x\pm c}^{\,2}-aa}{\overline{x\pm c}^{\,2}}}:bx\sqrt{\frac{1}{\overline{x\pm c}^{\,2}-aa}}=y$; but xy is a minimum by the question, ∴ $bxx\sqrt{\frac{1}{\overline{x\pm c}^{\,2}-aa}}$ or $\frac{x^4}{xx\pm 2cx+cc-aa}$ is also a minimum, which in fluxions is $2x^5\dot{x}\pm 6cx^4\dot{x}+4c^2x^3\dot{x}-4a^2x^3\dot{x}=0$, ∴ $x^2+3cx+2cc-aa=0$, or $xx\pm 3cx=2aa-2cc$, whence $x=\sqrt{2aa+\frac{cc}{4}}\pm\frac{3c}{2}$, the theorem for finding the least triangle.

Again, let $d=3{\cdot}1416$, and $\frac{b^2dx^3}{3}\times\frac{1}{\overline{x\pm c}^{\,2}-a^2}$, or $\frac{x^3}{\overline{x\pm c}^{\,2}-a^2}$ is a minimum. In fluxions $x^4\dot{x}\pm 4cx^3\dot{x}+3c^2x^2\dot{x}-3a^2x^2\dot{x}=0$, ∴ $x^2\pm 4cx+3c^2-3a^2=0$, or $x^2\pm 4cx=3a^2-3c^2$, whence $x=\sqrt{3a^2+c^2}\pm 2c$, the theorem for finding the least cone.

This

This gentleman informs us that ſeveral uſeful corollaries may be drawn from the above general theorems, which he promiſed to communicate for public benefit. And as we find him exceedingly well qualified, we ſhall diſtinguiſh his performances, with a due regard had to them. And we hope no ingenious contributor will take offence at our preferring what excels in this Diary, as it is the only means of improvement.

The ſame Queſtion *anſwered by Mr.* William Bevil.

The ratio of the dividing ordinate to the conjugate being given, their diſtance from each other is readily found, which diſtance call d, and let a = ſemi-tranſverſe, b = ſemi-conjugate of that ellipſis, and $x = OV$. Then, per conics, $x : a :: a : \frac{aa}{x} = OF$, $a - \frac{aa}{x} = GF$, $x - \frac{aa}{x} = FV$, $aa : bb :: aa - \frac{aa}{xx} : \frac{bb}{aa} \times \overline{aa - \frac{aa}{xx}} = EF^2$, $\frac{xx - aa}{x} : \frac{b}{x}\sqrt{xx - aa} :: x \pm d : \frac{x \pm d}{xx - aa} \times b\sqrt{xx - aa} = HG$, then $\frac{\overline{x \pm d}|^2}{xx - aa} \times b\sqrt{xx - aa}$ = area $\triangle\, HVI$ or HIv, which is a minimum; ſquared and put into fluxions, $4x^3\dot{x} + 12dx^2\dot{x} + 12d^2x\dot{x} + 4d^3\dot{x} \times \overline{xx - aa} - 2x\dot{x} \times \overline{x \pm d}|^4 = 0$. By reduction, $xx \pm dx = 2aa$, whence $x = \frac{1}{2}\sqrt{8aa + dd} \pm \frac{d}{2}$, a theorem for the leaſt triangle.

Now put $\cdot 2618 = c$, then $\frac{\overline{x \pm d}|^3}{\overline{xx - aa}|^2} \times \overline{xx - aa} \times 4bbc$ = the cone's ſolidity, which, or $\frac{x \pm a}{\overline{xx - aa}|^{\frac{1}{3}}}$, is a minimum. In fluxions, and reduced, $xx \pm 2dx = 3aa$, whence $x = \sqrt{3aa + dd} \pm d$, a theorem for the leaſt cone.

IV. QUESTION 328 *anſwered by the propoſer, the Rev. Mr.* Baker, *only.*

Put r = 4000 miles = earth's radius, $\frac{68444}{3 \times 1760} = c$, ſpace aſcended = x, air's denſity at the earth's ſurface = d. Then, per

per question, that of the vapour there will be $= \frac{1}{2}d$. And $x^{\frac{1}{3}} : 1^{\frac{1}{3}} :: \frac{1}{2}d : \frac{d}{2x^{\frac{1}{3}}} =$ its density at x height. But from page 96th of Mr. Emerson's Fluxions, we have $d \times$ number belonging to this log. $\frac{-rx}{cr+cx} = \frac{d}{2x^{\frac{1}{3}}}$, which reduced, according to the nature of logarithms, gives $\frac{3rx}{cr+cx} - 1.8 =$ l. x, whence, by a table of logarithms, $x = 7.763$ miles, the height required.

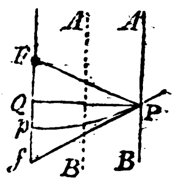
V. QUESTION 329 *answered by Mr.* Widger.

For want of room for the process, we only insert the numbers, viz. $\left\{\begin{matrix}6.0614\\3.2696\\2.669\end{matrix}\right\}$ pounds supported by the globe of $\left\{\begin{matrix}8\\9\\10\end{matrix}\right\}$ inch. diameter.

Mr. *Powle*, the proposer, did not send us his solution, as he proposed.

VI. QUESTION 330 *answered by* Waltoniensis.

Let F be the greater fire, n times bigger than f the lesser; $d = Ff$, their distance.



At any distance, draw AB parallel to Ff, then, since the philosopher must pass every line AB, we have only to find that point therein, at which if he stood still he should feel the least heat possible from both the fires. Suppose P to be that point, and drawing $PQ \perp Ff$; call FQ, x; PQ, y; then $\frac{n}{xx+yy} + \frac{1}{\overline{d-x}^2 + yy}$, expressing the heat of both fires, must be a minimum, y being given, or some constant quantity, while x is variable, the fluxion made $= 0$ will always shew the relation betwixt x and y.

The fluxion of this expression, when $y = 0$, is $\frac{-2n\dot{x}x}{x^4} + \frac{2d\dot{x} - 2x\dot{x}}{\overline{d-x}^4} = 0$, where $x = \frac{dn^{\frac{1}{3}}}{1+n^{\frac{1}{3}}}$; whence $\frac{d}{1+n^{\frac{1}{3}}}$ is the philosopher's distance from the least fire, directly betwixt both,

both fires, moving along the curve Pp, to be roasted on both sides alike. Mr. *Powle*'s solution gives the same; the heat emitted being directly as the two fires, and inversly as the squares of their distance.

Mr. *Sutton* and Mr. *Bevil* sent us their solutions.

N. B. The point p being found for the vertex of the curve, and Fp and fp being in a given ratio to each other, if any other distances from the curve to the greater or lesser fires, PF and Pf, be supposed in the same ratio, the path of the curve will be a circle, as observed by Fluxionensis.

VII. QUESTION 331 *answered by* Britannicus.

200 inches, the breadth of the greater channel by 45 its depth = 9000 square inches the area of the section; and 90 inches the breadth of the lesser channel, by 24 its breadth = 2160 square inches the area of its section; the velocity of water moving along each channel is as the square roots of its depth, respectively, viz. as $\sqrt{45} = 6{\cdot}708$ and $\sqrt{24} = 4{\cdot}899$ fere; therefore $9000 \times 6{\cdot}708$ and $2160 \times 4{\cdot}899$, or 60372 and 10582 the momenta, are nearly as the water respectively discharged by each channel, in the same time; therefore the greater channel pays 85 l. 1 s. 8 d. $\frac{47320}{70954}$, and the lesser 14 l. 18 s. 3 d. $\frac{23634}{70954}$, required.

N.B. Greater / Lesser } Bridge { 88 l. 13 s. / 11 7 — 100 0 { are the proposer's numb. who sent no process.

Our ingenious friend Mr. *Hulse*, corresponding with the proposer, has sent us 88 l. 14 s. and 11 l. 6 s. nearly, for each bridge to pay; correcting the proposer's numbers.

VIII. QUESTION 332 *answered by* Newtoniensis.

Since $y = \frac{Xx}{\sqrt{rr+xx}}$, therefore $y\dot{x} = \frac{Xx\dot{x}}{\sqrt{rr+xx}}$, and fluent of $y\dot{x} = X\sqrt{rr+xx} - s$, whence (p. 58 Emerson's Flux.) $\dot{s} = \dot{X}\sqrt{rr+xx} = \frac{\dot{x}}{x}\sqrt{rr+xx} = \frac{rr\dot{x}}{x\sqrt{rr+xx}} + \frac{x\dot{x}}{\sqrt{rr+xx}}$; and $s = \sqrt{rr+xx}$ + fluent $\frac{rr\dot{x}}{x\sqrt{rr+xx}}$; and fluent

$rr\dot{x}$

$\frac{rr\dot{x}}{x\sqrt{rr+xx}}$ or $\frac{rrx^{-2}\dot{x}}{\sqrt{1+rrx^{-2}}}$ by the table is $= \frac{-L}{r} \times$ log. $\overline{\frac{r}{x} + \sqrt{\frac{rr+xx}{xx}}}$. Therefore the fluent of $\frac{X\dot{x}x}{\sqrt{rr+xx}} =$

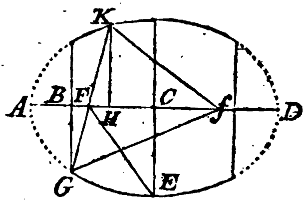
$\overline{X-1} . \sqrt{rr+xx} + \frac{2{\cdot}3024}{r} \times \text{log.}\ \overline{\frac{r}{x} + \sqrt{\frac{rr+xx}{xx}}}$.

N. B. There is no more difficulty where surds are concerned, the process being the same as in simple quantities.

Mr. *Powle* sends $\overline{X-1} . \sqrt{rr+xx} + 2{\cdot}3025r \times \text{log.}\ \frac{\overline{r+\sqrt{rr+xx}}}{x} =$ area.*

IX. QUESTION 333 *answered by the Proposer only.*

Let F, f, be the foci of the gener. ellipse, $BG=b$, $CE=h$, $\frac{1}{2}=n$, $3{\cdot}1416=p$, $AF \times DF = r$, $AC=v$, $Fc=z$, $BF=x$, $BC=y$. Then, by the figure, $v^2 - z^2 = EF^2 = b^2+z^2$, $\because z=\sqrt{\tfrac{1}{2}v^2 - \tfrac{1}{2}b^2}$, and $r=\frac{1}{2}v^2+\frac{1}{2}b^2$; and, because $FG \times Gf = AF \times DF$, and $x^2+y^2 = FG^2$, also $\overline{2z+x}|^2 + y^2 = GF^2$, by elliptic property; therefore $\overline{x^2+y^2} \times \overline{\overline{2z+x}|^2+y^2} = r^2$. Hence $y^2 = \sqrt{r^2+4z^4+8z^3x+4z^2x^2} - 2z^2 - 2zx - x^2$, is the equation of the curve. But when $y=h$, then, per quest. $x+z=nv$, $\because x=nv-z$. And substituting the above values of r, x, y, and z, in the said equation, we get, by reduction, $v^4 + \frac{b^2-n^2b^2-2n^2h^2-h^2}{n^2-n^4}v^2 = \frac{h^4-b^2h^2}{n^2-n^4}$, whence, in the present case, $v=12{\cdot}148$, or $32{\cdot}9267$, which last value is the true, because v cannot be less than b. Hence we have $r=742{\cdot}08365$, $z=18{\cdot}4955$; and by



* Mr. Powle's fluent is right, and Newtoniensis's would be the same if the latter part of it were drawn into r^2 as it ought.

by substitut. for the known quantities $y^2 = \sqrt{a^2 + bx + c^2x^2} - d - cx - x^2$. Wherefore $py^2\dot{x} = p\dot{x}\sqrt{a^2 + bx + c^2x^2} - dp\dot{x} - cpx\dot{x} - px^2\dot{x} =$ the fluxion of solidity between *A* and *F*; whose fluent, when properly corrected, is $\frac{2c^2x + b}{4c^2} \times p\sqrt{a^2 + bx + c^2x^2} - \frac{abp}{4c^2} + \frac{4a^2c^2 - b^2}{8c^3} \times p \times$ hyp. log. $\frac{2c^2x + b + 2c\sqrt{a^2 + bx + c^2x^2}}{2ac + b} - dpx - \frac{1}{2}cpx^2 - \frac{1}{3}px^3$, which, when $x = BF$, will be 5445·9815 inches. Again, putting $x = FH$, $y = HK$, and proceeding as before, we get $py^2\dot{x} = p\dot{x}\sqrt{a^2 - bx + c^2x^2} - dp\dot{x} + cpx\dot{x} - px^2\dot{x} =$ the fluxion of the solidity between *F* and *C*, which differs from the other fluxion in nothing but the signs, consequently its correct fluent is discovered to be $\frac{2c^2x - b}{4c^2} \times p\sqrt{a^2 - bx + c^2x^2} + \frac{abp}{4c^2} + \frac{4a^2c^2p - b^2p}{8c^3} \times$ hyp. log. $\frac{2c^2x - b + 2c\sqrt{a^2 - bx + c^2x^2}}{2ac - b} - dpx + \frac{1}{2}cpx^2 - \frac{1}{3}px^3$ $= 22116·2054$ when $x = CF$. Hence, the content of the whole cask in $\left\{\begin{matrix}\text{ale}\\ \text{wine}\end{matrix}\right\}$ gallons $\left\{\begin{matrix}195·4758\\ 238·63278\end{matrix}\right.$ *Q. E. I.*

X. QUESTION 334 *answered by Mr.* Steph. Hodges *of* Wellingborough.

The proportion of the quantities of wine drank by the pedant, priest, and alderman, in the same time, are as 1, 1·2, ·8, whose sum = 3. Then say, inversely, as 1 : 12 :: 3 : 4 days = 48 hours when the days are 12 hours long = 4 days 8 hours when the days are but 10 hours.

By direct proportion, say,

$$\text{As } 3 \text{ gall.} : 31·5 :: \left\{\begin{matrix}1·\\ 1·2\\ ·8\end{matrix}\right\} : \left\{\begin{matrix}10·5\\ 12·6\\ 8·4\end{matrix}\right\} \text{ gall.} \begin{matrix}\text{pedant's}\\ \text{priest's}\\ \text{alderman's}\end{matrix} \Big\} \text{ allowance.}$$

$$\overline{31·5}$$

Mr. *T. Cowper* of *Wellingborough* answered this question in the same concise and artificial manner, as did Mr. *Joseph Orchard*, of *Gosport*, in *Hampshire*.

The same answered by Mr. Will. Smith, *at* Churchdown, *near* Gloucester.

Put $x =$ time they will all drink it in? $12 \times 12 = 144 = c$, $12 \times 10 = 120 = d$, $12 \times 15 = b$, $w =$ half the hogshead.

Say, $c : w :: x : \frac{wx}{c}$; $d : w :: x : \frac{wx}{d}$; $b : w :: x : \frac{wx}{b}$, the several shares collected $= \frac{wx}{c} + \frac{wx}{d} + \frac{wx}{b} = w$.

Whence $x = \frac{cbd}{bd + cd + cb} = 48$ hours when the day is 12 hours $= 4$ days 8 hours when the day is 10 hours.

	gall.			gall.	
Hence	$\frac{48}{144}$ pedant's	Share		10·5	as above.
	$\frac{48}{120}$ priest's			12·6	
	$\frac{48}{180}$ alderman's			8·4	

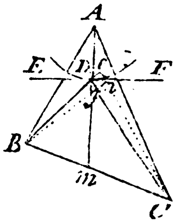
Mr. *William Dod,* of *Brampton,* in *Cumberland,* solved the same, exactly to the truth, and so did several others.

XI. QUESTION 335 *answered by Master* Billy Branch, *of* Rochester.

If 10 p. cards + 3 p. knaves = 9 p. knaves + 4 p. cards. Then, by reduction, 6 packs of knaves is of equal value with 6 packs of cards; whence the value of a pack of knaves is only a pack of cards.

The PRIZE QUSTION *answered by* Newtoniensis.

Let A, B, C, be the three towns, D the place of meeting sought; and suppose any of the distances, as AD to be given, and with the radius AD, describe the circular arch $GDnH$, and let EDF be a tangent at D; draw ADm, and take Dn infinitely small, and draw the lines BD, Bn, CD, Cn, and with the radii Bn, Cn, describe the small arcs ne, nf; then De is the increment of BD, and Df the decrement of CD.



Let $BD = x$, $CD = y$, $AD = z$, and a, b, c, three given numbers 2, 3, 4, in proportion as per question, (4000 people living in the town A, as the author informs me, instead of 1000 the printed number) so that $ax + by + cz$ may be a minimum.

minimum. Then, since z is supposed constant, we have $a\dot{x}+b\dot{y}=0$, and $a\dot{x}=-b\dot{y}$, or $\dot{x}:-\dot{y}::b:a$. In the two right-angled triangles Dne, Dnf, whose common hypothenuse is Dn, it is as $De(\dot{x}):Df(-\dot{y})::$ s. Dne : s. Dnf $::b:a$; but $\angle Dne = eDA = BDm$, and $\angle Dnf = mDf$ or mDC. Whence s. BDm : s. mDC $::b:a$, and $\frac{\text{s. } CDm}{a}=\frac{\text{s. } BDm}{b}$.

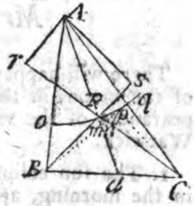
After the same manner it may be proved, that if y be supposed constant, $\frac{\text{s. } CDm}{a}=\frac{\text{s. } CDB}{c}$; ∵ when $ax+by+cz$ is a min. $\frac{\text{s. } CDm}{a}=\frac{\text{s. } BDm}{b}=\frac{\text{s. } CDB}{c}$. Therefore, if s. $CDm=v$, then $\frac{b}{a}v=$ s. BDm, and $\frac{c}{a}v=$ s. $CDm+BDm$ their sum. Therefore the problem comes to this, *To find the* ∠ CDm *whose sine is* v, *and* ∠ BDm *whose sine is* $\frac{b}{a}$v, *so that* $\frac{c}{a}$v *may be the sine of the sum* CDm + BDm. And ∠ CDm is easily found by Mr. Heath's method in the Diary 1738.

N. B. ∠ CDm is nearly 29°.

Now, all the parts of △ ABC being given, together with the angles at D, all the distances AD, BD, CD are easily found, viz. $AD=2{\cdot}6$, $BD=5{\cdot}02$, $CD=7{\cdot}64$. (See Ronayne, prob. 11 p. 363.)

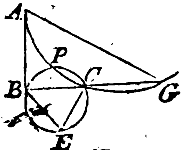
The same answered by Waltoniensis.

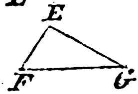
Let ABC represent the three towns, and let the number of people in A be $a=4000$ (correcting the printed number); in B, $b=2000$; in C, $c=3000$.



P the place of meeting sought. Draw AP, BP, CP, and with the rad. AP, on the center A, describe the arc oPq. A point being now supposed to move along the said arc, lines drawn to it from B and C will be continually variable. Let the point have moved from P to p, over the indefinitely small arc Pp, draw Bp, Cp, and with the radii Bp, Cp, describe the small arcs Pm, pn.

 Then





Then b times BP plus c times CP being a minimum, b times mp, the increment of BP, will be equal to c times Pn, the decrement of CP; therefore, the right-angled triangles APr, APs, made by letting fall the perpendiculars Ar, As, being respectively similar to the small right-angled triangles Ppn, pPm, the sine of the angle APr, or CPd, will be to that of $\angle APs$, or BPd, as b to c. In like manner it is proved, that the sine of $\angle CPs$ is to that of $\angle CPd$, as a to b. Whence it follows, that if a $\triangle EFG$ be constructed, whose sides FG, EF, and EG, are as a, b, and c respectively, and two circular arcs described without $\triangle ABC$, one upon the side BC, capable of the $\angle E$, and the other upon AC, capable of the $\angle G$, the supplement of those arcs, when completed into circles, will intersect each other at the point P sought: And AP will be found $= 2{\cdot}596$ miles, $BP = 5{\cdot}009$, $CP = 7{\cdot}623$. *Q.E.F.*

COROLLARY. If a, b, and c be equal, then two segments of circles described within the $\triangle ABC$, on any two scales, each capable of an angle of 120°, will intersect at the point required, according to Mr. Simpson's new doctrine and application of Fluxions, p. 26 and 27. He has inserted and solved this our question at p. 505 of his doctrine aforesaid.

Mr. *W. Jepson* sent his solution, which, with Dr. *Quibus*'s and two or three more were all the solutions we received.

The Prize of 12 *Diaries was won by* Newtoniensis, *and that of* 8 *by Dr.* Quibus.

The Eclipses calculated for 1751, *by Mr.* William Sutton.

There will happen four eclipses, two of the sun, and two of the moon, in the following order: The times and appearances of the visible ones according to the meridian of Warwick.

1. The sun eclipsed on tuesday May 14th, at 43 minutes in the morning, apparent time of conjunction in the moon's orbit; will be central and total to the eastern parts of Asia bounding on America; but invisible at London, or to any part of Europe.

The

2. The moon eclipsed wednesday May 29th, at one in the morning.

		h. m.		
Beginning 28th May		11 44 P. M.	}	apparent time.
Middle 29th	—	1 26 A. M.	}	Digits eclipsed on the south side 10° 47′.
Ecliptic opppofit.		1 31	}	
End	— — —	3 8	}	
Duration	— —	3 24		

Calculated by		Beg. h. m.	Mid. h. m.	End h. m.	Dur. h. m	Dig. ° ′
Mr. Randles,	London	12 21	2 3	3 4[illegible]		
	Barbadoes	8 28	10 10	11 5[illegible]	3 23	10 3[illegible]
	Wem, Shropsh.	12 11	1 53	3 34		
Mr. Hulse,	Chester	0 28	1 0	3 32	3 4	10 3
Mr. Cowper,	London	11 58	1 38	3 1[illegible]	3 18	10 19
	Wellingboro.	11 56	1 35	3 14		

3. Sun eclipsed on thursday November 7th, at 57 minutes in the morning, equal time of conjunction in the moon's orbit; invisible at London, but will be a central eclipse to some parts of the globe.

4. The moon eclipsed on thursday November 21st, at 9 h. 45 m. equal time.

		h. m.		
Beginning	—	8 26	}	apparent time.
Middle	——	9 48	}	Digits eclipsed on the north side 8° 27′
Ecliptic opposit.		9 54	}	
End	— —	11 10	}	
Duration	—	2 24		

Calculated by		Beg. h. m.	Mid. h. m.	End h. m	Dur. h. m.	Dig. ° ′
Mr. Randles,	London	8 30	9 52	11 14		
	Petersburgh	11 6	12 28	1 5		
	Lisbon,	7 53	9 15	10 37	2 44	8 21
	Wem, Shropsh.	8 20	9 42	11 4		
Mr. Hulse,	Chester	8 20	9 48	11 16	2 56	8 4
Mr. Cowper,	Wellingborough	8 13	9 36	10 59	2 46	8 38

With others, which for want of room we are obliged to omit.

New Questions.

I. QUESTION 337, *by Mr.* T. Cowper *of* Wellingborough, *Surveyor.*

The latitude explore,
And time laſt winter, when
Day broke exact at four,
And the ſun roſe at ten?

II. QUESTION 338, *by Mr.* William Leighton.

Two perſons, *A* and *B*, playing at hazard, *A* wins from *B* a certain number of guineas, conſiſting of three places, whoſe digits are in arithmetical progreſſion, in ſuch manner, that, if the number of guineas be divided by the ſum of their digits, the quotient will be 48; and, if from the ſaid number of guineas you take 198, the digits will be inverted: Quere the number of guineas won?

III. QUESTION 339, *by Mr.* William Bevil.

From what height muſt a ball of 4 ounces weight fall, to have $49\frac{67}{100}$ pounds force, on an inclining plane, whoſe angle of incidence is 40°?

IV. QUESTION 340, *by Mr.* Davis, *Teacher of the Mathematics, at* Painſwick, Glouceſterſhire.

In latitude of forty-eight,
A monument ſtood tall and ſtraight,
Which finiſh'd was o'th' firſt of June,
At five o'clock i'th' afternoon;
When, by repeated trials made,
The length of its extended ſhade
Was found in ratio to its height,
As ninety-two to twenty-eight;
And, where its baſe with earth did join,
An angle form'd of ninety-nine?
What was the Julian period then?
If 'twas erected ſince, and when?
Or, if before erected found,
How many years have ſince gone round?

QUES-

V. QUESTION 341, *by Mr.* Bevil.

Two men bought an equal number of ſheep, and it being demanded of them what they gave a-piece for each parcel; it was anſwered, that if the number of ſheep each of us bought be ſeverally multiplied by $\frac{24}{27}$ and $\frac{54}{27}$, 49 being reſpectively added to and ſubtracted from each product, both the ſum and remainder will be equal to the ſquare of the number of ſhillings given for each reſpective parcel; How many ſheep did each perſon buy? And what did each parcel coſt?

VI. QUESTION 342, *by Mr.* Steph. Hodges, *the younger.*

In an exciſeman's round,
An oblong ciſtern's found,
The ſum of one ſide and one end being given,
With diag'nal below,
The contents you're to ſhow,
Whoſe breadth's to the depth twenty-five is to ſeven.

84 *inches = the ſum of one ſide and one end.* 60 = *the diagonal.*

VII. QUESTION 343, *by Mr.* John Randle, *of* Wem, *in* Shropſhire.

A gentleman has a piece of ground in form of a geometrical ſquare, the difference between whoſe ſides and diagonal is 10 poles; he would convert two-thirds of the area into a garden of an octagonal form, but would have a fiſh-pond at the center of the garden, in the form of an equilateral triangle, whoſe area muſt be equal five poles. Required the length of each ſide of the garden, and of each ſide of the pond?

VIII. QUESTION 344, *by* Upnorenſis.

To determine the ſides of the leaſt right-angled triangle in whole numbers, whoſe legs are in proportion as 7 to 11?

IX. QUESTION 345, *by* χρονονμονονπυβλικος.

If a bookſeller buys a copy for 21 l. pays 21 l. for paper, 21 l. for printing 500 impreſſions, and 10 l. for advertiſements and other contingent expences, amounting in all to 73l. and sells 100 books yearly of the history, at 5s. each: What is his gain per cent. allowing compound interest, for the time he lies out of his money?

The PRIZE QUESTION, *by Mr.* T. Cowper, *of* Wellingborough, *Surveyor.*

Admit the moon, on the 17th of February, 1750, rose four minutes sooner in the latitude 51° 32′ north, than in the latitude 52° 20′ north, and was observed to come upon the meridian in the former latitude on the same morning 42 minutes after four, and the preceding morning 54 minutes after three; from whence her longitude and latitude at rising, in the latitude of 51° 32′, are required?

1752.

Questions answered.

I. QUESTION 337 *answered by Mr.* T. Cowper, *the proposer.*

PUT a = sine ☉'s ascensional difference, 60°; b = sine hour from 6, at day-break, or 30°; d = sine ☉'s depression at 18°; and x and y the sine and cosine of the latitude: also e and v the sine and cosine of ☉'s declination: By spherics $bvy + ex = d$ and $ex = avy$, and substituting avy for ex in the first equation $vy = \frac{d}{a+b}$: And also in the other equation putting $\frac{d}{a+b}$ for vy, and we have $ex = \frac{ad}{a+b}$; therefore $\frac{1-a}{a+b}d = \cdot0303074$ = cos. sum of lat. and sun's declin. 88° 15′ 48″. And $\frac{1+a}{1+b}d = \cdot4221252$ the cos. of their diff. or 65° 1′ 52″. Hence the lat. 76° 38′ 50″, and declin. 11° 36′ 58″; nearly agreeing with Mr. Gibbons's answer.

THEOREM.

THEOREM. As the ſum of the ſines of the ſun's aſcenſional difference and arch of time from day-break to ſix o'clock, is to the ſine of the ſun's depreſſion at day-break, ſo is the verſed ſine of the arch of time from ſun-riſe to noon, to the ſine of the meridian altitude; and ſo is the verſed ſine of the time from midnight to ſun-riſe, to the ſine of the ſun's depreſſion at midnight.

Mr. *William Bevil* has curiouſly and conciſely ſolved the ſame, exactly agreeing with the above. We wiſh we had more room, to inſert all he ſends us.

The ſame ſolved by Mr. Charles Smith *of* Rugby.

Put r and n for the coſines of the hour angles from midnight till day-break, and from ſun-riſe till noon, reſpectively; $d =$ coſ. of $108° = Z\odot$, [*ſee fig. p.* 136] x and $y =$ ſine and coſ. of the required latitude; u and $z =$ thoſe of declination. Then, in ſpheric triangles $\odot PZ$ and OZP, by common theorems, $rzy + ux = d$, and $nzy - ux = 0$, from whence $zy - ux = d \times \frac{1-n}{r+n} = {}^{\cdot}0303072 =$ coſ. 88° 15′ 48″ the ſum of lat. and declin. and $zy + ux = d \times \frac{1+n}{r+n} = {}^{\cdot}422125$ $=$ coſ 65° 1′ 52″ the diff. Hence the lat. 76° 38′ 50″ N. and the declin. 11° 36′ 58″ S. (anſwering to 7th Feb.) required; proving the truth of Mr. *Cowper*'s anſwer.

Mr. *John Aſh*, Mr. *Sutton*, Mr. *William Spicer*, Mr. *James Terey*, Mr. *Charles Maſon*, *Obadiah Wittam* of *Whitby*, Mr. *William Cottam* at his Grace the Duke of Norfolk's, and ſeveral others, ſolved the ſame.

II. QUESTION 338 *anſwered by Mr.* Rich. Gibbons.

Let x, y, and z repreſent the three digits; then, by the queſtion, we have $x + z = 2y$, $\frac{100x + 10y + z}{x + y + z} = 48$, and $99x - 99z = 198$; whence $x = 4$, $y = 3$, and $z = 2$; alſo number of guineas 432, required.

N. B. This queſtion is the 21ſt of the Miſcellanea Curioſa Math. vol. I. inverted.

Philotheoros, putting $a = 198$, $c = 48$, makes $x = \frac{a}{198} \times \frac{c-4}{c-37} = \frac{44}{11} = 4$; whence the number $= 432$.

Mr. *Joſeph Orchard* ſolved the ſame; alſo Mr. *John Fiſh* of *Dartford*, and ſeveral others.

III. QUES-

III. QUESTION 339 *anſwered by Mr.* John Aſh.

Ecce homo!

As ſine 40° : rad. : : 49·67 pounds force : 77·2728 pounds, the momentum or force of the falling body $= m$. Put n for the given weight $=$ ·25 pounds, and x for the required height; then, by the laws of motion, $\frac{m}{n}$ will be the velocity of the ball arrived at the plane of the horizon; and (if Deſaguliers's experiment, Philoſ. Tranſactions, No. 375, p. 269, can be depended on) we have $\sqrt{x} = \frac{m}{n}$; whence $x = \frac{m^2}{n^2} = 9553{\cdot}7$ feet, required.

Mr. *Richard Gibbons* ſolves this queſtion in the ſame manner: Thus,

As the ſine of the angle of incidence 40° : 49·67 pounds force : : rad. : 77·273 pounds force on the plane of the horizon, being let fall from the ſame height. By Dr. Deſaguliers's experiments, an heavy body deſcending four feet will have twice the quantity of motion it had when it began to fall (i. e. *we obſerve at the end of one foot fallen*) the time of its falling half a ſecond. Now, the force is always as the velocity and quantity of matter, *i. e.* $\sqrt{\text{ſpace}} \times$ matter, perpendicularly deſcended; putting $m = 77{\cdot}273$ the momentum, perpendicularly deſcended; $q = 0{\cdot}25$ pounds the quantity of the ball; and $s =$ ſpace required to run through: Then $q\sqrt{s} = m$; whence $\sqrt{s} = \frac{m}{q}$, and $s = \frac{m^2}{q^2} = 9553{\cdot}7$ feet, as before.

We received numerous other learned ſolutions to the foregoing queſtion. Mr. *Harland Wid*, of *Whitby*, makes the diſtance to be deſcended by the ball no leſs than 386622·4536 feet, or 73·223 miles; and ſome about as far as from the moon's orbit.

A Diary-Critic, obſerving our remark in laſt year's Diary where Newtonienſis points out an impropriety in propoſing theſe ſort of queſtions, endeavours to make the diſcovery his own, and is very angry at our ignorance, that we ſhould ſuffer ſuch a queſtion to be printed (ſee London Gazetteer for Dec. 13, 1750). But he ſhould have firſt conſidered, that Mr. *John Turner* and the Rev. Mr. *Baker*, who ſolved a like queſtion (in Diary 1750), the propoſer Mr. *Bevil* (in Diary 1751), and Mr. *Landen* (in the ſame Diary, and in what is called

called Gentleman's Diary), are equally culpable: Though he frequently borrows inventions and obſervations, and borrowed our own remark from the place above-mentioned. See our prize queſtion for the Diary 1750, inſerted in a late book of fluxions, p. 505.

To a famous Doctor, *on his Diſcoveries.*

So modern 'pothecaries, taught the art
By doctors bills to play the doctor's part,
Bold in the practice of miſtaken rules,
Preſcribe, apply, and call their maſters fools!

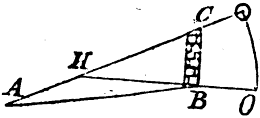
POPE.

Mr. *Landen* alſo. who refined upon Mr. *Turner*'s and Mr. *Baker*'s ſolution to a queſtion of this kind laſt year, now detects our modern philoſophers; but firſt ſaw the remark that we inſerted from *Newtonienſis*.

The relative forces of falling bodies being as the $\sqrt{\text{ſpace}}$ × quantity of matter, perpendicularly deſcended (*i. e.* as the rectangle of the velocity and matter) it will follow, that o and x, diſtances deſcended by two balls, whoſe weights are W and w, will have forces, as $o^{\frac{1}{2}} \times W$ to $x^{\frac{1}{2}} w$, that is, when W is at reſt, it will have no force in compariſon with the force of w at any diſtance deſcended: which is contrary to what is ſuppoſed in the 321ſt and 339th diary queſtions, where force compounded of weight of matter and velocity, is ſuppoſed equal to a degree of preſſure of matter unſuſpended, tho' at reſt.

IV. QUEST. 340 *anſwered by the propoſer, Mr.* Davis.

Let BC repreſent the monument, AB the ſhade's length, HO the horizon; then $\angle OH\odot$ = ſun's alt. The ſides AB to BC as 92 to 28, and $\angle ABC = 99°$. By trigon. $\angle OH\odot = 25°\ 0'\ 37''$ ſun's alt. and allowing $17'\ 37''$ for ſun's ſemi-diam. and refraction, $25°\ 0'\ 37'' - 17'\ 37'' = 24°\ 43'$ = ſun's true alt.



Now, from comp. lat. 42° 0', comp. ſun's alt. 65° 17', and hour angle from noon of 5 h. = 75°, the complement of ſun's declin. will be found 69° 53' 26", and declin. 20° 6' 34", which anſwers to ♉ 29° 38' the ſun's place in the ecliptic, or longitude from ♈ 59° 38'; and, by making proportion, I find, June 1ſt, 5 hours P. M. anno 965 ante chriſtum, the ſun's place is ♉ 29° 38' 0", as may be proved from Leadbetter's tables; being the time when the monument was erected.

Mr.

Mr. *Charles Mason*, jun. of *Sapperton*, *Gloucestershire*, sends us word, that the above solution was done by him, though sent us in Mr. *Davis*'s name.

Some make the time long before creation, when there were no men to build; and others in the time of the first Chinese emperors, who reigned before the European time of creation.

V. QUESTION 341 *answered by the proposer, Mr.* Bevil, *of* Harpswell, *near* Gainsborough, Lincolnshire.

It happened, through haste or inadvertency, that this question wanted a word or two to make it properly understood, which are now supplied in contrary characters: *Two men bought an equal number of sheep; and it being demanded of them what they gave a-piece for* their sheep in *each parcel, it was answered, that, if the number of sheep each of us bought be severally multiplied by* $\frac{24}{27}$ *and* $\frac{54}{27}$, 49 *being respectively added to or subtracted from each product, both the sum and remainder will be equal to the square of the shillings given* a-piece *for* sheep in each respective parcel. *How many sheep did each person buy? And what did each parcel cost*, at the cheapest price? for so every man would buy. Or, it had been better proposed, *Two men bought an equal number of sheep and hogs, &c* to distinguish one parcel, and the price of each hog and sheep in each parcel, the better from one another.

Put $x =$ number of sheep; then $\frac{24}{27}x + 49$ and $\frac{54}{27}x - 49$ are square numbers, whose roots are the shillings a-piece the sheep in each respective parcel (of different value, though equal number) cost.

But a square number, multiplied by a square number, will produce a square number. The expressions being multiplied respectively by 9 and 4, two square numbers will be $\frac{216}{27}x + 441$, and $\frac{216}{27}x - 196$, whose difference is 637.

To find two square numbers having that difference.

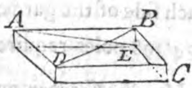
RULE. Resolve the given difference into any two factors; then the half sum and half difference of those factors will be the sides of the squares having the difference given: $637 = 13 \times 49 = 7 \times 91$. Therefore $\frac{49+13}{2} = 31$, and $\frac{49-13}{2} = 18$ will

will be the ſides of the ſquares: Conſequently $\overline{\frac{24}{27}x+49}\times 9 = 31^2$, and $\overline{\frac{54}{27}-49}\times 4 = 18^2$; from either of which equations $x = 65$ the number of ſheep: And, conſequently, $\frac{24}{27}\times 65+49 = \frac{31^2}{9}$, and $\frac{54}{27}\times 65-49=\frac{18^2}{4}$, whoſe ſquare roots are $\frac{31}{3}$ and $\frac{18}{2}$, or 10s. 4d. and 9s. the ſheep coſt a-piece, in each parcel of 65; whence $65\times$ 10s. 4d. = 33l. 11s. 8d. one parcel coſt; and $65\times$ 9s. = 29l. 5s. the other parcel coſt: the true anſwer.

The ſame method may be purſued with the factors $7\times 91 = 637$, when the ſides of the ſquares will be 49 and 42, and the number of ſheep in each parcel 245; conſequently $\frac{49}{3}$ and $\frac{42}{2}$, or 16s. 8d. and 21s. the ſheep in each parcel coſt a-piece, and 257l. 5s. and 204l. 3s. 4d. the price of each parcel; being dearer, and therefore not the true anſwer.

VI. QUESTION 342 *anſwered by Mr.* Joſeph Orchard *of* Goſport.

Given $AB+AD=84=a$; $AE=60=d$; and $\frac{BE}{AD} = \frac{7}{25} = \cdot 28 = r$. Let $AD = x$, then $AB = a-x$, and $Be = rx$: But $AB^2 + BE^2 = AE^2$, i.e. $aa-2ax+xx+r^2x^2=dd$, $\therefore xx - \frac{2ax}{rr+1} = \frac{dd-aa}{rr+1}$,



and $x = \frac{a-\sqrt{\overline{aa+dd-aa\times rr}}}{rr+1} = 24\cdot 39$ nearly, the breadth; and the length is $59\cdot 61$; alſo depth $6\cdot 8292$: whence the content $\begin{cases}35\cdot 208 \text{ ale}\\ 42\cdot 982 \text{ wine}\end{cases}$ gallons.

But, if by "*diagonal below*" is meant DB at the bottom or top of the ciſtern, then this is the ſolution:

Given $AB+AD=84=a$, $DB=60=d$; let $AD=x$. Then $AB=a-x$: But $AB^2+DA^2=DB^2$ i. e. $aa-2ax+2xx=dd$; ſolved $x=\frac{a-\sqrt{\overline{2ad-aa}}}{2}=36$ the breadth; the length is 48; and depth 10·08; whence the content $\left\{ \begin{array}{l} 65\cdot312 \text{ ale} \\ 75\cdot403 \text{ wine} \end{array} \right\}$ gallons.

Mr. *T. Cowper* ſolves it trigonometrically: As 60 : 84 :: coſ. 45° ; coſ. 8° 8′ half the diff. of ∠s; conſequently the ∠s are 53° 8′ and 36° 52′: Whence the length 48; breadth 36; depth 10·08; and content 8·1 buſhels.

Mr. *Fiſh*, of *Dartford*, by a ſhort proceſs, ſolved this queſtion, and finds the length, breadth, and depth of the ciſtern, exactly as above.

Mr. *W. Bevil*, of *Harpſwell*; Mr. *Randle*, of *Wem*, *Shropſhire*; and Mr. *Samuel Smith*, of *Cambden*, *Glouceſterſhire*, alſo ſolved it: as did *Obadiah Wittam*, of *Whitby*; Mr. *Hopkinſon Farmer*; Mr. *William Cottam*, at his Grace the Duke of Norfolk's; and others.

VII. QUESTION 343 *anſwered by Mr.* Orchard.

Let $d = 10$ the difference between the ſide and diagonal of the ſquare; then $\overline{1+\sqrt{2}} \times d =$ the ſide of the ſquare; two-thirds of the ſquare of which are 388·5618 &c. = the area of the octagonal garden: And, if x be the ſide thereof, then $xx \times 4\cdot8284$ &c. = 388·5618 &c. the ſaid area (*ſee Palladium for 51, p. 24*), $\because x = \sqrt{\frac{388\cdot5618 \text{ \&c.}}{4\cdot8284 \text{ \&c.}}} = 8\cdot9707$ poles, each ſide of the garden: And each ſide of the pond is $2\sqrt{\frac{5}{\sqrt{3}}}$ = 3·398 poles required.

Mr. *T. Cowper* anſwers it thus, very conciſely and elegantly: As $3 - 2\sqrt{2} : 10 :: 10 : \frac{100}{3 - 2\sqrt{2}} = 582\cdot842696$; two-thirds of which = 388·561797 = area of the octagonal garden; then $\sqrt{\frac{388\cdot561797}{4\cdot8284272}} = 8\cdot9707$ poles, the ſide thereof; and 3·398 = ſide of the triangular pond.

Obadiah Wittam ſolved the ſame in an elegant manner; ſo did Mr. *Cottam*, at his Grace the Duke of Norfolk's, and ſeveral others.

VIII. QUESTION 344 *answered by* Upnorensis, *the proposer, only.*

1. To find two such square numbers, whose roots may represent one leg and the hypothenuse of a right-angled triangle; and the difference of those squares to be a square number, whose root may represent the other leg.

Put x for one leg, or side of the square, $x+d$ for the hypothenuse, or side of the other square; then the squares will be denoted by xx and $xx+2xd+dd$, whose difference will be $2xd+dd=\overline{2x+d}\times d=yy$ for the square of the other leg, by question. It is evident, that $2x+d$ and d must be square numbers. Let $2x+d=n$, then the leg $x=\frac{1}{2}\times\overline{n-d}$, and hypothenuse $x+d=\frac{1}{2}\times\overline{n-d}$. Now, if $rr=n$, $ss=d$, then $\frac{1}{4}\times\overline{rr+ss}|^2-\frac{1}{4}\times\overline{rr-ss}|^2=rrss=yy$, per 47 E. 1. and, by transposition and reduction, $\overline{rr+ss}|^2=4rrss+\overline{rr-ss}|^2$. Whence we have this theorem: $2rs$ and $rr-ss$ will express the legs of a right-angled triangle, and $rr+ss$ the hypothenuse, r and s being assumed any rational or whole numbers at pleasure.

The ratio of the legs, as 7 to 11, being given so far as in whole numbers (for, exactly given, it would be no question, and an impossible one, if the sum of their squares were not a square) by a trial or two, r will be found $=3\frac{1}{2}$, and $s=1$, by the theorem; when the complete ratio of the legs will be 7 to $11\frac{1}{4}$, the nearest to the given numbers, and the corresponding hypothenuse as $13\frac{1}{4}$, four times which values will be 28, 45, and 53, the sides of the least right-angled triangle, in whole numbers, required. See p. 186 of Dodson's Mathematical Repository, requiring two numbers in the complete ratio of 8 to 15, the sum of whose squares shall be a square number; where the required is given, and a superfluous theorem that finds the numbers 576 to 1080, being 72 times 8 to 15; whereas 2, 3, 4, 5, &c. 8 to 15, had been a direct answer. And there was no way to propose this question, but as it was proposed, without giving what was required (or to the same effect) or else proposing an impossibility.

N.B. The foregoing answers a scandalous and false advertisement in the London Gazetteer of Dec. 13, 1750.

IX. QUESTION 345 *answered by* Upnorensis.

The bookseller buys an annuity of 25 l. a year, to continue five years, for 73 l. ready money—To find his gain per cent. according to the allowance of compound interest.

Let a, r, t, z signify: a the annuity. r 1 l. and its interest for one time or year. t the number of times the annuity is to be paid. z the whole amount of the annuity's pres. worth.

Say,

$r : 1 :: a : \frac{a}{r}$ present worth of a payable at the end of 1st time.

$r : 1 :: \frac{a}{r} : \frac{a}{r^2}$ present worth of a payable at the end of 2d time.

$r : 1 :: \frac{a}{r^2} : \frac{a}{r^3}$ present worth of a payable at the end of 3d time.

Conseq. $\frac{a}{r^t}$ present worth of a payable at the end of t^{th} time.

The sum of all which progressionals $\frac{1}{r}, \frac{1}{r^2}, \frac{1}{r^3}$, &c. $\frac{a}{r^t}$, multip. into a, $\frac{\overline{r^t - 1} \times a}{\overline{r - 1} \times r^t} = z$ the whole present worth of all the payments of a, from 1 to t times.

For, if z be the greatest term, a the least, and r the ratio, or any term decreed by the next lesser, $\frac{rz - a}{r - 1} =$ the sum, universally.

The aforesaid equation reduces to $r^t + 1 - \frac{a + z}{z} r^t + \frac{a}{z} = 0$; in which, according to Dr. Halley, if t the number of years be great, 40 or upwards, and the rate of interest be high, $1 + \frac{a}{z}$ will be nearly, or more accurately $\frac{z + a}{z} - \frac{z^t}{z + a} \times \frac{a}{z}$, the value of r, when $\frac{a}{r^t \times \overline{r - 1}}$ will be exceedingly near the value of the reversion, which, if it be called x, then $1 + \frac{a}{z + x}$ will approach the value of r sufficiently. *See Dr. Halley's method, at p. 33 and 34 appendix to Sherwin's Logarithms.* But t the years being small, this

rule

rule fails; in which case, if $\frac{at}{z}\Big|^{\frac{2}{t+1}} - 1 = y$, and $\frac{6}{t-1} = b$, then $1 + b - \sqrt{bb - 2by}$ is sufficiently equal to r, and will still be nearer the truth, as t the years be of the smaller value, the small error being always in excess; viz. $r = 1 + b - \sqrt{b \times b - 2 \times \frac{at}{z}\Big|^{\frac{2}{t+1}} - 1}$. And, putting capitals for the logarithms of quantities denoted by small letters, then $2 \times \frac{A + T - Z}{t+1} = D$, and $\frac{B + L.\overline{b - 2 \times \overline{d-1}}}{2} = E$, $\because r = 1 + b - e$; and consequently $r - 1 = b - e =$ ·21094 &c. the rate of interest of 1 l. per year, and 21l. 1s. 10½d. for 100l. a year, bookseller's profit.

Mr. *Flitcon* elegantly considers z paid down as a principal put to interest, whose account at first year's end $= zr$; when, a becoming due, $zr - a$ is the principal running on; which, drawn into r, is $zr^2 - ar$ the amount at second year's end; and, a being again paid, $zr^2 - ar - a$ will be the principal running on; which at the end of five years will be $zr^5 - ar^4 - ar^3 - ar^2 - ar - a = 0$ principal running on: the bookseller then being repaid all his money at first laid out, with the interest thereof running on as a principal; consequently the value of r in this equation shews the rate of interest as before.

N. B. The sum of all the terms, except the first, $= a \times \frac{r^5 - 1}{r - 1}$, by the universal rule aforesaid for summing geometrical progressions; and $\because zr^5 - a \times \frac{r^5 - 1}{r - 1} = 0$, which reduces to $zr^6 - \overline{z + a} \times r^5 + a = 0$ the same equation with the first.

Mr. *Terey* of *Portsmouth*, Mr. *Alexander Rowe*, Mr. *John Honey* of *Redruth*, *Cornwall*, and some others, solved this question.

The PRIZE QUESTION *answered by the Proposer.*

Put $t =$ co-tang. 51° 32′, $T =$ co-tang. 52° 20′, $x =$ sine, and $y =$ cos. of the ☽'s ascensional diff. in lat. 51° 32′; $s =$ sine, and $c =$ cos. of the diff. between the ascen. diff. $= 1°$; then will $cx + sy =$ sine of ascens. diff. in lat. 52° 20′. Now,

 by

by ſpheric trigon. as $1 : t :: x : tx =$ tang. ☽'s declination. Again, $1 : T :: cx + sy : Tcx + Tsy$; hence $tx = Tcx + Tsy$; i. e. $\frac{y}{x} = \frac{t - Tc}{Ts} =$ 1·6808489 the co-tang. of ☽'s aſcenſ. diff. (in lat. 51° 32′) = 30° 45′. Hence her declin. is found = 22° 6¼′. Then, to right aſcen. ☉ 341° 12′ add 250° 30′ (= time ☽'s ſouthing) the ſum, rejecting 360°, will be 231° 42′ = right aſcen. ☽ at ſouthing. By the rule of proportion, the diff. of right aſcen. from ☽'s riſing to ſouthing = 1° 58½′; conſequently her right aſcen. at riſing, in lat. 51° 32′, is = 229° 43½′. Thus, having her right aſcen. and declin. I find her true place to be ♏ 23° 7′ 11″, lat. 3° 38′ 27″ S.

Obadiah Wittam, of *Whitby*, (whoſe letter followed one we received from Mr. *Harland Wid* on the ſame ſubject) makes the moon's longitude ♏ 23° 44′ 30″, and her lat. 3° 29′ 30″ S. which is not the truth: Conſequently Mr. *T. Cowper*, the propoſer, claims the prize, he having no competitor.

We approve Mr. *William Cottam*'s method, at his Grace the Duke of Norfolk's, who makes ☽'s long. ♏ 22° 36′ 25″, and lat. 2° 55′ 23″ S. allowing for the moon's parallax and refraction at riſing—whoſe anſwer had been very near the propoſer's, had he not made a ſmall overſight in his tabular computation.

The Prize of 12 *Diaries was won by Mr.* T. Cowper, *and that of* 8 *by Mr.* C. Maſon.

The Eclipſes calculated for 1752, *by Mr.* Ralph Hulſe.

There will be but two eclipſes of the ſun for this preſent and both inviſible, as follows:

1. On Saturday May 2d, at 6h. at night. At the Bay of Honduras, in North America, the ſun will be totally eclipſed in 23° of ♉. The beginning 4h. 30m. Middle 6h. 0m. End 7h. 34m. Total duration 3h. 4m. Digits eclipſed 12° 8′.

2. On Friday November 17th, at 2 morn. At Carpentaria, in South America, at 6h. 6m. in the morning, the ſun is ſeen eclipſed 14° of ♏ lat. 16° N. long. 148°. E. 9h. 52m. where it will appear very formidable to its inhabitants. The begin. 0h. 28m. Middle 2h. 2m. End 3h. 36m. Total Dur. 3h. 6m. Digits eclipſed 12° 4′.

	Beg.	Mid.	End	Dur.	
Mr. Al. Man, London, May 2d		5 52			
Jamaica, Nov. 17th	11 44	1 20	2 48	3 4	10 45
Boſton ———	10 30	1 44	2 50	2 20	6 15

Mr.

Mr. *John Child*, of *Barnet*, *Hertfordshire*, sends us his observations on the moon's eclipse that happened on sunday Dec. 2d, 1750, in the morning, by a clock exactly set.

		h.	m.	
Barnet, Hertford.	Beginning — — —	4	36½	app. time.
	Beg. total darkness	5	36	
	End of tot. darkness	7	14¾	
	Dark — — —	1	38¾	

By which our astronomical tables may be proved.

Of the Alteration of the Style.

By 365 days, 6 hours, the mean Julian year, being long reckoned for 365 d. 5 h. 48 m. 54 s. 41 th. and 27 fourths, the year by the sun, according to Dr. Halley, (see Palladium, 1750, p. 53.) The account of time has each year run a-head of time by the sun 11 m. 5 s. 18 th. 33 fourths, or 44 m. 21 s. 14 th. 12 fourths, every 4 years, and consequently 3 d. 1 h. 55 m. 23 s. 40 th. in 400: And so from the council of Nice, when the kalendar was settled, in the year 325, to this present year 1752, being 1427 years, the time by account is forward of that by the sun 10 d. 23 h. 43 m. and therefore 11 days is left out of account in this month [*September the 3d being accounted the 14th day*] as the most convenient for reducing the kalendar or year to its first established order. And for keeping the shortest and longest days (or the solstices) and also the days of 12 hours long (or the equinoxes) on the same nominal days of the month for the future, it is ordained by act of parliament, that every fourth hundred year is to consist of 366 days as usual, but all other whole hundred years of 365 days only: The years between which whole hundreds to be common and bissextile as formerly, and the date of the year henceforward to begin on the first of January [*instead of the 25th of March.*]

New Questions.

I. QUESTION 347, *by Mr.* T. Cowper *of* Wellingborough, *Surveyor.*

By a meand'ring limpid brook,
In the blithe month of May,
Early one morn a walk I took,
And did some land survey:

Trian-

Triangular its form I found,
The base, then measur'd o'er,
On horizontal verdant ground,
Made perches thirty-four.
From midst the perpendicular,
The base's ends I view,
And find the angle forming there,
Degrees just ninety-two.
The vertex angle I behold
Just fifty-five degrees,
From whence the unknown sides are told,
And acres, if you please.

II. QUESTION 348, *by Mr.* James Terey *of* Portsmouth.

The greatest spheroid, and parab'lic conoid,
Inscrib'd in a cone are by art,
From whence as below*, the contents you're to show,
Of each separate solid apart.

* *Diam. of cone's base = 35 inches, and its altitude = 30 inches.*

III. QUESTION 349, *by Mr.* Obadiah Wittam, *of* Whitby.

On what two days of the year 1752 will the sun rise at the same instant of time at Petersburgh and Jerusalem?

IV. QUESTION 350, *by Mr.* William Honnor.

Required a theorem for determining the length of a lever of the first kind (supposed of no weight) capable of being divided into two brachias, y the greater, and x the lesser, so that $y^m - x^m = y^n \times x^n$; on whose ends two given weights being suspended, w the greater, and v the lesser, shall equipoize each other?

V. QUESTION 351, *by* Taptinos.

In a right-angled triangle, there is given the distance from the angle at the base to the center of an inscribed circle 4 chains; and if it be prolonged 2 chains further it will touch the cathetus: To find the sides?

VI. QUESTION 352, *by Mr.* Randles.

A gentleman has an orchard of fruit trees, one-half of the trees bearing apples, one-fourth pears, one-sixth plumbs, and 50 of them bearing cherries: How many fruit trees in all grow in the said orchard?

VII. QUES-

VII. Question 353, *by* Taptinos.

In a plain triangle there is given the rectangle of the ſides 195, the rectangle of the ſegments of the baſe 45, and the perpendicular 12; to find the ſides.

VIII. Question 354, *by* Philotheros.

Given the area of the greateſt trapezium that can be inſcribed in an apolonian parabola, whoſe abſciſſa and ſemi-ordinate are as 3 to 2, equal 256: Required the dimenſions of the parabola and trapezium by a ſimple equation.

IX. Queſt. 355, *by* Anagramenſis Holy in Heart, Ebor.

There are two cities in the ſame parallel of latitude, whoſe difference of longitude is 144° 15′, and their diſtance in the arch of a great circle 6797′4 ſtatute miles: Required their latitude, and what day of the year the ſun riſes to the one city exactly at the ſame time he ſets to the other?

X. Question 356, *by Mr.* John Williams, *of* Mold, *in* Flintſhire.

A gentleman would make a corn mill to be turned by a current of water that runs a tun in a minute, and has 16 feet fall or perpendicular deſcent: It is required to know the diameter of the water-wheel, ſo that the iſſuing water may give the wheel the greateſt power, or force poſſible?

XI. Question 357, *by the Rev. Mr.* Baker, *of* Stickney, Lincolnſhire.

Let $ABCE$ repreſent a compound barometer, filled with mercury from B to C, and with water from C to E: How then muſt the bores of the two tubes $ABCF$, and FEK be adjuſted, or proportioned, ſo that the ſcale of variation in the leſſer tube of this barometer may be to the common ſcale, as 10 to 1? [*See the fig. to the ſolution.*]

XII. Question 358, *by* Upnorenſis.

To determine the ſolidity and ſuperficies of an elliptical ring*, of any dimenſions (c the conjugate and t the tranſverſe inches) the ſubſtance filling whoſe periphery is circular of p inches diameter?

XIII. Ques-

XIII. QUESTION 359, *by* Honorius.

Miss's apron grown short, she is full of complaint,
And to merit your pity she looks like a saint!
On the floor falls her tea; then her screams you may hear,
And fainting she sinks in a fit on the chair.
Mamma for the doctor immediately sends,
Who, in honour to miss, in his chariot attends;
He examines her pulse, and appearing so wise,
Descants on the languishing looks of her eyes—
But alas! neither spirits, nor letting miss blood,
Specifics, nor preaching, are found to do good:
For a surgeon came in, who the cause did declare,
And the doctor's finesse, and his art made appear.
Mamma now was told Miss's hoop was too small,
Therein lay her grievance, disorder, and all;
The question was ask'd—Polly sighing, reply'd,
A French hoop will cure me, and so will a bride.
A hoop of the fashion to cure her disease,
Extends from her center quite round to her knees:
In the right and left wing a French placket * is made,
To her elbows advancing, and forms a parade.
Miss Polly to church now, or play can repair,
And wherever she goes is admir'd for her air!
At the sight of a beau, how her heart beats alarms!
While the winds swell her pride, and her legs tell their [charms:
Her hidden perfections she knows will invite,
Or ensnare the beholder, should chance give them sight.
By the pow'r of her hoop Polly steps into fame,
By out-priding the rest she conceals her own shame;
In the country she reigns o'er the 'squire and the clown,
O'er the lords and the fops she's triumphant in town.
Her hoop is the secret—and if you would know
What it holds with her petticoat, seek from below †.

* *Opens and shuts, forms a pair of bellows, and rises and falls by the means of strings or bowlings.*

† *Form of the hoop is the lower frustum of an ellipsoid, with its vertex next the head.*

Transverse	*diams.*	42 *inches*	*above,*	48 *inch.*	*below.*
Conjugate		26		29	

Altitude of the frustum 12 *inches.*

From the lower part of the hoop's circumference to the bottom of the petticoat, the form is an elliptic cylinder, by the petticoat hanging nearly perpendicular from thence: the altitude of which elliptical cylinder is 18 *inches: Quære the content of the whole concavity in wine gallons?*

The

The Prize Question, *by* χρονονμονονκπυβλικος.

Archimedes, the renowned mathematician of Sicily, once bathing himself, observed the water to rise so much higher on his going into the bath. It was from thence he first took the hint for measuring the solidities of all irregular bodies, not measurable by the known rules of art; and also for determining the different specific gravities of bodies. For, being transported with the discovery, he came out of the bath, forgetting he was naked, and ran home, crying out, Εὕρηκα, Εὕρηκα, signifying, *I have found it*; and, afterwards, discovered the quantity of silver mixed with the gold in King Hiero's crown, which the workman confessed.

It is proposed to determine by the best method, the nearest superficial content in inches of a modern mathematician, of a middle age, weighing 160 pounds avoirdupois, being naked, all his parts middle-sized, and meanly proportioned; and his muscles not rigidly swelled, nor yet quite unbraced?

N.B. The same rule will hold good for male or female mensuration; and man and woman being microcosms, expressions of many elegant and useful curves may thence be discovered; and several improvements made in the rectifications of curve lines, and quadrature of curvilinear spaces; besides cubation of several important solids; whose forms of fluxions, with their fluents, we shall insert in our new Harmonia Mensurarum.

New Paradoxes.

Paradox I. *by Mr.* Honnor.

A truss of hay weighing but half a hundred weight in a scale, weighed two hundred weight stuck upon the end of a fork carried on Hodge's shoulder: How could that be?

Paradox II. *by Mr.* James Collingridge.

How can a mechanic file a square hole with a round file? and fill up an oval hole with a round stopper?

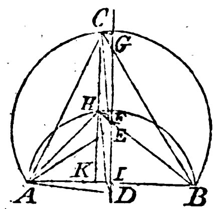
Questions

1753.

Questions answered.

I. QUEST. 347 *answered by Mr.* Terey *of* Portsmouth.

LET ABC be the triangle; then on the base AB (per 33 Euc. 3) describe two segments of a circle, ACB and AHB, containing the given angles; through the centers D, E draw DG; also draw DH, EC to the $\perp CK$; and let fall CG and HF $\perp$s to GD. By trigonometry, $AD = DH = 17{\cdot}0104 = a$; $AE = EC = 20{\cdot}7531 = b$; $ID = {\cdot}593651 = c$; $IE = 11{\cdot}9035 = d$. Put $x = CK$, then (by 47 Euc. 1) $aa - cc - cx - \frac{1}{4}xx = bb - dd + 2dx - xx = CG = HF$; hence $x = 2c + 4d \pm$



$$\frac{\sqrt{3 \times \overline{b-a} \times \overline{b+a} + \overline{2c+d}^2}}{3} = 32{\cdot}5340 = CK;$$ whence $KI = 2{\cdot}2524$; $AC = 35{\cdot}7204$; $CB = 37{\cdot}8036$; and the area $= 3$ a. 1 r. $33{\cdot}078$ p. required.

Mr. *Widd*'s solution agrees. Mr. *F. Holden* solves it by the same method, and brings out the same correct numbers; with which Mr. *T. Cowper*'s solution, by another method, exactly agrees. *Philotheros* solved it.

The same answered by Nichol Dixon *of* Blackwell.

Put $t =$ tangent $\angle ACB = 55°$, and $T =$ tangent $\angle AHB = 92°$, and $2b = AB = 34$ poles, $y = HK = HC$, $x = KI$, the distance from the $\perp CK$ to the middle of the base. Then, by trigonometry, as $y : 1 :: b + x : \frac{b+x}{y} =$ tangent $\angle KHB$; and $y : 1 :: b - x : \frac{b-x}{y} =$ tangent angle AHK: Now, by prob. 8 p. 21 of Mr. Emerson's excellent Trigo-

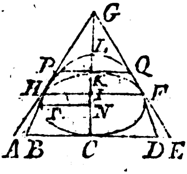
Trigonometry, As $1-\frac{b^2-x^2}{y^2} : 1 :: \frac{2b}{y} : \frac{2by}{y^2-b^2+x^2}$ $= T$; and, by the same reasoning, $\frac{4by}{4y^2-b^2+x^2}=t$; from which equations $y=\frac{4b}{3t}-\frac{2b}{3T}=\frac{2}{t}-\frac{1}{T}\times\frac{2}{3}b$: But rad. divided by the tangent = co-tangent: Therefore $y = \overline{2\times\text{co-tangent }55^\circ - \text{co-tangent }92^\circ}\times\frac{2}{3}b = 16{\cdot}2666$; hence $AC = 35{\cdot}718$, $CB = 37{\cdot}804$, and area = 3 a. 1 r. 33·06 p. &c.

Mr. *Joseph Orchard*, of *Gosport*, putting b and x as above, tang. $\angle AHB = 92^\circ = -v$, tang. $\angle ACB = t$, rad. 1, makes the tangents of the respective angles $\frac{2by}{yy-bb+xx} = -v$, and $\frac{4by}{4yy-bb+xx}=t$, the equations brought out of fractions, the former multiplied by t and the latter by v; and, taking the sum of both, we get $\overline{2v+t}\times 2b = 3vty$, whence $y=\frac{2v+t}{3vt}\times 2b = 16{\cdot}2665$, whence the area as before.

Mr. *Bevil* solves it by the same method, whose numbers exactly agree; as do the solutions of Mr. *Stephen Hartley*, Mr. *Cottam*, at his Grace the Duke of Norfolk's, Mr. *Cha. Smith*, Mr. *Richard Gibbons*, Mr. *John Wigglesworth*, Mr. *John Cross*, and some others.*

II. QUESTION 348 *answered by Mr.* J. Orchard, *Writing-master and Teacher of the Mathematics at* Gosport.

Let $GC = a = 30$ inches, $AE = 2b = 35$ inches, $m = {\cdot}7854$, and $KC = x$. Then $KG = a-x$; and per sim. $\triangle$s GCA, GKP, $a : b :: a-x : \frac{a-x}{a}\times b = PK$; per conics, $PK\times AC = TN^2 = bb\times\frac{a-x}{a}$. Then $8mbb\times\frac{ax-xx}{3a}$ = the solidity of the spheroid, which, or $ax-xx$, is a maximum. In fluxions



* A construction is given in p. 313 British Oracle.

fluxions $aa\dot{x} - 2x\dot{x} = 0$, whence $x = \frac{1}{2}a$. Again, let $GL = LI = x$; per sim. Δs GAC and GHI, $a : b :: 2x : \frac{2bx}{a} = HI$; per conics $x : \frac{4bbxx}{aa} :: a - x\ (= LC) : 4bb \times \frac{ax - xx}{aa} = BC^2$; then $\frac{8mbbx \times \overline{a-x}|^2}{aa}$ is the conoid's solidity, which, or $aax - 2axx + x^3$, is a maximum. In fluxions $aa\dot{x} - 4ax\dot{x} + 3x^2\dot{x} = 0$; whence $x = \frac{1}{3}a$. These values of x substituted in the maximums, and dimensions, by proper theorems for the segments and frustums, the content of each solid, with the ratio they are in, are exhibited in the following table, by Mr. *Joseph Orchard*.

Generating lines from the cone's axis	Ratio of each solid to the cone.		Content of each solid.
GHL	4	: 54	712·677
LHK	5		890·847
KHC spheroid.	27		4810·575
CHBC	16		2850·712
AHB	2		356·338
sum = whole cone.	54		9621·15

Philotheros solved it. Mr. *Terey*'s solution: He puts $AE = b = 35$, $CG = a = 30$, and $z = \cdot 78539$, &c. when the cone's solidity $= \frac{1}{3}bbaz = S = 9621\cdot 0274$.

1. Then put $CK = x$: As $a : b :: a - x : \frac{a-x}{a}b = PQ$; but $PQ \times AE = \square$ diam. spheroid; whence its content $= \frac{bbax - bbxx}{a} \times \frac{2}{3}z$, is a max. when in fluxions, $a\dot{x} - 2x\dot{x} = 0$, and $x = \frac{1}{2}a$, by writing which value in the above expression, its content $= \frac{1}{2}$ of $S = \frac{1}{2}$ content of the cone.

2. *To find* CI. (N is the center of the spheroid) $NG : NK :: NK : NI$; i. e. $\frac{3}{4}a : \frac{1}{4}a :: \frac{1}{4}a : \frac{1}{12}a = NI$. Whence $CI = \frac{1}{3}a$, $IK = \frac{1}{6}a$. By fluxions, content of the spheroidal segment $= \overline{\frac{2ccxx}{t} - \frac{\frac{4}{3}ccx^3}{tt}} \times z$; for t put $\frac{1}{2}a$; for cc, $\frac{1}{4}bb$; and for x, $\frac{1}{6}a$; then the segment $HKF = \frac{7}{54}$ of S, and segment $HCF = \frac{10}{27}$ of S.

3. To

3. *To find the greatest parab. conoid* BLD. Let $LC = x$, then $LG = LI = a - x$; say, $a : b :: 2a - 2x : \frac{2ab - 2bx}{a} = HI$, and per known property, $a - x : 2b \times \frac{a-x}{a} \times 2b \times \frac{a-x}{a} :: x : \frac{4bbax - 4bbxx}{aa} = BD^2$; whence the content $BLD = \frac{2aabb - 2bbx^3}{aa} \times z$, a max. from whence, by fluxions, $x = \frac{2}{3}a = CL$, and $LG = \frac{1}{3}a$; consequently, the contact, in this case, cuts off $\frac{1}{3}$ of the axis, viz. IC, the the same as of the spheroid: For x put $\frac{2}{3}a$, and the conoid $BHLFD = \frac{8}{9}$ of S.

4. Conoid $HLF = \frac{4}{9}bb \times \frac{1}{3}az = \frac{2}{9}$ of $S = 2139{\cdot}006$.

5. (3 and 4) $BHLFD - HLF = BHFD = \frac{2}{3}$ of $S = 6414{\cdot}0182$.

6. Cone $HGF = \frac{8}{27}$ of S: Consequently, frustum $AHFE = \frac{19}{27}$ of $S = 6770{\cdot}3526$.

7. (4 and 6) cone HGF — conoid HLF = solid $HGFL = \frac{2}{27}$ of $S = 712{\cdot}6687$.

8. (2 and 4) conoid HLF — segment HKF = solid $HKFL = \frac{5}{54}$ of $S = 890{\cdot}8358$.

9. (2 and 5) frustum $BHFD$ — segment HCF = solid $HCFDB = \frac{8}{27}$ of $S = 2850{\cdot}6748$.

10. (5 and 6) frustum $AHFE$ — frustum $BHFD$ = solid $HABDEF = \frac{1}{27}$ of $S = 356{\cdot}3343$. *W. W. R.*

Mr. *Thomas Cowper*'s solution and numbers are very correct, as they always are: So is Mr. *James Hartley*'s solution, Mr. *Bevil*'s, Mr. *F. Holden*'s, Mr. *Charles Smith*'s, Mr. *William Cottam*'s, and those by some others, who are every one complete artists.

III. QUEST. 349 *answered by Mr.* T. Cowper, *Teacher of the Mathematics, at* Wellingborough.

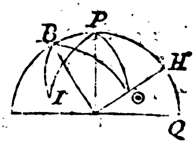
By Dr. Halley's astronomical tables, lat. of Petersburgh = 60°, lat. of Jerusalem = 31° 55′, their diff. of long. = 5°. Put a = tang. 60°, b = tang. 31° 55′, and s and c = sine and cos. 5°, also x = tang. sun's declination. Then, by spherical trigonometry, $1 : a :: x : ax$ = sine ascensional diff. at

Petersburgh; cos. $= \sqrt{1-a^2x^2}$; also $1 : b :: x : bx =$ sine ascensional diff. at Jerusalem. But $acx - \sqrt{s^2 - a^2s^2x^2} = bx$, or $\overline{ac-b} \times x = s\sqrt{1-a^2x^2}$: Therefore, $a^2c^2x^2 - 2abcx^2 + b^2x^2 = s^2 - a^2s^2x^2$, or (because $1 = s^2 + c^2$) $a^2x^2 - 2abcx^2 + b^2x^2 = s^2$; whence $x^2 = \frac{s^2}{a^2+b^2-2abc}$; But writing $v =$ sine sun's declination, we have $\frac{v^2}{1-v^2} = \frac{s^2}{a^2+b^2-2abc}$, or $2v^2 = \frac{2s^2}{a^2+b^2-2bac} = \cdot0121914$ = the versed sine of 8° 57′ 21″ = twice the sun's declination; consequently the sun's declin. = 4° 28′ 40½″, corresponding to the 26th of February and the 20th of March, and the 31st of August and the 23d of September, O. S. *W. W. R.*

Mr. *John Peachy* found the same: Mr. *Thomas Allen*, of *Casterton* School, *Lincolnshire*, solved it: Also *Philotheorus*.

N. Dixon's *Answer*.

Let B be Petersburgh, I Jerusalem, P the pole, by Gordon's Geographical Grammar, $BP =$ 38° 35′; $PI = 57°$ 16′; $\angle BPI = 5°$ 25′, the diff. long. Let $B\odot = 90°$ be perpendicular to BI, then $\odot$ is the place of the sun at rising. In the triangle BPI, is given BP, PI, and $\angle P$, to find the $\angle B = 169°$ 54′, from whence take 90°, and you have $\angle PB\odot$. Then in the quadrantal triangle $BP\odot$ there is given BP 30° 35′, $\angle B$ 79° 54′, to find $P\odot$ the comp. of the sun's declin. = 84° 53′, whence the sun's declin. = 5° 7′. And the days correspondent thereto are the 22d of March and the 28th of August, O. S. on which the sun will rise at both places nearly at the same time. *W. W. R.*



Mr. *Widd*, the proposer's solution, gives the same days.

Mr. *James Hartley*, of *Yarum*, solves it thus: Lat. Jerusalem = 32° 30′, tang. = t; lat. Petersburgh = 60° 4′, tang. = T; diff. of merid. = 3° 30′; let its sine = s, rad. = 1, and x = tang. sun's declin. Then $1 : t :: x : tx$. Again, $tx + s : x :: T : 1$, whence $\frac{s}{T-t} = x = 3°$ 10′ 40″, answering to the 1st and 17th of March, and the 3d and 19th of September, O. S. *W. W. R.*

Mr. *Cottam* brings out the same days, and so does Mr. *F. Holden*, by making the sun's declination 3° 8′ 15″. For we did

did not give the latitudes of Jerusalem and Petersburgh that we might see the answer from different authorities of those latitudes. A gentleman (without name) with a large figure stuck on with a wafer, also solved it 5° 6′ 44″ declination.

IV. QUESTION 350 *answered by Mr.* James Hartley *of* Yarum.

Take $R = \frac{v}{w}$ = the ratio of the given weights. Then, $Ry = x$; and the given equation will become $y^m - R^m y^m = R^n y^{2n}$, which being reduced, we have the following theor.

When m is greater than $2n$, $\left(\frac{R^n}{1 - R^m}\right)^{\frac{1}{m-2n}} = y$.

When $m = 2n$, $R^m + R^n = 1$. Here R is limited, and y indeterminate.

When m is less than $2n$, then $\left(\frac{1 - R^m}{R^n}\right)^{\frac{1}{2n-m}} = y$.

Mr. John Honey, *of* Redruth, *in* Cornwall, *solves it thus:*

Put N = length of the lever, and $s = w + v$, then $x = \frac{Nv}{s}$, and $y = \frac{Nw}{s}$; also $\frac{N^m w^m - N^m v^m}{s^m} = \frac{N^{2n} w^n v^n}{s^{2n}}$, per question; reduced $N = \left(\frac{s^{2n-m} \times w^m - v^m}{w^n v^n}\right)^{\frac{1}{2n-m}}$ = length required.

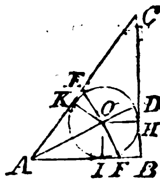
Mr. *Charles Smith's* theorem is $\left(\frac{w^n}{v^n} - \frac{v^{m-n}}{w^{m-n}}\right)^{\frac{1}{2n-m}} + \left(\frac{w^{m-n}}{v^{m-n}} - \frac{v^n}{w^n}\right)^{\frac{1}{2n-m}} = y + x$. Who says, if $m = 3$; and $n = 2$, that $y + x = \frac{w}{v} \times \overline{1 + \frac{w}{v}} - \frac{v}{w} \times \overline{1 + \frac{v}{w}}$.

Mr. *F. Holden*, Mr. *Prisson*, Mr. *Rob. Butler*, *Philotheoros*, and some others, concisely solved the same.

V. QUES-

V. QUESTION 351 *answered by Mr.* Joseph Orchard, *of* Gosport.

Geometrical constructions of problems being always valuable, where they are to be had, we insert the construction first, as follows: Draw $AD = 6$, making $AO = 4$, and $OD = 2$; through O draw EF, at right angles to AD, making $OE = OF = OD$; draw AB and AC through F and E, and BC through D perpendicular to AB. Then ABC will be the triangle required, as is evident; which is the greatest of demonstration.



Calculation: In $\triangle AOF$ we have $AO = 4$, $OF = 2$, whence $AF = \sqrt{20} = 2\sqrt{5}$. Per sim. triangles, $AF : AO :: AD :$ $AB = \frac{12}{\sqrt{5}}$. Per trigon. tang. $\angle FAO = \frac{1}{2}$, and (per schol. to prop. 2 of Mr. Emerson's Elements of Trigonometry) tang. $\angle BAC =$ tang. $2\angle FAO = \frac{4}{3}$, $\because 1$ (rad.) $: \frac{12}{\sqrt{5}} (AB) :: \frac{4}{3} : BC = \frac{16}{\sqrt{5}}$; and $AC = \frac{20}{\sqrt{5}}$; the sides are in arithmetical progression, and are $AB = 5.366$, $BC = 7.155$, and $AC = 8.944$. *W.W.R.* With which Mr. *T. Allen's*, of *Gosberton* school, in *Lincolnshire*, and Mr. *Widd's* solutions agree.

Mr. *Thomas Cowper*, Teacher of the Mathematics, at *Wellingborough*, computes thus: Put $x =$ sine, and $y =$ cosine $\angle DAB = \angle DOH$; then $1 : 4 :: x : 4x = OI = OH$. And $1 : 2 :: y : 2y = OH$, $\because \frac{x}{y} = \frac{2}{4} = .5$, the tangent of $\angle OAI = 26° 33' 54.2''$; whence $AB = 5.3665626$, $BC = 7.1554168$, and $AC = 8.944271$. *W.W.R.*

Mr. *Cottam* answers it elegantly in the same numbers, as likewise does Mr. *John Williams*, Mr. *James Hartley* of *Yarum*, Mr. *William Bevil*, Mr. *Robert Butler*, Mr. *Hollingsworth*, Mr. *F. Holden*, Mr. *Richard Gibbons*, Mr. *Chr. Tate* of *Hull*, Mr. *John Adams*, (who constructs the solution) Mr. *William Honnor*, our old friend Mr. *John Ramsay*, (mathematician and enigmatist) Mr. *John Hampson*, Mr. *Alex. Roe*, Mr. *Joseph Hilditch*, Mr. *Brownbridge*, Mr. *J. Hirst*, Mr. *Stephen Hartley*, Mr. *Honey*; and, in a beautiful and incomparable hand-writing, Mr. *Thomas Huntley* solved the same in latin diction; as he has done many other problems in in our Diary, fit for posterity to look it) putting at the bottom of his elegant latin letter, *Dabam Burfordiæ, in comitatu Oxoniensi, pridie kalendas 4ti mensis, anno 1752.*

Mr. *Joseph Orchard*, Mr. *Thomas Allen*, Mr. *Cha. Smith*, Mr. *Tho. Cowper*, Mr. *James Terry*, Mr. *James Hartley*, Mr. *Wm. Bevil*, Mr. *Holden*, and some others, add to the value and correctness of their performances by propriety of diction, and hand-writing, being clear, full, and concise; from whose compositions we find pleasure to collect; as we do to encourage all useful and correct correspondents in general.

VI. QUESTION 352 *answered by Mr.* Henry Watson, *of* Gosberton *School, in* Lincolnshire.

Put x = the number of fruit trees unknown; then $\frac{x}{2}+\frac{x}{4}+\frac{x}{6}+50=x$, and, by transposition, $x-\frac{x}{2}-\frac{x}{4}-\frac{x}{6}=50$; by division $x=\frac{50}{1-\frac{1}{2}-\frac{1}{4}-\frac{1}{6}}=\frac{50}{1-\frac{11}{12}}=\frac{50}{\frac{1}{12}}=600$, the number of fruit trees required.

The same concisely and elegantly answered by Mr. John Fish, *of* Dartford.

$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=\frac{24}{48}+\frac{12}{48}+\frac{8}{48}=\frac{44}{48}=\frac{11}{12}$ wanting $\frac{1}{12}$ of the whole. Therefore $\frac{1}{12}=50$ trees; consequently the whole $=600$ trees. Now $\frac{1}{2}=300$ apple trees, $\frac{1}{4}=150$ pear trees, $\frac{1}{6}=100$ plumb trees, whose sum $=550$, to which adding 50 trees, the sum total $=600$ trees, the proof.

Mr. *John Nicholson* of *Rochester*, answers it in the same concise and easy manner; as did Mr. *James Hartley* of *Tarum*, (who solved all the problems) Mr. *F. Holden*, Mr. *Hollingsworth*, Mr. *Peter Brooke*, Mr. *Thomas Trimingham* of *Hull*, Mr. *Richard Gibbons*, Mr. *John Adams*, Mr. *John Williams*, Mr. *John Ramsay*, Mr. *Joseph Hilditch*, Mr. *Robert Butler*, Mr. *William Cottam*, Mr. *John Hampson*, (who also sent the times of eclipses for Leigh) Mr. *A. Brook*, Mr. *T. Cowper*, Mr. *Brownbridge*, Mr. *John Potter*, of *Duke-street*, in *Southwark*, Mr. *John Peachy*, Mr. *Honnor*, Mr. *Henry Watson*, and Mr. *Thomas Allen*, of *Gosberton* school, in *Lincolnshire*, and others.

But Mr. *Thomas Huntley* of *Burford*, putting $12x$ = trees; then $6x+3x+2x+50=12x$; whence $x=50$, and $12x=600$, required.

VII. QUES-

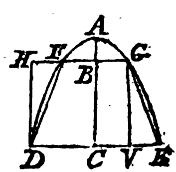
VII. QUESTION 353 *answered by Mr.* Henry Watson, *of* Gosberton *School, in* Lincolnshire.

Put $x = AD$ the greater, and $y = DB$ the lesser segments; [*see the fig. p. 223*] $z = AC$, and $u = BC$, the sides of the triangle; $a = 45$, $b = 195$, and $c = 12$. Then $xy = a$, $zu = b$, per quest. and $xx + cc = zz$, and $yy + cc = uu$, by 47 Euc. 1. Whence $zz = \frac{bb}{uu} = xx + cc$, $uu = \frac{bb}{xx + cc} = yy + cc$, and $yy = \frac{bb}{xx + cc} - cc = \frac{aa}{xx}$; from which last $-x^4 + \frac{bbxx - aaxx}{cc} - ccxx = aa$: Put $d = \frac{bb - aa}{cc} - cc$, then $-x^4 + dxx = aa$, or $x^4 - dxx = -aa$; whence $xx = \frac{d}{2} \pm \sqrt{\frac{dd}{4} - aa} = 81$, and $x = 9$; whence $y = 5$, $z = 15$, and $u = 13$. *Q.E.F.*

Most of the gentlemen beforn-mentioned solved this question, and particularly Mr. *Thomas Allen* and Mr. *John Williams*; all agreeing with *Taptinos* the proposer's solution. And Mr. *Thomas Huntley* solved it in latin.

VIII. QUEST. 354 *answered by Mr.* Terey *of* Portsmouth.

Let $AC = a$, $DC = b$, $BC = x$. Per property of the curve, $a : bb :: a - x : \frac{a-x}{a} \times bb = FB^2$; and $FB = b\sqrt{\frac{a-x}{a}}$. But $bx + bx\sqrt{\frac{a-x}{a}}$ = area of the trapezium $DFGE$ to be a maximum. In fluxions and reduced, $x = \frac{8}{9}a$ (let b be what it will). Now, substituting this value of x in FB above, $FB = \frac{1}{3}b$, and also $\frac{2}{3}a$ for b (per quest.),



$FB = \frac{2}{9}a$: But $\overline{DC + FB} \times BC = \overline{\frac{2}{3}a + \frac{2}{9}a} \times \frac{8}{9}a = \frac{8}{9}a \times \frac{8}{9}a = 256$; whence, by extraction, $\frac{8}{9}a = 16$. And $a = 18 = AC$, and $DC = 12$, $BC = 16$, $FB = 4$; the area of the parabola $= \frac{4}{3}ab$; and of the trapezium $\frac{4}{3}b \times \frac{8}{9}a = \frac{32}{27}ab$. Hence every parabola is to the greatest inscribed trapezium, as $\frac{4}{3}$ to $\frac{32}{27}$, *i. e.* 9 to 8. *W.W.R.*

Mr.

Mr. James Hartley *solves it thus* :

'The ratio of the abscissa and semi-ordinate being as 3 to 2, the shortest side of the greatest inscribed trapezium will be the parameter, and its area will be equal to the square, whose side HG will be double the parameter. Put $2x = DC$, then $3x = AC$, and from the nature of the curve $\frac{DC^2}{AC} = FG = \frac{4x}{3}$. And $AB = \frac{x}{3}$; but $AC - AB = BC = \frac{8x}{3}$. And $DC + BG = DV = \frac{8x}{3} = \sqrt{256}$; whence $x = 6$: Consequently $DE = 24$, $FG = 8$, and $FD = GE = \sqrt{256 + 64} = 17\cdot8885$; and lastly, $AC = 18$. *W. W. R.*

The above solutions are short and elegant, as the proposer's solution; as is likewise the solution by Mr. *Cottam*, and those by Mr. *Charles Smith*, Mr. *Joseph Orchard*, Mr. *F. Holden*, and several others, which 'tis needless to publish, being of a species with the two solutions above, sufficient to satisfy the curious.

IX. QUEST. 355 *answered by Mr.* T. Cowper, *Teacher of the Mathematics, at* Wellingborough.

First, $\frac{6797\cdot4}{69\cdot5} = 97° 48'$, the distances of the two places, in the arch of a great circle, half of which $= 48° 54'$. By spherics, As sine ½ diff. long. $= 72° 7\frac{1}{2}'$: sine ½ the dist. $= 48° 54'$ :: rad. : sine $52° 24'$ = comp. lat. Hence the lat. $= 37° 39'$. Again, as radius : co-tang. lat. $(37° 39')$ :: cos. ½ diff. long. $(72° 7\frac{1}{2}')$ = sun's semi diurnal arch : tang. sun's declination $= 21° 42'$ almost; answering to the 29th of November and the 12th of January, also the 29th of May and the 14th of July, N. S. *W. W. R.*

Mr. *Peter Brooke* found the same, as did Mr. *John Williams*.

Mr. *F. Holden* solves it thus: $\frac{6707\cdot4 \text{ miles}}{69\cdot5} = 97° 48' 15''$, the dist. of the cities in degrees. If a perpendicular be let fall from the pole, it will bisect that dist. and also the diff. of long. Therefore, by opposite sides and angles, as sine ½ diff. long. $72° 7\frac{1}{2}'$: sine ½ dist. $48° 54\frac{1}{8}'$ :: rad. : cos. lat. $37° 38' 38''$. Now, the sun's semi diurnal arch = ½ diff. long. = $72° 7\frac{1}{2}'$. Therefore by right-angled spheric triangles, as tang. lat. : rad. :: cos. $72° 7\frac{1}{2}'$: tang. sun's declin. $= 21° 42'$.

But

But if the apparent time of rising to one city and setting to the other be required, Say, as sine ½ dist. cities 48° $54\frac{1}{2}'$: sine sun's refraction 33′ :: rad. : 43′ 48″, which being added to the above-found declin. gives 22° 25′ 48″ declin. at the time of rising and setting required.

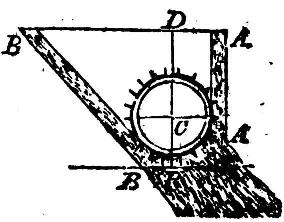
Mr. *James Hartley*, of *Yarum*, solves it in the very same manner, and determines the situation of the cities upon the globe thus:

1753 { 29th of May and 14th of July, the sun rises at Japan when he sets in Spain.
12th of January and 30th of November, the sun sets at Japan when he rises in Spain.

And vice versâ. With which Mr. *Richard Gibbons* agrees, and Mr. *Thomas Allen* nearly.

Mr. *Cottam* determines the same latitude, and days of the year, very near; and properly observes, that, in the question, there should have been expressed, *on what days*, instead of *on what day* of the year, &c. But, we observe, as the declination can seldom, if ever, correspond with the true time of sun-rising or setting in the required latitude, there is no propriety in this sort of questions, which only admit of answer near the truth. And, for the same reason, the 349th question (where the days of sun-rising at Petersburgh and Jerusalem, at the same instant, are required) contains the same geometrical absurdities; since the required declination is hardly ever possible to hit the true time of sun-rising in both those different latitudes; the declination being still variable every moment of time. And, if the sun's declination be supposed the same for the space of twenty-four hours, in this sort of questions, still the declination and time of sun-rising, on a particular day of the month, cannot exactly correspond, according to computation of the sun's place for that day, at noon, or time of rising, by astronomical tables.

X. QUESTION 356 *answered by Mr.* William Cottam, *at his Grace the Duke of Norfolk's*.



When the water is conveyed by the trough $AA = DC$, then 'tis called an over-shot mill, and the diameter of the wheel is thus found: Let $DP = 16$ feet $= a$, the distance of the fall; $CP = CA = x$. By mechanics $nx = n\sqrt{a - x}$, where $n =$ force of water at A; therefore

$x =$

$x = \sqrt{a + \frac{1}{4}} - \frac{1}{2} = 3.5313$, whence 7.06226 = diam. of the wheel required.

If the water be conveyed by the trough *BB*, then 'tis called an under-ſhot mill; and the greater the diameter of the wheel, the greater will be its force, and conſequently will have more force than the over-ſhot wheel.——And, on ſecond conſideration, I make the force of the water drawn into the radius of the over-ſhot mill a max. i.e. $nx\sqrt{a - x}$, a maximum; whoſe fluxion $\frac{2a\dot{x} - 3x\dot{x}}{2\sqrt{a-x}} = 0$, being reduced $x = \frac{2}{3}a = 10\frac{2}{3}$, and therefore $21\frac{1}{3}$ feet = wheel's diameter.

But Mr. *James Hartley*, of *Yarum*, ſolves it thus: Let *DP* = 16 feet = a; $16\frac{1}{12}$ feet = c; x'' = time of deſcent of the water from *D* to *C*; then $cx^2 = DC$, and $2cx$ = velocity at *C*. But $\frac{a - cx^2}{2}$ = ſemi-diam. of the wheel, which multiplied into the velocity is $= acx - c^2x^3$, and its fluxion made = 0, and reduced gives $\frac{a}{3} = cx^2 = DC = 5\frac{1}{3}$ feet, whence $CP = 10\frac{2}{3}$; though I don't ſee the neceſſity of the queriſt's mentioning "*a tun a minute.*"

Mr. *John Rickerby*, of *Wooburn, Bucks*, ſays, he has ſpent great part of his life among the beſt paper mills in the nation; and obſerves, that a ſwing wheel, which receives its force of water eight inches above its breaſt center, exceeds an over-ſhot wheel; provided the current of water and fall are alike: And ſays, though Mill-wrights differ in their opinions concerning the true pitch of the water-wheel, that this opinion of his own is true. He ſpeaks of a pen, to give the diſcharge of water the greater force, at that part of the fall where the water-wheel receives its impetus, or depreſſing force; and computes the diameter of ſuch a wheel to be 26 feet 8 inches; but on principles a little doubtful.

XI. QUES-

XI. QUESTION 357 *answered by Mr.* James Hartley, *of* Yarum.

Let AB be the length of the whole ſcale in the common barometer $= s$; while the mercury deſcends from A towards B, it will equally aſcend from D towards C; ſo that $DC = \frac{s}{2}$. Let M be the place of the water and mercury at a middle ſtate of the atmoſphere; then per queſt. $MH = MK = 5s$. Put $d =$ diam. greater tube, $y =$ diam. leſſer, and let $Ci = x$; then $\overline{\frac{s}{4}+x} \times d^2 = \overline{\frac{21s}{4}+x} \times y^2$; and, as mercury is about fourteen times as heavy as water, $14x \times d^2 = \overline{\frac{21s}{4}+x} \times y^2$; whence $\frac{s}{52} = x$. Now, if inſtead of x, in the firſt equation, we ſubſtitute its value, and aſſume $d = 1$, we get, by reduction, $y^2 = \frac{7}{137}$, whence $y = \cdot 243$; if therefore d be taken at pleaſure, it will be as $1 : \cdot 243 :: d : y$. *W. W. R.*

The Rev. Mr. *Baker*'s ſolution is thus: s to 1 the ſpecific gravity of mercury to that of any fluid in a leſſer tube; r to 1 the ratio of the tubes; v a given variation in the common barometer, and x the correſpondent variation in the leſſer tube. Then $r^2 : x :: 1^2 : \frac{x}{r^2} =$ variation at the upper ſurface C of the greater tube (being reciprocally as ſquares of the tubes diameters); the whole variation of the leſſer tube $= \frac{r^2x+x}{r^2}$; the variation of the mercury's ſurface at C, in the greater tube, the ſame with that of the water in the ſame place, is $\frac{x}{r^2} =$ variation at B, of the ſame diameter. The whole variation of the greater tube $BG = \frac{2x}{r^2}$. The preſſure of the mercury and water together upon the air at K is from the length of the tubes, the contained fluids in GLC and CmI being ſuſpended in equilibrio; the variation in the preſſure of the different columns depend on their weights

weights $\frac{r^2x+x}{r^2}$ and $\frac{2x}{r^2}$: Say, $s : 1 :: \frac{r^2x+x}{r^2} : \frac{r^2x+x}{r^2s} =$ weight of that column, in reſpect of a column of mercury of the ſame length; $\frac{2x}{r^2} - \frac{r^2x+x}{sr^2} = v$, the variation in the common barometer; whence $x = \frac{vr^2s}{2s-r^2-1}$, i.e. $x : v :: r^2s : 2s - r^2 - 1$; and per queſt. $10 : 1 :: 14r^2 : 28 - r^2 - 1$, (here $s = 14$ nearly) whence $r = 3{\cdot}3541$, and the diameters of the tubes are as 3·3541 to 1. *W. W. R.*

COR. I. If $s = 1, 2, 3, 4, 5, 6$, &c. and $r = \sqrt{\frac{1}{2}}, \sqrt{\frac{3}{3}}, \sqrt{\frac{5}{4}}, \sqrt{\frac{7}{5}}, \sqrt{\frac{9}{6}}, \sqrt{\frac{11}{7}}$, &c. correſpondent; or if $s = \frac{r^2+1}{2-r^2}$, or $r = \sqrt{\frac{2s-1}{s+1}}$, the variations in this barometer will equal thoſe in the common ſort.

COR. II. If $s = 1, 2, 3, 4$, &c. and $r = \sqrt{1}, \sqrt{3}, \sqrt{5}, \sqrt{7}$, &c. or if $s = \frac{r^2+1}{2}$, or $r = \sqrt{2s-1}$, the variation will be infinite. Hence,

COR. III. The ſcale of variation in this barometer may have any aſſignable ratio to the variation in the common barometer.

Mr. *J. Williams* ſays, that Mr. Rowning (in his Compendious Syſtem, p. 112) determines the ratio of the variation of x, in the leſſer tube, to the common ſcale by $x = \frac{vmd^2}{2m-s^2-1}$, whence $d = \sqrt{\frac{2mx-x}{vm+x}}$.

XII. QUESTION 358 *anſwered by* ΦΙΛΟ-משקד.

t being the tranſverſe, and c the conjugate diameters of the inner ellipſis of a ſolid elliptical ring, whoſe circumference is circular, of p inches diameter; then $t+p$ and $c+p$ will be the tranſverſe and conjugate diameters of the ellipſis paſſing through the middle of the ring, whoſe circumference is in the center of gravity; which circumference put $= a$; then, ſince the ſolid or ſurface generated is equal to the product of the generating plane, or circular line, reſpectively multiplied into the way made by the center of gravity, therefore $\cdot 7854ppa$ is the ſolidity, and $3{\cdot}1416pa$ the ſurface, of the elliptical ring, required.

The same answered by Mr. John Hartley, *of* Yarum.

If d be taken $= \frac{tt - cc}{tt}$, in the quest. and $m = 3 \cdot 1416$, also A, B, C, &c. = the next preceding term in the following series, then $1 - \frac{d}{2.2} - \frac{3d}{4.4}A - \frac{3.5d}{6.6}B - \frac{5.7d}{8.8}C - \frac{7.9d}{10.10}D$, &c. $\times \left\{ \begin{array}{l} m^2pt = \text{surface} \\ \frac{1}{4}m^2p^2t = \text{solidity} \end{array} \right\}$ of the ring.

N. B. The above series are taken from Mr. Emerson's excellent Doctrine of Fluxions, p. 174.

XIII. QUESTION 359 *answered by Mr.* John Honey, *of* Redruth, Cornwall.

Put a and $b = 42$ and 26 inches, the transverse and conjugate diameters of the hoop above; and c and $d = 48$ and 29, the dimensions of those below; also $m = 12$, the frustum of the ellipsoid's altitude; and $n = 18$ inches, the elliptical cylinder's altitude: Then, by a known theorem, $\frac{ab + cd + \sqrt{abcd}}{882 \cdot 36} \times m = 50 \cdot 54$ wine gallons, the content of the hoop's concavity; and $\frac{cdn}{294 \cdot 12} = 85 \cdot 18$ wine gallons, the content of the cylindrical concavity; whence the concavity of both $= 135 \cdot 72$ wine gallons.

Mr. *John Wigglesworth* answers it thus: Let $a = 42$ inches, $b = 26$ = transverse and conjugate diameters above; $t = 48$, $c = 29$ = transverse and conjugate diameters below; $h = 12$, $p = 18$, and $m = \cdot 2618$, then the content of the whole concavity $= \frac{mh \times \overline{ct + \frac{1}{2}bt + ab + \frac{1}{2}ac} + 3mtcp}{231} = 135 \cdot 7416$ wine gallons.

The solutions by Mr. *T. Cowper*, Mr. *J. Hartley*, Mr. *J. Adams*, and Mr. *Cottam*, agree the same; but they missed the secret of half Miss Polly's solidity, which being taken from the last-found concavity, leaves the concavity sought. Mr. *Adams* observes, the frustum of the ellipsoid may be reduced, with advantage, to the frustum of a cone, and the elliptic cylinder to a round cylinder.

The PRIZE QUESTION *answered by Mr.* F. Holden, *at* Westhouse, *near* Settle, Yorkshire.

Take a piece of wood or a stone, of a known superficies, and, dipping it into a vessel full of melted tallow, you may, by by trying the weight of the tallow and dipping-veſſel in a ſcale before and after dipping, know the quantity of tallow, in weight, taken up by the piece of wood or ſtone. Then take about 18 or 20 modern mathematicians, (the more the better) ſtrip them ſtark-naked, and ſuſpend them (like Abſaloms) by the hair of their heads, as chandlers hang their candles, or elſe by ſoft bandages under the chin and behind the nape of the neck, ſo that they may be raiſed or let down by pullies without hurting; dip them alſo, one by one, in the ſame veſſel where the wood or ſtone was lately dipped, and mark the tallow they all take up, by weighing the veſſel and tallow, before and after they are all dipped, (keeping the tallow juſt melted and of an equal warmth): Then ſay, as the quantity of tallow, in weight, taken up by the wood or ſtone, is to the known ſuperficies of either, ſo is the weight of tallow taken up by all the mathematicians to the ſuperficies of all the mathematicians. But, by all means, take care that they are kept naked till they are ſhivering, and almoſt as cold as the wood or ſtone itſelf, before they are dipped, elſe this proportion will not hold good.

When they are all dipped, well ſcoured with ſoap, and cleanſed from the tallow, let them be weighed, (or they may be all weighed before dipping) and ſay, as the weight of them all in pounds is to the late-found ſuperficies of them all in ſquare inches or ſquare feet, ſo is 160 pounds weight to the ſuperficies of the modern mathematician required to be known, (= 14½ ſquare feet, nearly, as we find by another method). *W. W. R.*

Paradoxes anſwered.

I. PARADOX *anſwered by Mr.* Richard Gibbons, *of* Plymouth.

The fork was as the ſteelyard, Roger's ſhoulder as the fulcrum, ſuſtaining the burthen, between the two powers, acting at both ends of the fork.

II. PARADOX *anſwered by Mr.* Edward Griffiths, *of* Elleſmere, Shropſhire, *and others.*

Caſe 1. A piece of pliable metal being doubled, by applying a round file to the doubled edge, and filing a half-ſquare gap, on opening the metal, a ſquare hole will appear.

Or, the ingenious Mr. *Gato* informs us, that, if two corners and an edge, at the end of a miſer's iron cheſt, be filed away, with a round or any other file, there may be an exact ſquare hole left.

Caſe

Case 2. A cylindrical body being cut obliquely, the plane of the section will be an oval; and, consequently, a round body, situated obliquely in an oval hole, will completely fill it. *Archimedes*.

The Eclipses calculated for 1753, *by* *Mr*. Cowper, *of* Wellingborough.

1. Moon rises eclipsed $4\frac{1}{4}$ digits, on Tuesday, April 17th, N. S. at 7h. 3m. apparent time in the evening, visible at London. End 7h. 47m.

2. Sun eclipsed, Thursday, May 3d, 7h. 40m. N. S. in the morning; invisible at London. But (according to Mr. *Hulse*) will be near 8 digits eclipsed on the south side ♉ in 13°; being vertical to the Arabian sea, lat. 16° south, long. 62° east; and visible to the inhabitants of Madagascar.

3. Moon eclipsed, Friday the 12th of October, 9h. 40m. morning, N. S. invisible at London. But (according to Mr. Hulse) eclipsed on the south side in ♈ 19° vertical to the great ocean, west of America, where it is partly visible to that quarter of the earth.

4. Sun eclipsed, Friday the 26th of October, N. S. visible. Beginning 8h. 35m. 5s. Middle 9h. 43m. 36s. Visible conjunction 9h. 44m. 34s. End 10h. 56m. 39s. Duration 2h. 21m. 34s. Digits on the sun's lower limb 8° 20'.

1. Eclipse calculated by		Beg. h. m.	Mid. h. m.	End h. m.	Dur. h. m.	Dig. ° '
Mr. Cowper, by Dr. Halley's corrected Tab.	London	5 27	6 37	7 47	2 20	5 2
	Wellingbor.	5 25	6 35	7 44		
		M.r.Lo.4°4' dig. at 7h. 3m. Ev.				
Mr. Man,	London	4 59	6 17	7 30	2 31	2 30
	Plymouth	4 43	6 1	7 14	2 31	2 27
	Constantinople	6 55	8 13	9 26	2 31	5 8
Mr. Hulse,	London	7 58	8 22	9 42	2 52	5 44

4. Eclipse calculated by		Beg. h. m.	Mid. h. m.	End h. m.	Dur. h. m.	Dig. ° '
Mr. Cowper, by Dr. Halley's corrected Tab.	London	8 35	9 $43\frac{1}{2}$	10 $56\frac{1}{2}$	2 $21\frac{1}{2}$	8 22
	Wellingb.				2 26	
Mr. Man,	London	8 42	9 57	11 8		8 15
	Plymouth,	8 30	9 47	10 59	2 29	8 29
	Lisbon	7 50	9 0	11 20	3 30	11 49
	Constantinople	11 24	12 46	2 8	2 44	7 45
Mr. Hulse,	London	8 30	9 48	10 30	2 26	8 36

There

There is a Transit of ☿ on May 6th, N. S. (Mr. *Hulse* says 5th). Beginning 2 h. 47 m. 24 s. morn. Ending 10 h. 43 m. 20 s. according to Mr. *Cowper*, from Dr. Halley's tables, being 2 hours sooner than by Leadbetter.

For proving Astronomical Tables.

The moon's eclipse, June 8th, O. S. appeared thus at Rome. Beginning 8 h. 2 m. 2 s. Immersion 9 h. 6 m. 10 s. Emers. 10 h. 35 m. 19 s. End 11 h. 39 m. 49 s. *Alexander Man.*

New Questions.

I. QUESTION 361, *by a Person of Honour.*

A water mill is to be built where there is a fall of water of 24 feet.—It is required to determine whether a wheel of 18 feet with 6 feet fall, or a wheel of 16 feet with 8 feet fall, will grind the most corn with least water?

II. QUESTION 362, *by Mr.* John Fish, *of* Dartford.

A ball weighing 4 pounds upon the surface of the earth, to what height, in the air, must it be carried to weigh but 3 pounds, and how long would it be in falling to the ground?

III. QUESTION 363, *by* Nichol Dixon, *of* Blackwell.

In Craven as I walk'd alone*,
Three objects once appear'd in stone,
I do protest I ne'er saw bigger.
And stood in right triang'lar figure.
These stones (as being told to me)
Go by the names of *A*, *B*, *C*;
From *C* to *A* I measur'd, then,
In English miles exactly ten.
From *A*, for *B*, due north I stride,
Till I the rising sun espy'd
Appearing in a line with *C*,
Directly, as I stopt at *D*:
And there old Bob (who came in sight)
Told me "the angle *C* was right;
"That three miles further on stood *B*,"
And said "that course was true for me:"

* Lat. 54°.

The time this happen'd, I may say,
Was on the 28th of May.
Now, without meas'ring, I don't doubt
But you'll the miles to *C* find out.
Ye, who to cards or dice pretend!
This problem solv'd, the game will end:
Tho. Simpson sent it first for fun,
Now, solve it you, some son of a gun!
For cards nor dice can Simpson charm,
Like old Sir John, that keeps him warm.

[*See the fig. to the solution*]

IV. QUESTION 364, *by Mr.* T. Cowper, *Teacher of Mathematics, at* Wellingborough.

On the 14th of last March, at half an hour after 11 in the forenoon, being in latitude 52° 22′, I observed 10 beats of my pulse between the time of a small cloud shading me, and that when the shadow thereof reached a tree, at the distance of 88 yards, easterly from me; immediately before, I likewise observed that the angle formed by the shadow of a stick perpendicular to the horizon, and a line drawn from the tree to the place of observation was 120°. Now, admitting 70 pulsations in a minute, the hourly velocity of the cloud, its direction, and what point of the compass the wind blew, are thence required.

V. QUESTION 365, *by Mr.* John Williams.

What pounds principal, being put out at its equal value per cent. at simple interest, for an equal number of years, will raise an interest equal to half the principal?

VI. QUESTION 366, *by the Rev. Mr.* Baker, *of* Stickney, Lincolnshire.

If the thickness of two microscopic glasses be three-eighths of their respective radii of convexity, and these be in the ratio of 10 to 3, how must those glasses be disposed, in a compound microscope, so that an object, eight inches distant from the eye, shall be thereby magnified a thousand times?

VII. QUESTION 367, *by Mr.* T. Cowper, *of* Wellingborough.

On the 19th of September, 1751, at night, the vertical angle between Jupiter and the star Castor was observed to be

35° 48′, and that between Jupiter and the bright ſtar in the whale's tail, 78° 29′; it is required to determine the latitude of the place and hour of the night, where and when thoſe obſervations were made?

VIII. QUESTION 368, *by Mr.* Chriſtopher Maſon, *of* Eaſtburn, *near* Petworth, *in* Suſſex.

A conſtant quantity being put for a factor, in Monſ. Ozanam's Mathematical Recreations, to be multiplied by variable factors, in order to produce 3 ones, 3 twos, 3 threes, &c. through all the digits, I deſire to know both the conſtant and variable factors, that will produce 6 ones, 6 twos, 6 threes, &c. alſo 9 ones, 9 twos, 9 threes, &c. through all the digits?

IX. QUESTION 369, *by Mr.* Terey, *of* Portſmouth.

Required the ſuperficial content of a ſcalenous cone, whoſe longeſt ſide is 12 inches, ſhorteſt 9, and baſe diameter 6 inches.

X. QUESTION 370, *by Mr.* Joſ. Hilditch, *at* Handal, *near* Shrewſbury.

The three diſtances from an oak, growing in an open plain, to the three viſible corners of a ſquare field, lying at ſome diſtance, are known to be 78; 59, 161, &c. and 78 poles, in ſucceſſive order: Quere the field's dimenſions, and the acres it contains?

XI. QUESTION 371, *by Mr.* Terey, *of* Portſmouth.

What is the content of a caſk, in ale gallons, whoſe ſtaves are exactly circular, and dimenſions of the head and bung diameters, and alſo length 24, 36, and 48 inches?

XII. QUESTION 372, *by Mr.* W. Bevil, *of* Harpſwell, Lincolnſhire.

Suppoſe the moon's diameter to be 2170, the earth's diameter 7970, and the diſtance of each other's ſurface 240000 miles, where muſt I view them on a line drawn betwixt their centers, to ſee the greateſt quantity of ſurface of both bodies poſſible?

XIII. QUES-

XIII. QUESTION 373, *by Mr.* John Williams, *of* Mold, *in* Flintshire.

Mathematicians take pains to describe curves and solids that never existed, yet say little or nothing to the properties of those things that are in nature; especially the sections, solidities, and curve superficies of the egg, which is one of nature's principal productions. If any of the problematic problemists would be pleased to give the solution of the quantity of curve superficies and solidity of the egg, when its axis is $2\frac{1}{2}$ inches, greatest ordinate $1\frac{1}{2}$, and the distance from that ordinate to the nearest end 1 inch, they would be intitled to a maximum of applause, instead of the minimum acquired, by puffing and cavilling about their superior dignity, who are odd fishes at foot-ball.

XIV. QUESTION 374, *by* Philotheoros.

My Lord Mayor's gold chain being 50 inches in measuring round it, at what distance must it be hung over two pins, horizontally fixed in a wall, covered with crimson damask, for a spectator to behold the most damask possible, within the circumference of the chain?

PRIZE QUESTION, *by Mr.* James Hartley, *of* Yarum.

Suppose *DAFE* to be a vessel in the form of a conical frustum, [*see the fig. to the solution*] whose top diameter $DA = 20$, bottom diameter $= 10$, and depth $= 15$ inches, suspended by an inflexible line $Ca = 100$ inches (the line and vessel being supposed of no weight) and the vessel full of water to weigh 100 pounds weight, when the vessel is perpendicularly suspended at *B*. Let the vessel be drawn aside by a cord fastened at (*a*) the bottom of the vessel, while at the other end of the cord is a weight *W* suspended, the cord freely sliding over the pulley at *P*, placed at such a distance from *C*, in the horizontal line *PC*, as to require the least weight, *W*, possible, to equipoize the vessel when the tension of the cord *Pa* is a maximum: Required the weight, *W*, and distance *PC*?

New

New Paradoxes.

PARADOX I. *by Mr.* Cottam, *at his Grace the Duke of Norfolk's.*

Where'er my Lord a journey takes,
Or to a friend a visit makes,
His nearest road south always lies,
(Directly south) which all surprize!
Remote the place, adjacent, either,
This side or that, no matter whether.
Hereof the reason make appear
In your fam'd Diary for next year.

PARADOX II. *by* John a Stiles, *Clerk of* H——n.

A man's lands, which he's supppofed to have purchased, devolved to his only daughter, and then to her eldest son (according to common descents) and the only son of the purchaser, who was begotten in lawful wedloc, as well as the daughter; yet he had nevertheless no manner of right to the inheritance after his decease: How could this be?

1754.

Questions answered.

I. QUESTION 361 *answered by Mr.* J. M.

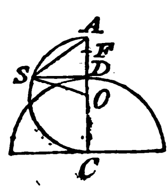
THE spaces passed through by falling bodies being as the squares of the acquired velocities, the velocity of the water falling 8 feet is to its velocity acquired in falling 6 feet, as $\sqrt{8}$ to $\sqrt{6}$. Therefore by the property of the lever, the force of the same water to turn a wheel of 16 feet, with a fall of 8 feet, is to its force to turn a wheel of 18 feet, with a fall of 6 feet, as $16\sqrt{8}$ to $18\sqrt{6}$, or as $8\sqrt{4}$ to $9\sqrt{3}$; that is, as $\sqrt{256}$ to $\sqrt{243}$. Whence, it is evident that the wheel of 16 feet diameter has the greatest advantage.

It was answered in like manner, and on the very same principles, by several other contributors. But these principles, though true in themselves, do not appear to us sufficient to give a right and full determination of the problem under consideration. To have the true quantity of the effect, not only the height of the fall, and the diameter of the wheel, but also the weight and force of the water in the wheel ought to be regarded; and consequently the different positions of the buckets with respect to the horizon.——As this is a subject of much importance, it is hoped our ingenious correspondents will think it worthy of a farther consideration, and communicate their thoughts thereon, for the benefit of the public, in our next Diary; to which we shall refer for a fuller discussion of this matter.

II. QUESTION 362 *answered by* Timothy Doodle.

Let CD be the earth's semi-diameter, and DA the required height from whence the ball must fall: Then $3 : 4 :: CD^2 : CA^2$; and consequently $CA = CD \times \sqrt{\frac{4}{3}} = 3980\sqrt{\frac{4}{3}} = 4596$ miles. Whence DA is given $= 616$ miles, or 3252480 feet.

Now the distance descended in the first second of time being always as the force, it will here be $= \frac{3}{4}$ of $16\frac{1}{12}$ feet $= 12\frac{1}{16}$ feet: And consequently the time of descent through AD, with the same force uniformly continued, $= \sqrt{\frac{3252480}{12\frac{1}{16}}} = 519$ seconds. But, supposing a semi-circle ASC to be described upon AC, and DS perpendicular to AC, the true time of descent through AD will be in proportion to 519 seconds, the time just now found, as half the sum of the sine DS and the arch AS is to the chord AS (as is proved by the writers on fluxions). Now $AC : CD$ $(:: \sqrt{4} : \sqrt{3}) :: 2$ (twice the radius of the tables) $: \sqrt{3} = 1\cdot732 =$ the versed sine of the angle $SOC = 137° 4'$. Whose supplement AOS is therefore given $= 42° 56'$: The natural sine of which will be $\cdot6811$, and the measure of the angle itself $= \cdot7494$; the half sum of these is $\cdot7152$: But the chord of $42° 56'$ $(2 \times$ sine $21° 28')$ is $\cdot7319$. Hence it will be $\cdot7319 : \cdot7152 :: 519 : 507$ seconds, or 8m. 27s. the true time required.



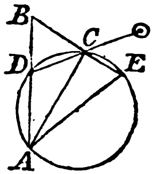
The same answer'd by Anthony Shallow, *Esq.*

If the earth's semi-diameter (CD) be denoted by a, it is plain that $a\sqrt{\frac{4}{3}} - a$ will express the required height AD from which the ball must fall. To determine therefore the time of the descent through AD, let the velocity of the ball, per second, acquired in falling through any distance AF ($= x$), be denoted by v; putting $c = AC$; and d ($= \frac{1}{2} \times 16.1$) $=$ the distance descended in the first second of time from A: Then, $2d$ being the measure of the velocity acquired in one second, with the force at A, it will be as $\overline{c-x}|^2 : c^2 :: 2d : \frac{2dcc}{\overline{c-x}|^2}$, the velocity generated, per second, by the force at F: Therefore $v : \frac{2dcc}{\overline{c-x}|^2} :: \dot{x} : \dot{v}$; and consequently $\frac{v\dot{v}}{2d} = \frac{cc\dot{x}}{\overline{c-x}|^2}$. Hence, by taking the fluent, $\frac{vv}{4d} = \frac{cc}{c-x} - c = \frac{cx}{c-x}$. Therefore $v = 2\sqrt{dc} \times \frac{\sqrt{x}}{\sqrt{c-x}}$, and consequently $\frac{\dot{x}}{v} = \frac{\dot{x}\sqrt{c-x}}{2\sqrt{cdx}} = \frac{1}{2\sqrt{cd}} \times \frac{c\dot{x} - x\dot{x}}{\sqrt{cx-xx}} =$ the fluxion of the required time. Whose fluent is $= \frac{1}{4}\sqrt{\frac{c}{d}}$ multiplied by the sum of the sine and arch whereof the corresponding versed sine is $\frac{2x}{c}$ (unity being the radius). But $\sqrt{\frac{c}{d}}$ (expressing the time of descent through AC, with an uniform force equal to that at D) is given $= 1418$ seconds. And $\frac{2x}{c}$ is $= 0.26795$; answering to an arch of $42° 56'$; whose length is $= 0.7494$; and that of its sine $= 0.6811$. Hence we have $\frac{0.6811 + 0.7494}{4} \times 1418 = 507$ seconds $= 8$ m. 27 s. *W. W. R.*

III. QUES-

III. QUESTION 363 *answered by Mr.* Rich. Gibbons.

Construction. From the latitude of the place and the sun's declination, the sun's horizontal azimuth BDC is given $= 48° 35'$. Having therefore made the angle $BD\odot = 48° 35'$, and $DB = 3$; let a square, whose side AC is 10, be formed, and let the angle thereof ACB be so moved along the line $D\odot$, that the end A, of the side CA, may, at the same time, pass over the line BDA, till the other side of the square passes through the given point B. In which position draw BC and AC, and the thing is done, as is evident by inspection.



Algebraic solution. Put $b = AC = 10$, $c = BD = 3$, and $s =$ sine of $BDC =$ sine ADC; also put $d = \frac{s}{b} = 0{\cdot}074992$, and $x =$ sine of ACD: Then is $\sqrt{1 - xx} =$ sine BCD; and we have $b : s :: x : dx = AD$; also $\sqrt{1-xx} : c :: s : \frac{cs}{\sqrt{1-xx}} = BC$. And, by 47 Euc. 1, $b^2 + \frac{c^2 s^2}{1 - xx} = c^2 + 2cdx + d^2x^2$. Reduced, $x = 0{\cdot}5517 =$ the sine of $33° 29'$. From whence $CD = 3{\cdot}4738$ miles.

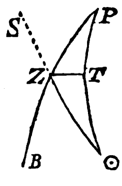
The same answered by Anthony Shallow, *Esq.*

Suppose the circumference of a circle to pass through A, D, and C, cutting BC produced in E. Then, AE being drawn, in the triangle ACE will be given (besides the right angle) the side AC and the angle $E = BDC = 48° 34' =$ the sun's horizontal azimuth from the north: Whence CE is given $= 8{\cdot}8265$; which put $= a$, making $AC = b$, $BD = c$, and $BC = x$: Therefore $AB = \sqrt{bb + xx}$; and, by the property of the circle, $c\sqrt{bb + xx} = x \times \overline{x + a}$. Hence $x = 2{\cdot}6966$; and consequently $DC = 3{\cdot}473$ miles.

It was also answered, in a curious manner, by Mr. *J. Ash*, Mr. *Sam Bamfield*, *Bathonius*, Mr. *W. Bevil*, Mr. *T. Coates*, Mr. *J. Hollingworth*, Mr. *J. Milbourn*, Mr. *B. Talbot*, and Mr. *J. Wigglesworth*.

IV. QUES-

IV. QUESTION 364 *answered by Mr.* John Wigglesworth.



Let P represent the north pole, Z the zenith, ZS the shadow of the stick, and ZT the direction of the cloud; then the angle $TZ\odot = 60°$; and, in the oblique spheric triangle $\odot ZP$, is given PZ the complement of latitude, $P\odot$ the complement of the sun's declination, with the contained angle P; whence the angle $\odot ZR$ is found $= 9° 42' 30''$; which added to $TZ\odot$, gives $TZR = 69° 42' 30''$. Therefore the wind blew W. b. N. $\frac{3}{4}$ N. nearly. Again, by allowing 70 pulsations to a minute, the shade of the cloud will move over a space = 88 yards in the time of 10 pulsations, or $8\frac{4}{7}$ seconds; which is at the rate of 21 miles per hour.

The same answered by Mr. James Robinson.

During 10 pulsations, we may suppose the sun does not sensibly change its place; and, considering the immense distance of the sun, when compared with that of a cloud, we may take the rays proceeding from the sun to the cloud, at both observations, to be parallel: Then, the sun's azimuth being (per spherics) 9° 26′ from the south, the direction of the cloud was E. b. S. $\frac{3}{4}$ S. fere; and the hourly velocity of the wind, blowing from the opposite point, 21 miles.

W. W. R.

Mr. *Ash*, *Bathonius*, Mr. *W. Bevil*, Mr. *T. Coates*, Mr. *Richard Gibbons*, and some others, also answered this problem, and agree, exactly, in the velocity of the cloud; but make some little difference in its direction, arising from having taken the sun's declination from different authors.

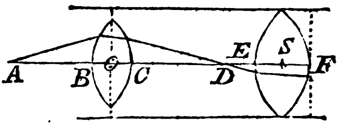
V. QUESTION 365 *answered by Mr.* J. Milbourn.

Let 100l. principal $= a$, and for the required principal, &c. put x; then $a : x :: x : \frac{xx}{a}$, the interest for one year; and as 1 (year) is to x (years) so is $\frac{x^2}{a}$ to $\frac{x^3}{a}$, the interest for x years: Whence (per quest.) $\frac{x^3}{a} = \frac{x}{2}$; reduced, $x = \sqrt{\frac{a}{2}}$ $= 7{\cdot}07106$ pounds $= 7$l. 1s. 5d.

In the very ſame manner it was anſwered by Mr. *J. Aſh*, Mr. *S. Bamfield*, Mr. *G. Dickinſon*, Mr. *J. Colinge*, Mr. *Gibbons*, Mr. *Holingworth*, Mr. *Jonah Milford*, Mr. *W. Newman*, Mr. *Joſ. Peil*, Mr. *D. Roberts*, Mr. *James Robinſon*, and Mr. *Ch. Tate*.

VI. QUESTION 366 *anſwered by* Κυβερνήτης.

Let the radius of convexity of the lens O, next the object A, be put $= a$, thickneſs $BC = b$, the radius of the eye-glaſs $S = d$, and its thickneſs $EF = e$; and let the ſine of incidence be to that of refraction, out of air into glaſs, as 1 to r; putting $q = 1 - r$, $t =$ the given linear amplification, and $x =$ the diſtance CD of the place of the image from the lens O.



Then, A being the place of the image of an object at D, by a known theorem in optics, AB will be $= \frac{rAQ}{qQ - a}$; Q being put $= b + \frac{x}{r - \frac{qx}{a}}$. And, by another known theorem (which is a corol. to the former) the principal focal diſtance ED of the lens S, will be $\frac{rd}{q} \times \frac{d - qe}{2d - qe}$.

But, by the queſtion, $b = \frac{3a}{8}$, $e = \frac{3d}{8}$, and $d = \frac{10a}{3}$.

Therefore, by the ſubſtitution of equal values, $Q = \frac{3a}{8} + \frac{x}{r - \frac{qx}{a}}$, and $DE = \frac{10ra}{3q} \times \frac{1 - \frac{3}{8}q}{2 - \frac{3}{8}q} = a \times \frac{10r}{3q} \times \frac{8 - 3q}{16 - 3q}$.

Whence DS $(= DE + ES) = a \times \frac{10r}{3q} \times \frac{8 - 3q}{16 - 3q} + \frac{5a}{8}$ $= ca$, by putting $c = \frac{10r}{3q} \times \frac{8 - 3q}{16 - 3q} + \frac{5}{8}$. Again, by the queſt. $\frac{OD}{OA} \times \frac{FA}{SD} = t$; that is, in ſpecies, $\frac{x + \frac{3a}{16}}{\frac{raQ}{qQ - 1} + \frac{3a}{16}} \times$

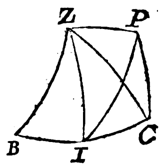
$ca +$

$\frac{ca + x + \frac{3a}{8} + \frac{ra\mathcal{Q}}{q\mathcal{Q} - a}}{ca} = t$. Put $\frac{x}{a} = z$, and $\frac{3}{8} + \frac{z}{r - qz} = y$; then will $\mathcal{Q}$ $(= \frac{3a}{8} + \frac{az}{r - qz}) = ay$; and our equation by ſubſtitution, &c. will be reduced to $\frac{z + \frac{3}{16}}{\frac{ry}{qy - 1} + \frac{3}{16}} \times \frac{c + \frac{3}{8} + z + \frac{ry}{qy - 1}}{c} = t$; or, $\overline{z + \frac{3}{16}} \times \overline{c + \frac{3}{8} + z + \frac{ry}{qy - 1}} = ct \times \overline{\frac{3}{16} + \frac{ry}{qy - 1}}$. From whence, and $\frac{3}{8} + \frac{z}{r - qz} = y$, the value of z, will be found by an equation of three dimenſions: And then, the value of FA being given in terms of a, by putting that value $= 8$ inches, a itſelf will be found, and from thence every thing elſe, required.

But as the finding of z this way (the terms being numerous) will be ſomewhat troubleſome, the known method of approximation, by trial-and-error, may be here uſed with advantage. According to which, having aſſumed for the value of z, that of y $(= \frac{3}{8} + \frac{z}{r - qz})$ will be immediately found; and then, by ſubſtituting theſe two values in the other equation, the error will be determined, &c. &c.

VII. QUESTION 367 *anſwered by Mr.* W. T—t.

Let P be the pole of the world, Z the zenith of the place, and B, I, C, the three ſtars: From the given longitudes and latitudes of which, or from their right aſcenſions and declinations, the diſtances BI and IC, and the angle BIC, may be found, by common trigonometry.



Aſſume the value of ZI as near as you can to its true value: Then, having two ſides and one angle, in each of the triangles BIZ, CIZ, the angles BIZ and CIZ may be found, and conſequently their ſum BIC. Mark how much this value of BIC differs from the given value of the ſame angle: Then make a ſecond aſſumption for ZI; and find, again, the value of the angle BIC, marking the error, as before.

 From

From these two errors a new value of IZ, by the known methods of approximation, may be found; and so on, till you arrive to what degree of exactness you please. Having thus determined ZI and ZIC, from the latter of these deduct PIC, the remainder gives the angle ZIP: From which, and the two given sides including it, both ZP and ZPI will become known.

VIII. QUESTION 368 *answered by Mr.* J. Robinson.

Let $999999 = a$ be the constant factor, in order to produce 6 ones, 6 twos, &c. The other factor call x, and the first product p; then $xa = p$, and consequently $x = \frac{p}{a}$.

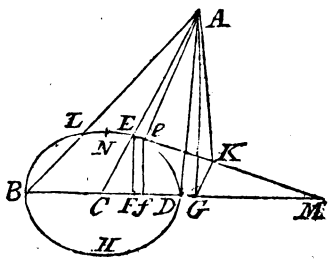
Universally, putting the constant factor (which is arbitrary) for the denominator, and the given product for the numerator, the fraction, or fractions, thence arising, will be the variable factor, or factors, required.

If any number of nines be taken for the denominator of a fraction, and the same number of any of the digits for a numerator, the fraction, when reduced to a decimal one, will have the very same figures as the numerator, repeated to infinity. Thus, for example, $\frac{1234}{9999}$ is $=$ ·1234123412341234, &c. ad infinitum. Thus also, $\frac{444444}{999999} =$ ·44444, &c. and consequently $999999 \times$ ·4444, &c. $= 444444$; and so of others. The variable factors, derived by this general method, are fractions; but there are particular answers to be had in whole numbers. Thus, because $\frac{111111}{3} = 37037$, and $\frac{111111111}{3} = 37037037$, it is evident that, if 3 be taken for the constant factor, the respective variable factors, to produce 6 and 9 ones, will be 37037 and 37037037. The multiples of which by 2, 3, 4, 5, &c. will consequently be the other variable factors required. In like manner, 37 being assumed for the constant factor, the variable ones will be 3003 and 3003003, together with their multiples.

IX. QUES-

IX. QUESTION 369 *answered by* Κυβερνήτης.

Let $ABHDEC$ be the given cone, and AG its perpendicular height: Let EM be a tangent to the circular base $BEDH$ at any point E; and, supposing e to be another point in the curve indefinitely near to E, let EA and eA be drawn in the surface of the cone: And from the same points E, e, upon the diameter BD, passing through G, let fall the perpendiculars EF and ef: Draw the radius CE, and the line GK parallel thereto, meeting the tangent EM, at right angles, in K; to which point from the vertex of the cone draw KA, which will be perpendicular to the tangent EM; because (being equal to $\sqrt{AG^2 + GK^2}$) it will be the least possible, in this position, where GK is the least possible.



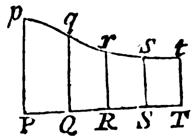
Put now $CE = a$, $CG = b$, $AG = c$, and $CF = x$, $BE = z$, and $Ee = \dot{z}$. Then, by the property of the circle, $CM = \frac{aa}{x}$: And, by similar triangles, $CM : CE :: MG$ $(\frac{aa}{x} - b) : GK = a - \frac{bx}{a}$. Whence $AK = \sqrt{AG^2 + GK^2}$ $= \sqrt{c^2 + a^2 - 2bx + \frac{bbxx}{aa}}$: Which, multiplied by $\frac{1}{2}\dot{z}$, gives $\frac{1}{2}\dot{z}\sqrt{cc + aa - 2bx + \frac{bbxx}{aa}}$ for the area of the triangle EAe, or the fluxion of the required superficies. Which fluxion, because $\dot{z} = \frac{a\dot{x}}{\sqrt{aa - xx}}$, is also $=$

$$\frac{\frac{1}{2}a\dot{x}\sqrt{cc + aa - 2bx + \frac{bbxx}{aa}}}{\sqrt{aa - xx}}$$

: Whose fluent will express the required superficies of the cone.

But as the finding of this fluent is extremely troublesome, and, when it is found, converges slowly (except where the cone is but little inclining) I ſhall therefore give the ſolution by a different, and more general method.

Let PT be a right line equal in length to the ſemi-circumference $BNED$; upon which, as a baſe, (or abſciſſa) ſuppoſe a curve $pqrst$ to be deſcribed, ſuch that, taking PS always $= BE$, the ordinate Ss ſhall be every-where equal to half the correſponding perpendicular AK: Then it is plain, that the area $PTtp$ of this curve, will be exactly equal to half the convex ſuperficies of the cone. To approximate which, conceive the axis PT of the curve and the ſemi-circumference $BNED$ of the circle, to be divided, each into four equal parts; and let the ſucceſſive values of AK ($= \sqrt{c^2 + \overline{a - \frac{bx}{a}}|^2}$) anſwering to the points of diviſion B, L, N, E, D (or P, Q, R, S, T) be computed, and repreſented by d, e, f, g, and h, reſpectively. Then, theſe values being the doubles of the correſponding ordinates Pp, Qq, Rr, Ss, Tt, it is evident, by the method of equidiſtant ordinates, that $\frac{7 \times \overline{d+h} + 32 \times \overline{e+g} + 12f}{90} \times \frac{1}{2}PT$, will expreſs the area of the curve, or half the ſuperficies of the cone, very nearly.



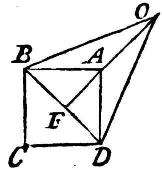
Now, in the caſe propoſed, AB being $= 12$, $AD = 9$, and $BD = 6$, we have $CE = 3 = a$; DG ($= \frac{AB^2 - AD^2 - BD^2}{2BD}$) $= \frac{9}{4}$; $CG = 5\frac{1}{4} = b$; and $AG^2 = 75{\cdot}9375 = c^2$. Here, therefore d ($= AB$) $= 12$; e ($= \sqrt{cc + aa + \frac{1}{2}bb + ab\sqrt{2}}$) $= 10{\cdot}9996$; f ($= \sqrt{cc + aa}$) $= 9{\cdot}2162$; g ($= \sqrt{cc + aa + \frac{1}{2}bb - ab\sqrt{2}}$) $= 8{\cdot}7432$; and h ($= AD$) $= 9$.

From whence ($\frac{7 \times \overline{d+h} + 32 \times \overline{e+g} + 12f}{90} \times NED$) the content of the whole, required, ſuperficies comes out 93·13 ſquare inches. By taking a greater number of ordinates, the anſwer may be brought out to any degree of exactneſs deſired, however great the inclination may be.

Mr. *Bamfield*, by an eaſy approximation, brings out nearly the ſame numbers with the above.

X. QUES-

X. QUESTION 370 *answered by Mr.* J. Robinson.



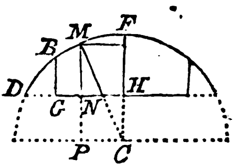
Let $a = OD = OB = 78$, $b = OA = 59{\cdot}161$, and $x = AF$. Then (per Eu. 47, 1) $bb + 2bx + 2xx = aa$. Reduced, $x = \sqrt{\frac{aa}{2} - \frac{bb}{4}} - \frac{b}{2}$: From whence the side of the square will be found = 24 poles, and the area of the field = 3 acres, 2 roods, and 16 perches.

The same answered by Master John Birks, *a Tyro at* Gosberton *School.*

The three visible corners of the field being represented by A, B, and D, it is evident, because the given distances of the two last from the oak, at O, are equal, that the line OAF bisects the angle BAD, and consequently that the angle $BAO = 135°$. From which, and the given sides AO and BO, the angle ABO is found (by trigonometry) $= 32°\ 26'$; $AOB = 12°\ 34'$; and the side of the square AB = 24 poles: Whence the area of the field = 3 acres, 2 roods, and 16 perches, which was required.

According to this last method it was also answered by Mr. *Richard Gibbons*, and Mr. *J. Milbourn*. Mr. *C. Tate*, Mr. *J. Wigglesworth*, Mr. *J. Ash*, and some others, solved it by different methods.

XI. QUESTION 371 *answered by Mr.* T. Coates, *Writing-master, at* Bristol.



Put r = the radius CF of the generating circle, which is found (by common properties) = 51; also put $c = CH = 33$, $DH = b$, $HN\ (= CP) = x$, and $MN = y$.

Then, universally, $y = \sqrt{rr - xx} - c$: Therefore $yy = rr - xx + cc - 2c\sqrt{rr - xx} = bb - xx - 2cy$; and $py^2\dot{x} = p \times \overline{bb\dot{x} - xx\dot{x} - 2yc\dot{x}} = \dot{S}$; whose fluent is $p \times \overline{bbx - \frac{1}{3}x^3 - 2c \times \text{area } MNHF} = S$: Which, when $x = 24$, gives the content 139·42 ale gallons.

The

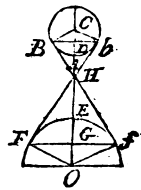
The same answered by Mr. John Wigglesworth.

Let $GH=$ half the length of the cask $=24=a$, $BG=12$ $=b$, $HF=18=c$; also put $r=$ rad. $CM=CF=\frac{aa+\overline{c-b}^2}{2c-2b}$ (by the nature of the circle) $m=PN=CH$, $p=3.14159$ &c. $x=HN=CP$, and $y=MN$: Then $y=\sqrt{rr-xx}-m$; and therefore $py^2\dot{x}=p\dot{x}\times\overline{r^2-x^2+m^2}-2mp\dot{x}\sqrt{rr-xx}$ = the fluxion of the solidity: Whose fluent $px\times\overline{r^2-\frac{1}{3}x^2+m^2}$ $-2mp\times$ area $CFMP=$ the solid described by the rotation of $HFMN$ about the line HN: And, if L be put for the length of the arc whose radius is r, and right sine x, then $\frac{rL+\quad x+xy}{2}=$ area $CFMP$. Whence the solidity of the whole cask, when $x=a$, and $y=b$, is $2p\times$ $\overline{ar^2-\frac{1}{3}a^3-mrL-abm}=39316.99$ cubic inches, or 139.4219 ale gallons.

Mr. *Richard Gibbons*, putting $x=12$, makes the content, by Shirtcliff's Gauging, (p. 201) $=0.080685x^3=139.4237$ ale gallons. Mr. *J. Milbourn*, by a different method, makes it 139.44; and Mr. *Charles Tate*, of Hull, 139.43.

XII. QUESTION 372 *answered by* Timothy Doodle.

Let O and C be the centers of the earth and moon, and H the place required: Suppose HF and HB to touch the two surfaces in F and B, and let FGf and BDb be perpendicular to OC.



Put $a=OE=3985$, $b=CA=1085$, $c=OC=245070$, and $x=OH$; and let $p=2\times3.14159$ &c. So shall the circumference FEf, &c. $=pa$, and the circumference BAb, &c. $=pb$: And therefore the parts FEf, BAb of the two surfaces visible to an eye at H, are equal to $pa\times EG$ and $pb\times AD$, respectively.

But, by similar triangles, $OH\,(x):OF\,(a)::OF\,(a):OG=\frac{aa}{x}$: Whence $EG=a-\frac{aa}{x}$: And, in the very same manner, $AD=b-\frac{bb}{c-x}$. Therefore, by substitution, FEf $+BAb=pa\times\overline{a-\frac{aa}{x}}+pb\times\overline{b-\frac{bb}{c-x}}$: Which being a maximum,

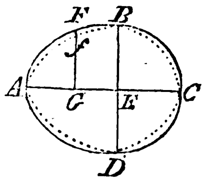
maximum, $\frac{a^3}{x} + \frac{b^3}{c-x}$ must be a minimum; and its fluxion $-\frac{a^3\dot{x}}{xx} + \frac{b^3\dot{x}}{\overline{c-x}|^2} = 0$. Hence $\frac{x^2}{a^3} = \frac{\overline{c-x}|^2}{b^3}$; or $\overline{\frac{b}{a}}|^{\frac{3}{2}} \times x = c - x$, and consequently $x = \frac{c}{1 + \overline{\frac{b}{a}}|^{\frac{3}{2}}} = 214585$. Therefore the place where friend *Bevil* must take his view is 210600 miles above the surface of the earth, if he can find his way up so high.

Mr. *Ash*, Mr. *Wigglesworth*, and Mr. *Bevil* the proposer, (proceeding upon the same principles) bring out the same conclusion.

XIII. QUESTION 373 *answered by* Anth. Shallow, *Esq.*

This question, in the form it is proposed, is indeterminate: The figure of the egg, as well as its principal dimensions, ought to have been given; since, of an infinity of curves that may be described through the same, given, points, experience is not sufficient to direct us which to choose: it not being known that ever two eggs were exactly of the same figure.

Let *AFBCD* be a section of the egg through its axis *AC*, and let *BD* be the given position of the greatest ordinate. It is visible that innumerable curves, *AFBCD*, *AfBCD*, &c. may be described thro' the given points *A*, *B*, *C*, and *D*, to cut *AC* and *BD* at right angles, conformable to the nature of the problem. But, the greatest ordinate *BD* dividing the axis *AC* unequally, no curve of a lower order than the second can possibly answer these conditions.



Let, therefore, *AFBC* be a curve of this order, whose equation is $yy = bx + cx^2 + dx^3$ (being the most simple the data will admit of): Also let $AC = p$, $AE = q$, $BE = r$: Then, by making $x = p$, and $y = 0$, our general equation becomes $bp + cp^2 + dp^3 = 0$, or $b + cp + dp^2 = 0$.

Also, by making $x = q$, and $y = r$, we have $bq + cq^2 + dq^3 = r^2$, or $b + cq + dq^2 = \frac{r^2}{q}$.

Lastly,

Lastly, by making $b\dot{x} + 2cx\dot{x} + 3dx^2\dot{x}$ (the fluxion of $bx + cx^2 + dx^3$) $= 0$, and writing q in the room of x, we have $b + 2cq + 3dq^2 = 0$.

Now, from the three equations thus derived, d is found $= -\frac{rr\times\overline{2q-p}}{q^2\times\overline{p-q}^{\,2}}$; $c = \frac{rr\times\overline{3qq-pp}}{q^2\times\overline{p-q}^{\,2}}$; and $b = \frac{rrpq\times\overline{2p-3q}}{q^2\times\overline{p-q}^{\,2}}$. Therefore the general equation, in known terms, is $yy = \frac{rr}{q^2\times\overline{p-q}^{\,2}}\times\overline{\overline{2p-3q}\times pqx+\overline{3qq-pp}\times xx-\overline{2q-p}\times x^3}$.

Whence, if a be put $= 3.14159$ &c. we get $ayy\dot{x} = \frac{arr}{q^2\times\overline{p-q}^{\,2}}\times\overline{\overline{2p-3q}\times pqx\dot{x}+\overline{3qq-pp}\times x^2\dot{x}-\overline{2q-p}\times x^3\dot{x}}$ for the fluxion of the solidity. Whose fluent, when $x = p$, will be found $= ar^2p^3\times\frac{6pq-6qq-pp}{12q^2\times\overline{p-q}^{\,2}}$, expressing the true content of the whole solid. Which therefore is to ($arrp$) that of the circumscribing cylinder, in the proportion of $p^2\times\frac{6pq-6qq-pp}{12q^2\times\overline{p-q}^{\,2}}$ to unity. This proportion, in the case proposed, (where $p = 2\frac{1}{2}$, $q = 1\frac{1}{4}$, and $r = \frac{3}{4}$) becomes as $\frac{275}{432}$ to unity. Therefore the solidity (according to the above assumption) comes out 2·8124 cubic inches.

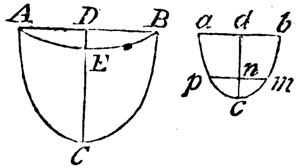
As to the superficies, or shell of the egg, it may be also found from the same general equation; but it is hoped the facetious proposer will himself determine that, and accept it as a proper reward for his trouble and industry in promoting useful science.

Some correspondents consider the egg as formed of two unequal semi-spheroids; but this does not seem to agree well with the true figure, it being hard to conceive that the curvature shall be immediately changed by more than one-half, in passing from one side of the greatest ordinate to the other.

XIV. QUES-

XIV. QUESTION 374 *answered by Mr.* W. Bevil.

Let acb be a curve similar to that (AGB) formed by the chain, such that its ray of curvature (a) at the lowest point c may $= 1$. Then, the area of the semi-curvilineal space, acd, will be truly defined by $y\sqrt{1+zz} - z$, (as is proved by the writers on fluxions) z being $= ca$, and y ($= ad$) $=$ hyp. log. $\overline{z + \sqrt{1+zz}}$.



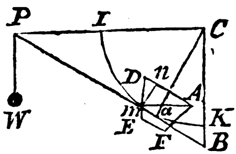
Hence, putting the length of the chain $= c$, we have (by the general property of similar figures) as $\overline{z+y}|^2$: $(\frac{1}{4}cc\,(\overline{AC+AD})|^2) :: y\sqrt{1+zz} - z : \frac{1}{4}cc \times \frac{y\sqrt{1+zz}-z}{\overline{z+y}|^2}$ $=$ area ACD: Which being a maximum, let its fluxion be therefore taken and made $= 0$; whence, after proper reduction, there will come out $\frac{1}{2}yz \times \frac{z+y}{1+\sqrt{1+zz}} =$ $y\sqrt{1+zz} - z$. From which equation (by the known methods of approximation) the values of z and y may be found. For, having assumed for z, y will be given from the equation $y =$ hyp. log. $\overline{z+\sqrt{1+zz}}$; and then, by substituting these values of z and y in the above equation, the error will be known; and from thence, by repeating the operation, &c. the true value of z; which comes out $= 5{\cdot}462$; and $y =$ $2{\cdot}399$. Then $7{\cdot}861\,(ac+ad) : 2{\cdot}399\,(ad) :: \frac{1}{2}c\,(AC+AD)$ $: AD = 0{\cdot}15259 \times c$: Whence $AB = 0{\cdot}30518 \times c = 15{\cdot}259$ inches.

Anthony Shallow, Esq; solves this problem exactly in the same manner. But Mr. *Timothy Doodle*, and Mr. *O'Cavanah*, taking the meaning of the question in a different sense [supposing the arch ACB, and not $ACB + AB$, to be given $= 50$] bring out $AB = 33{\cdot}575$ for the answer. In which case it appears that AD, CD, and AC, will be in the ratio of $0{\cdot}6715$, $0{\cdot}6656$, and 1, respectively; and that the area ACB is to the square of the arch ACB, as $0{\cdot}1549$ to unity.—— But there is yet another way in which the question may be taken, as it is not specified whether the chain is to be fastened to the pins, (in which case the area will be the greatest possible) or whether it is suffered to slide freely over them, till

till the two parts thereof (here repreſented by ACB and AEB) acquire an equilibrium. In this laſt ſenſe the ſolution will be ſtill more complex; but the beſt method will be to aſſume two arcs, $acb\,(=2Z)$ and $pcm\,(=2z)$, of the ſame given catenaria, ſimilar to the two parts ACB and AEB of the chain. From whence (by the properties of the catenaria, and the conſideration of ſimilar figures, and of the equal action of the two branches of the chain at B) there will be had $\frac{\sqrt{1+ZZ}}{\sqrt{1+zz}} = \frac{\text{hyp. log. } Z+\sqrt{1+ZZ}}{\text{hyp. log. } z+\sqrt{1+zz}}$; and $\frac{z\times\overline{1+ZZ}-Z\times\overline{1+zz}}{\overline{z\sqrt{1+ZZ}+Z\sqrt{1+zz}}|^{2}}$, a maximum. From which the values of z and Z may be determined.

The PRIZE QUESTION *anſwered by Mr.* J. Aſh.

'Tis plain that the leaſt weight will equipoize the veſſel in any degree of elevation when the cord Pa is perpendicular to Ca: Then, putting $s=$ ſolidity of the whole fruſtum, $c=\cdot7854$, $w=$ tenſion of the cord, $n=$ the ratio of Dn to mn, (Am being the horizontal ſurface of the water) and $x^2=$ diameter of the fruſtum at m, we have $\frac{a-x^2}{2}=Dn$,



$\frac{na+nx^2}{2}=mn$, $\frac{a+x^2}{2}=An$, and, per theorem Diary 1744, Queſt. 240, $\frac{ca^3-ca^{\frac{3}{2}}x^3}{2}\times\frac{n}{3}.=$ (becauſe $n=3$) $\frac{ca^3-ca^{\frac{3}{2}}x^3}{2}$, the ſolidity of the hoof $DnAm$; therefore $\frac{2s-ca^3}{2}+\frac{ca^{\frac{3}{2}}x^3}{2}=$ the ſolidity of the remaining water, which let $=qx^3-p$: Then (per ſtone) Ca : ſine of $Ka :: qx^3-p : w$; or, which is the ſame thing, (becauſe the triangles are ſimilar) $Am\left(\sqrt{\frac{10a^2-16ax^2+10x^4}{4}}\right) : mn\left(\frac{na-nx^2}{2}\right) :: qx^3-p : w = \frac{\overline{na-nx^2}\times\overline{qx^3-p}}{\sqrt{10a^2-16ax^2+10x^4}}$, the

the tension, a max. and consequently $\frac{\overline{a+x^2} \times \overline{qx^3-p}}{\sqrt{a^2-1{\cdot}6ax^2+x^4}}$, thrown into fluxions and properly reduced, gives $x^2 = 13{\cdot}6$ fere. Hence the weight required $= 24{\cdot}7054$, and $PC = 201{\cdot}57$.

The same answered by Mr. Patrick O'Cavanah.

Put $a = 15 =$ the depth of the frustum, $b = 20$ the greatest diameter AD, $c = 10 =$ the least diameter EF, $p = {\cdot}7854$, and $(2am) = x$. Then, supposing mn perpendicular to DA, it will be $b-c : b-x :: a : a \times \frac{b-x}{b-c} = mn$: Whence, by a well-known theorem for the content of a conical ungula, the solidity of ADm is $= \frac{1}{3}pab \times \frac{bb - x\sqrt{bx}}{b-c}$. Which, subtracted from $\frac{1}{3}pa \times \overline{bb+bc+cc}$, the content of the whole frustum $AFED$, leaves $\frac{1}{3}pa \times \frac{bx\sqrt{bx}-c^3}{b-c}$ for the part $AFEm$, in which the water is contained. Now the tension of the rope, or the force of the weight W, acting at right angles to Ca, so as to sustain the water in this position, is known to be in proportion to the weight of the water in the vessel, as the sine of the angle CPa (or DAm) to the radius; that is, as mn to Am, or, in species, as $a \times \frac{b-x}{b-c}$ to $\sqrt{\frac{a^2 \times \overline{b-x}|^2}{\overline{b-c}|^2} + \frac{\overline{b+x}|^2}{4}}$. Whence, by the question, $\frac{\overline{b-x} \times \overline{bx\sqrt{bx}-c^3}}{\sqrt{a^2 \times \overline{b-x}|^2 + \frac{1}{4} \times \overline{b-c}|^2 \times \overline{b+x}|^2}}$ (which is in a given ratio to $AFEm \times \frac{mn}{Am}$) must be a minimum. This expression, by putting $yy = bx$, $d = \frac{b-c}{2}$, and $f = \frac{aa-dd}{aa+dd} \times 2bb$, and dividing the denominator by $\sqrt{aa+dd}$, is reduced to $\frac{\overline{bb-yy} \times \overline{y^3-c^3}}{\sqrt{b^4-f^2y^2+y^4}}$. Which, in fluxions, &c. gives $\frac{3y}{y^3-c^3} - \frac{2}{bb-yy} + \frac{ff-2yy}{b^4-f^2y^2+y^4} = 0$. Hence $y = 16{\cdot}516$, $x = 13{\cdot}7578$, $CP = 202{\cdot}66$; and $W = 24{\cdot}709$ pounds.

Remark. As the taking of the fluxions of expressions compounded like that in the preceding solution, is somewhat trou-

troublesome, the following method may, in such cases, be of use.

Seeing $\frac{\overline{y^3-c^3}\times\overline{b^2-y^2}}{\sqrt{b^4-f^2y^2+y^4}}$ is to be a maximum, by the question, the logarithm thereof, or its equal, log. $\overline{y^3-c^3}$ + log. $\overline{b^2-y^2}$ − ½ log. $\overline{b^4-f^2y^2+y^4}$, must also be a maximum, and consequently its fluxion $\frac{3y^2\dot{y}}{y^3-c^3}-\frac{2y\dot{y}}{bb-yy}+\frac{ffy\dot{y}-2y^3\dot{y}}{b^4-f^2y^2+y^4}=0$: Whence, dividing by $y\dot{y}$, we have $\frac{3y}{y^3-c^3}-\frac{2}{bb-yy}+\frac{ff-2y^2}{b^4-f^2y^2+y^4}=0$, the same as before. Which equation stands in a much better form, for a solution, than that immediately resulting from the common method. In like sort, supposing

$$\frac{\overline{a^2-x^2}|^{\frac{1}{2}}\times\overline{b^3+x^3}|^{\frac{1}{3}}\times\overline{c^4-x^4}|^{\frac{1}{4}}}{\overline{a-x}|^2\times\overline{a^2+2dx+x^2}}$$ was to be a maximum,

or minimum, we should have ½ log. $\overline{a^2-x^2}$ + ⅓ log. $\overline{b^3+x^3}$ + ¼ log. $\overline{c^4-x^4}$ − 2 log. $\overline{a-x}$ − log. $\overline{a^2+2dx+x^2}$ a max. or min. And consequently $-\frac{x}{a^2-x^2}+\frac{x^2}{b^3+x^3}-\frac{x^3}{c^4-x^4}+\frac{2}{a-x}-\frac{2d+2x}{a^2+2dx+x^2}=0$. And so of the others.

No solution came in time this year to obtain the prize.

Paradoxes answered.

PARADOX I. *answered by Mr.* R Pearson.

Let Cottam tell his lord, he must
Dwell underneath the north pole just:
Then, let him visit age, or youth,
His course he'll surely steer full south.

PARADOX II. *answered by* Hodge, *the Miller.*

The man had his son and daughter by two several women; and the estate was settled on the daughter's mother and her heirs.——Mr. *J. Moreland* answers them both the same way.

The

The Eclipses calculated for 1754, *by* *Mr.* Ralph Hulse.

There will happen six eclipses this year; but, what is remarkable, not one of them will be visible to any part of Great Britain, or Ireland: At other places they will be seen according to the following order.

1. March 23d, at 6 afternoon, the ☉ is 2 digits eclipsed on the north side, vertical to a little sea, west of Terra Firma, lat. 8° north, long. 90° west. Visible in North America.

2. April 7th, at 4 in the morning, the ☽ is eclipsed totally 21 digits, vertical to the eastern borders of Peru, lat 6° south, long. 70° west. Visible to all America.

3. April 22d, at ten in the morning, the sun is 2 digits eclipsed on the south side, vertical to the eastern parts of Nigritia, in Africa, lat. 22° north, long. 30° east. Visible to the southern seas.

4. Sept. 16th, at 1 afternoon, the ☉ is eclipsed in ♍ 23°, towards the eastern ocean, beyond the Phillipine Islands, lat. 3° N. long. 163° E. This eclipse is still less than the former.

5. Oct. 1st, at 6 in the morning, the moon is totally eclipsed 21 digits, in ♈ 8°, visible to all America, vertical to the sea west of Panama, lat. 3° north, long. 90° west.

6. Oct. 16th, at 1 in the morning, the sun is eclipsed 2 digits on the north side in ♎ 22°, vertical to Medelzar, lat. 8° south, long. 130° west. Visible within the arctic circle.

As we do not know what tables Mr. *Hulse* made use of in these calculations, we cannot satisfy the public in that particular, nor take upon us to judge of their exactness, not having made any calculations of these eclipses ourselves, as they will be all invisible to us.

New Questions.

I. QUESTION 376, *by* Rusticus.

An honest man a horse did buy,
  That was both lame and poor:
A golden guinea was the price,
  And five good shillings more.
This horse he fed with corn and hay,
  Till he seem'd wond'rous sound:
When, meeting with another chap,
  He sold him for three pound.

By

By which he lost half the prime cost,
One-fourth o'th' keeping too.
What did the keeping stand him in?
What did he lose, say you?

II. Question 377, *by Miss* Maria A—t—s—n.

There are three cities, *A*, *B*, and *C*, lying in the same road; whereof the first is 136 miles distant from the second, and the second 104 miles distant from the third: From *A* to *B* a courier travelled in two days; and from *B* to *C* in two days more, diminishing his distance every day alike, from the first to the last. What number of miles did he travel each particular day.

III. Question 378, *by Mr.* Charles Tate.

My wife's a scold, a niggard, and a slut,
And ev'ry day she's sure to pay my scott;
And yet for what, no mortal e'er can tell,
Unless her courage rise from living well:
The which to tame, that I may live in quiet,
I am resolv'd henceforth to stint her diet,
In quantity, to what it was before,
As *e* to *a*; which, gentlemen, explore,
From the equations * that you see subjoin'd:
Else come and take my place—if you've a mind.

* Given $\begin{cases} \overline{aa+ee} \times \frac{e}{a} = b = 83{\cdot}2. \\ \overline{aa-ee} \times \frac{a}{e} = c = 1920. \end{cases}$

IV. Question 379, *by Mr.* John Morland.

Two persons, *A* and *B*, having an equal claim to an annuity of 100l. to continue for 30 years, agree to share it between them in this manner, viz. *A* for his part is to enjoy the whole annuity for the first 10 years; *B* and his heirs being to have the entire reversion thereof for the remaining 20 years. The question is, To find the rate of interest allowed in this contract, with the present value of the annuity corresponding.

V. Question 380, *by* W. T—t.

The sum of the squares of the two diagonals, of any trapezium, together with the square of twice the line joining their middle points, is equal to the sum of the squares of all the

the four sides of the trapezium. A demonstration of this is required.

VI. QUESTION 381, *by* Bathonius.

Two ships sail, at the same time, from two ports under the same meridian, whose difference of latitude is 1° 25′. That from the southermost port runs due east at the rate of 4½ miles per hour; and that from the northermost E. S. E. at the rate of 7 miles per hour: I demand the distance sailed by each ship, when they are at their nearest distance from each other, and also what the distance will be.

VII. QUESTION 382, *by Mr.* Thomas Moss.

To determine the least triangle that can be circumscribed about a given triangle, whereof the three sides are 8, 10, and 12 inches.

VIII. QUESTION 383, *by* Anthony Shallow, *Esq.*

To draw a right line parallel to a given line, which may cut three other lines given by position, in such sort, that the rectangle under the two parts thereof, intercepted by those lines, may be given in magnitude.

IX. QUESTION 384, *by Mr.* Tho. Moss.

Sailing due north, at the rate of 4 knots, in a current, a certain small island bore E. N. E. from us, at the distance of 40 miles: After running 12 miles (by the log.) it bore due east; and having run 16 miles more, upon the same course, its bearing was then found to be S. E. To determine, from these observations, the direction and velocity of the current.

X. QUESTION 385, *by* W T—t.

The vertical angle of a triangle being = 70°, and the sum of the two including sides = 100 feet; to determine the triangle itself, when the perpendicular is a mean proportional between the whole base and one of its two segments.

XI. QUESTION 386, *by Mr.* Timothy Doodle.

Within a rectangular garden, containing just an acre of ground, I have a circular fountain, whose circumference is 28, 40, 52, and 60 yards distant from the four angles of the garden. From these dimensions the length and breadth of the garden, and likewise the diameter of the fountain, are required.

XII. QUESTION 387, *by Mr.* Patrick O'Cavanah, *of* Dublin.

In the latitude of 51° 32′ north ſtand two pillars S.W. and N.E. of one another, at the diſtance of 200 feet: The height of the ſouthermoſt pillar is 60 feet, and that of the northermoſt 40 feet. At what time of the day, on June 20, do the ſhadows of their ſummits approach the neareſt to each other?

XIII. QUESTION 388, *by Mr.* Timothy Doodle.

Suppoſing p, q, r, s, t, &c. to repreſent the tangents of any number of arcs P, Q, R, S, T, &c. equal, or unequal: To determine a general expreſſion for the tangent of the ſum $(P+Q+R+S+T+$ &c.$)$ of all thoſe arcs; the common radius being unity.

XIV. QUESTION 389 *by Mr.* E. R——n.

To determine the ratio of the denſities of the ſun and earth, independent of the ſun's parallax.

XV. QUESTION 390, *by* Anthony Shallow, *Eſq.*

Having given any three computed viſible latitudes of the moon, in a ſolar eclipſe, together with the correſponding differences of longitude of the ſun and moon: To ſhew the manner of finding, from thence, the true time of the greateſt obſcuration, and likewiſe the neareſt approach of the two centers.

The PRIZE QUESTION, *by* Anthony Shallow, *Eſq.*

To determine the figure which the piers (or the ſtarlings) of a bridge ought to have, ſo that the length, and greateſt breadth of each, and their diſtances from one another, being given, the water in its paſſage through the bridge ſhall ſuffer the leaſt reſiſtance poſſible.

N.B. The perſon who gives the beſt ſolution to this queſtion will be intitled to a prize of ſix Diaries: And whoever truly anſwers it before Candlemas-day, will have a chance, by lot, to win the ſame number of Diaries.

Queſtions

1755.

Questions answered.

I. Question 376 *answered by Mr.* W. Lisson.

THREE pounds, two shillings, and eight-pence, the keeping it did cost;
One pound, eight shillings, and eight-pence, was what the poor man lost:

The same answered by Mr. Edward Gallyatt.

Put $4x$ = keeping, $2a = 26$, and $b = 60$: Then, per quest. $4x + 2a - b = a + x$; whence $x = \frac{b-a}{3} = 15$ s. 8 d. Consequently $4x = 3$ l. 2 s. 8 d. = the charge of keeping, and $a + x = 1$ l. 8 s. 8 d. = the money lost.

The same answered by Mr. Phil. Williams.

Let $a = 26$, $b = 60$, and x = the keeping; then $a + x - b = \frac{a}{2} + \frac{x}{4}$ (per quest.) whence $x = \frac{4b - 2a}{3} = 62\frac{2}{3}$.

The corn and hay appear from hence
To cost three pounds, two and eight-pence;
The money that the good man lost,
Was eight groats more than the prime cost.

Answers to this question were likewise received from Mr. *G. Brownbridge*, Mr. *T. Barker*, Mr. *W. Beer*, Mr. *T. Beston*, Mr. *R. Butler*, Mr. *T. Eston*, Mr. *W. Gawthorpe*, Mr. *E. Griffiths*, Mr. *G. Hicks*, Mr. *Samuel Koit*, Mr. *T. Lover*, Mr. *T. Padison*, Mr. *T. Prichard*, Mr. *William Richardson*, Mr. *T. Scholar*, Mr. *Benj. Thearle*, Mr. *Ja. Vacary*, Mr. *R. Younge*, and several others.

II. Question 377 *answered by Mr.* Samuel Koit.

Let $2a = 136$, $2b = 104$, and $2x$ = the common difference of each day's journey; then $a + x$, $a - x$, $b + x$, and $b - x$ will be the respective distances travelled each day: But the first + the third = twice the second, that is, $a + b + 2x = 2a - 2x$; whence $4x = a - b$, and $x = \frac{a-b}{4} = 4$: Therefore 72, 64, 56, and 48, are the four distances required.

The

The same answered by Mr. W. Gawthorpe.

Put $x =$ the first day's journey, and $y =$ the common difference: Then $2x - y = 136$, and $2x - 5y = 104$ (per quest.); and, by subtracting the latter equation from the former $4y = 32$: Whence $y = 8$, and $x = \frac{136 + 8}{2} = 72$.

According to the one, or the other, of the above methods, it was likewise answered by Mr. *R. Butler*, Mr. *G. Brownbridge*, Mr. *T. Barker*, Mr. *T. Boston*, Mr. *W. Beer*, Mr. *T. Drury*, Mr. *T. Eadon*, Mr. *T. Estob*, Mr. *E. Gillyatt*, Mr. *E. Griffiths*, Mr. *Ja. Giles*, Mr. *E. Johnson*, Mr. *Alex. Rowe*, Mr. *J. Richardson*, Mr. *G. Reed*, Mr. *T. Scholar*, Mr. *Ja. Vicary*. Mr. *Phil. Williams*, Mr. *R. Younge*, and many others.

III. QUESTION 378 *answered by* J. Milbourn.

Multiplying the given equations crosswise into each other, we have $\overline{aa - ee} \times \frac{ba}{e} = \overline{aa + ee} \times \frac{ce}{a}$; whence $a^4 - \frac{b + c}{b} ee \times aa = \frac{ce^4}{b}$; and, by compleating the square, &c. $aa = ee \times \frac{b + c + \sqrt{bb + 6bc + cc}}{2b} = ee \times 25$; consequently, $a = 5e$: Whence, by substitution, &c. $e = 4$, and $a = 20$.

The same answered by Mr. Tho. Todd.

Put $\sqrt{xy} = a$, and $\sqrt{\frac{x}{y}} = e$; then $\frac{1}{y} = \frac{e}{a}$, and $y = \frac{a}{e}$; whence, by substitution, $\overline{xy + \frac{x}{y}} \times \frac{1}{y} = b$, and $\overline{xy - \frac{x}{y}} \times y = c$: From the first of which we get $yy = \frac{x}{b - x} = \frac{c + x}{x}$, by the second. Hence $x = \sqrt{\frac{bc}{2} + \frac{\overline{c - b}|^2}{16}} + \frac{b - c}{4} = 4$. Therefore $y = 5$, $a = 20$, and $e = 4$.

The same answered by Mr. W. Enefer.

Let $re = a$; then, by substitution, $\overline{rree + ee} \times \frac{1}{r} = b$, and $\overline{rree - ee} \times r = c$: By multiplying these equations crosswise

crosswise, we have $br^4 - br^2 - cr^2 = c$, or (putting $1 + \frac{c}{b} = 2m$, and $\frac{c}{b} = n$) $r^4 - 2mr^2 = n$; whence $r = \sqrt{m + \sqrt{n + mm}} = 5$; and from thence $e \left(= \sqrt{\frac{br}{rr+1}}\right) = 4$, and $a (= re) = 20$.

The same answered by Sylvius.

Let $\frac{a}{e} = x$; then will $\frac{e}{a} = \frac{1}{x}$, and the given equations will become $\overline{eexx + ee} \times \frac{1}{x} = b$, and $\overline{eexx - ee} \times x = c$; therefore $ee = \frac{bx}{xx+1} = \frac{c}{x^3 - x}$; whence $bx^4 - bx^2 = cx^2 + c$; and, by reduction, $x = \left(\frac{c}{b} + \frac{\overline{b+c}|^2}{4bb} + \frac{b+c}{2b}\right)^{\frac{1}{2}} = 5$; hence $a = 20$, and $e = 4$.

Mr. *J. Ash*, Mr. *T. Barker*, Mr. *W. Bevil*, Mr. *T. Boston*, Mr. *A. Brooke*, Mr. *Hugh Brown*, Mr. *G. Brownbridge*, Mr. *R. Butler*, Mr. *W. Cottam*, Mr. *J. Eadon*, Mr. *E. Gillyatt*, Mr. *Ja. Hemingway*, Mr. *E. Johnson*, Mr. *Wm. Kingston*, Mr. *T. Moss*, Mr. *J. Nichols*, Mr. *G. Reed*, Mr. *W. Richardson*, Mr. *Alex. Rowe*, Mr. *W. Trott*, Mr. *Harland Widd*, and Mr. *R. Younge*, likewise answered the same in a neat, concise manner, by equations not exceeding a quadratic.

IV. QUESTION 379 *answered by Mr.* W. Kingston.

Let x = the rate of interest, and $u = 100$: Then per Ward's theorem, we have $\frac{u}{x-1} - \frac{u}{x^{10} \times \overline{x-1}}$ = present worth for 10 years, and $\frac{u}{x-1} - \frac{u}{x^{30} \times \overline{x-1}}$ = present worth for 30 years; hence $\frac{x^{30}-1}{x^{30}} = 2 \times \frac{x^{10}-1}{x^{10}}$, or $2x^{20} - x^{30} = 1$; which, solved, gives $x = 1{\cdot}049298$; from which the required value of the annuity comes out 1594l. 13s.

The same answered by Mr. Hugh Brown.

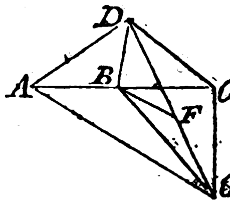
Let R be the amount of 1l. in one year; then, by the question and the doctrine of annuities, we have $2 \times \frac{1 - R^{\cdot 10}}{R - 1}$ =

$= \frac{1 - R^{30}}{R - 1}$; $\therefore R^{30} - 2R^{20} + 1 = 0$; which, divided by $R^{10} - 1$, gives $R^{20} - R^{10} - 1 = 0$: Whence $R = \sqrt[10]{\frac{1+\sqrt{5}}{2}}$ $= 1.049297$, &c. and the present value sought $= 1594$ l. 13 s. $0\frac{1}{2}$d.

Answers to this question were likewise received from Mr. *J. Ash*, Mr. *A. Brooke*, Mr. *S. Bamfield*, Mr. *W. Bevil*, Mr. *G. Brownbridge*, Mr. *R. Butler*, Mr. *W. Cottam*, Mr. *R. Gibbons*, Mr. *T. Moss*, Mr. *Ja. Robinson*, *Sylvius*, and Mr. *Harland Wild*.

V. QUESTION 380 *answered by Mr.* J. Randles, *Teacher of Mathematics at* Wem, *in* Shropshire.

Let the two diagonals AC and DG be bisected in B and F: Then, by the 12th of 2d of Simpson's Geometry, $AD^2 + CD^2 = 2AB^2 + 2BD^2$, and $AG^2 + GC^2 = 2AB^2 + 2BG^2$, and consequently $AD^2 + CD^2 + AG^2 + GC^2 = 4AB^2 + 2BD^2 + 2BG^2$; but DG by hyp. is also bisected by BF, and so $BD^2 + BG^2 = 2DF^2 + 2BF^2$ or $2BD^2 + 2BG^2 = 4DF^2 + 4BF^2$; whence, by equal substitution, $AD^2 + CD^2 + AG^2 + GC^2$ $(= 4AB^2 + 4DF^2 + 4BF^2) = AC^2 + DG^2 + 4BF^2$. *Q.E.D.*



In the very same manner it was demonstrated by Mr. *Brown*, Mr. *Moss*, Mr. *Watson*, and Mr. *Younge*.——Mr. *Cottam* and Mr. *Emesfer* proved it by different methods.

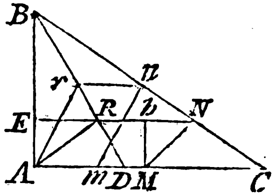
VI. QUESTION 381 *answered by Mr.* J. Ash.

Let A and B represent the two ports, a the distance between them $= 85$ miles, m and n the sine and cosine of the angle EBN, $q =$ the given ratio, and $x = BN =$ the distance sailed by one of the ships; then $mx = EN$, $nx = EB$, and $qx = AM$; whence $a - nx = AE = hM$, and $mx - qx$ (which let $= px$) $= hN$, and then $ppxx + aa - 2nax + nnxx = MN^2$, a minimum; which, put in fluxions and reduced, gives $x = \frac{an}{pp + nn} = 144.3$; whence $AM = 92.764$, and MN (the nearest distance) $= 50.371$.

The

The same answered by Mr. R. Younge, *Teacher of Mathematics in* Chester.

CONSTRUCTION. Supposing the two courses to intersect in C, take CD to CB in the given ratio of the celerity in AC to that in BC; and having drawn BD, make AR perpendicular thereto; make also RN parallel to AC, and NM parallel to AR, so shall M and N represent the required places of the two ships when they are the nearest possible to each other.



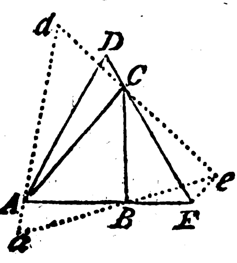
DEMONSTRATION. From any point r in BD, draw rA and rn, the latter parallel to AC; also draw nm parallel to Ar. By construction and similar triangles, $Bn : nr\ (Am) :: BC : CD ::$ the velocity in BC : velocity in AC; whence it is evident, that n and m are always contemporary positions: But Ar, or its equal mn, is, evidently, the least possible when Ar coincides with AR, or when $mn = MN = AR$. Q.E.D.

CALCULATION. As $7 + 4\frac{1}{2} : 7 - 4\frac{1}{2} ::$ co-tang. $\frac{1}{2}C$ (11° 15′) : tang. $\frac{CDB - CBD}{2} = 47° 32'$; whence $CBD = 31° 13'$, $ABR = 36° 17'$, $AR\ (= MN) = 50.302$, $RN\ (= AM) = 92.79$, and $BN = 144.34$. *W.W.R.*

Answers to the same were likewise received from Mr. *S. Bamfield*, Mr. *W. Bevil*, Mr. *G. Brownbridge*, Mr. *R. Butler*, Mr. *T. Cottam*, Mr. *W. Enefer*, Mr. *R. Gibbons*, Mr. *E. Gallyatt*, Mr. *E. Johnson*, Mr. *W. Kingston*, Mr. *J. Nichols*, Mr. *Ben. Thearle*, and several others.

VII. QUEST. 382 *answered by the Proposer, Mr.* Moss.

It is evident, that one side AE of the required triangle AED must fall upon, or coincide with, one side AB of the given triangle ABC; for, if another equilateral triangle aed be described about ABC, near to the former, the side thereof (ae) will be greater than the side AE of the former, because both the angles aAB and BEe being obtuse, Ba will be greater than BA, and Be greater than BE, and consequently ae ($Ba + Be$) greater than AE ($BA + BE$).



Now,

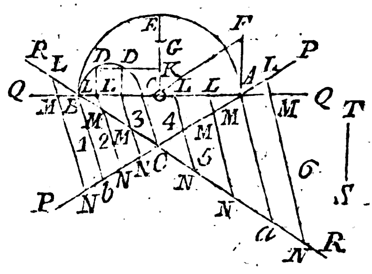
Now, all the three sides of the triangle ABC being given, the angle BAC may be found: Then it will be as the s. E (60°) : AC (12) :: s. ACE (120° — BAC) : AE; which quantity (as the two first terms are given) will be the least when 120° — BAC is the least, that is, when BAC is that angle of the given triangle which is nearest (below) the angle of the equilateral one. In the present case, the angle BAC (opposite to the mean side) being 56° 46′, AE will be found 12·48, and the area of the triangle 67·43 square inches. *W. W. R.*

Note, An equilateral triangle, constituted on the longest of the three given lines as a base, will be less than that above determined; but cannot be said to be described about the given triangle (as the question requires) since all the angles of the latter are not situate in the sides of the former.

This question being wrong printed (by omission of the word *equilateral*) no answer to it, besides the proposer's, has been received.

VIII. QUESTION 383 *answered by Mr.* H. Watson.

Let B, A, and C be the points of intersection of the three given lines QBQ, RCR, and PAP; and let Cc be the line to which the required line LMN is to be parallel; moreover, let ST be the side of a square, equal in area to the rectangle which is to be contained under the parts LM, MN, of the said line.



It is evident, that the problem (unless it be restrained by giving ST too large) will admit of six answers, or different positions of the line LN: But the determination of all these depends upon two cases only.

Construction of Case 1. (No. 1.) Upon AB let a semicircle be described, and at c erect the perpendicular cE, meeting the circumference in E; make AF also perpendicular to BA and equal to a fourth proportional to Cc, cE, and ST; and, from F to the center O, draw OF; take $OM = OF$, and through M draw LN parallel to Cc, and the thing is done.

Demon-

Demonstration. It is evident, (Euc. 6. 2.) that $AM \times BM$ $(= OM^2 - OA^2 = OF^2 - OA^2) = AF^2$: And, by similar triangles, $\begin{matrix} ML : BM :: Cc : Bc \\ MN : AM :: Cc : Ac \end{matrix}$; therefore $ML \times MN$: $AM \times BM$ (AF^2) :: Cc^2 : $Bc \times Ac$ (cE^2). But (by construction) $Cc^2 : cE^2 :: ST^2 : AF^2$; whence, by equality, $ML \times MN : AF^2 :: ST^2 : AF^2$, and consequently $ML \times MN = ST^2$. *Q.E.D.*

Construction of Case 2. (No. 2 and 3.) Draw Bb parallel to Cc, meeting PAP in b; upon Bc let a semicircle be described; and in cE, perpendicular to Bc, take cG a mean proportional between Bb and Cc, and take cK a fourth proportional to cG, Bc, and ST; draw KD parallel to BA, cutting the circumference of the circle in D, D; also draw DL perpendicular to BA, and then draw LMN parallel to Cc.

Demonstration. By similar triangles, $\begin{matrix} LM : BL :: cC : Bc \\ MN : cL :: Bb : Bc \end{matrix}$; therefore $LM \times MN : BL \times cL$ $(LD^2) :: cC \times Bb$ (cG^2) : Bc^2. But (by constr.) $cG^2 : Bc^2 :: ST^2 : cK^2$ (LD^2); whence, by equality, $LM \times MN : LD^2 :: ST^2 : LD^2$; and consequently $LM \times MN = ST^2$. *Q.E.D.*

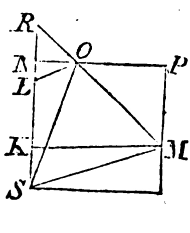
The sixth position of LN has the very same construction with the first; and the fourth and fifth are determined like the second and third: But these four, when ST exceeds a certain limit, are impossible.——Besides the two cases above, there are other particular ones; such as, when all the three lines meet in a point, or when two of them are parallel, &c. But these, being much easier than the above, need not, I presume, be insisted on.

Mr *Moss* sent a construction of this problem, very little different from the above.

*** *In the fig. for OE read* cE.

IX. QUESTION 384 *answered by Mr.* Rich. Gibbons.

Construction. By the first bearing and distance lay down the right-angled triangle SMK, so that M may represent the island; and S the ship, which, at the second observation, must be in the east and west line KM: Produce SK to R, so that $KR = KM$; and draw MR, in which the ship must be at the last observation. Moreover, as the given distances are in the ratio of 3 to 4, make $KN = \frac{3}{4}$ of SK; and draw NO parallel to KM, meeting RM in O, which is the ship's true place. From



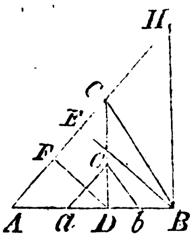
this construction (taking $SL = 28$) the distance LO, run by the current in seven hours, is found to be 18·25, and its direction $(NLO) = 64° 58'$.

The same answered by Mr. S. Bamfield.

Let S be the place of the ship at the first observation, SN the meridian, M the island, and MK a perpendicular to SN, &c. Then (per trig.) $MK (= NP) = 36·95$, and $SK = 15·31$; which, per log is but 12; say therefore $12 : 15·31 :: 16 : 20·413 = KN = MP = PO$ (because the angle $POM = 45°$). Hence $SN = 35·723$, LN $(= 35·723 - 28) = 7·723$, and $ON (= NP - OP) = 16·54$. Hence, per trig. $SO = 39·36$ = the true distance sailed, $NLO = 64° 59'$ = the current's direction, and $LO = 18·258$ = the distance run by the current in seven hours, being at the rate of 2·608 miles per hour. *W. W. R.*

In the same manner it was answered by Mess. *Ash*, *Brown*, *Brownbridge*, *Enefer*, *Kingston*, *Milbourn*, *Moss*, *Peart*, *Robinson*, *Thearle*, and *Widd*.

X. QUESTION 385 *answered by Mr.* Harland Widd, *of* Whitby.



Let c = tang. $70° = ACB$, $y = AB$, and $x = AD$; then, by the question, $\sqrt{xy} = CD$; and (per trig.) $\sqrt{xy} : 1 :: x : \frac{x}{\sqrt{xy}}$ = tangent ACD; also $\sqrt{xy} : 1 :: y - x : \frac{y-x}{\sqrt{xy}}$ = tang. BCD: Whence, by the known theor. for the tangent of the sum of two angles, we have $\frac{yy}{x\sqrt{xy}} = c$; and therefore $y = x\sqrt[3]{cc} = rx$. Again, (per Euclid 47. 1.) $\sqrt{xx + rxx} + \sqrt{xx - rxx + rrxx}$ $(= AC + BC) = 100$; whence $x = \frac{100}{\sqrt{1+r} + \sqrt{1-r+rr}} = 29·24$, and y $(= rx)$ $= 57·36 = AB$.

The same answered by Sylvius.

Let the tang. ACB $(70°) = t$; and in acb, similar to ACB, assume $cD = 1$, and $aD = x$; then will $ab = \frac{1}{x}$ (per quest.) and consequently $Db = \frac{1}{x} - x = \frac{1 - xx}{x}$: But

ab

$\frac{ab}{1-aD\times Db} = t$ (= tang. acb), that is, $\frac{1}{x^3} = t$; whence $x = 0.713984$ = tang. 35° 31′ 34″ = acD: From which the angle bcD = 34° 28′ 20″; and it will be $ca+cb$ (the sum of the secants of these two angles) : $AC+BC$ :: ac : AC = 50.3215; whence the rest are easily found.

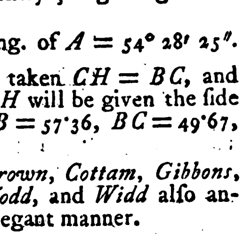
The same answered by Mr. Henry Watson.

Upon the side AC, of the required triangle ABC, conceive the perpendiculars BE and DF let fall; then, per similar triangles, $\begin{matrix} AC : AD :: AB : AE \\ AC : CD :: CD : CF \end{matrix}$. From whence, as the rectangles under the means of both proportions are equal (per quest.) we have $AE = CF$, and consequently $CE = AF$. But AF (CE) : DF :: AD : CD, and DF : BE :: AD : AB: Therefore, by composition, $CE : BE :: AD^2 : AB\times CD :: AD^3 : AD\times AB\times CD$ (= CD^3); whence $\frac{BE}{CE} = \frac{CD^3}{AD^3}$, and consequently $\frac{1}{3}$ log. tang. ECB ($\frac{1}{3}$ log. $\frac{BE}{CE}$ = log. $\frac{CD}{AD}$) = log. tang. of A = 54° 28′ 25″. Now, if in AC produced, there be taken $CH = BC$, and B, H be joined, in the triangle ABH will be given the side AH and all the angles; whence $AB = 57.36$, $BC = 49.67$, and $AC = 50.32$.

Messrs. *Ash*, *Bamfield*, *Bevil*, *Brown*, *Cottam*, *Gibbons*, *Johnson*, *Metcalf*, *Moss*, *Peart*, *Todd*, and *Widd* also answered the same, in a concise and elegant manner.

XI. QUESTION 386 *answered by Mr.* William Cottam, *at his Grace the Duke of Norfolk's.*

Let Bm, Ep, Dq, and An, (= 60, 52, 28, and 40) be denoted by m, p, q, and n, respectively; and let the radius of the fountain = x: Then (per figure) $\overline{m+x}|^2 - \overline{p+x}|^2$ (= $BI^2 - EI^2 = AH^2 - DH^2$) = $\overline{n+x}|^2 - \overline{q+x}|^2$; whence $x = \frac{1}{2}\times\frac{m^2+q^2-p^2-n^2}{p+n-m-q} = 10$.



Now let m, p, q, n, (= 70, 62, 38, and 50) = BC, EC, DC, and CA, respectively, and let A = 4840 = the area of an acre in yards; also let x, now, = GC; and then we have $\overline{\sqrt{pp-xx}+\sqrt{qq-xx}}\times\overline{x+\sqrt{nn-qq+xx}} = A$;

$= A$; which, ſolved, gives $x = 15{\cdot}11145$: Hence $DE = AB = 94{\cdot}99631$, and $AD = BE = 50{\cdot}94936$. *W.W.R.*

The ſame anſwered by Mr. Hugh Brown.

It is manifeſt that the radius of the pond muſt be 10; becauſe $BC^2 + DC^2 = EC^2 + AC^2$; conſequently $BC = 70\ (=a)$, $EC = 62\ (=b)$, $AC = 50\ (=c)$, $DC = 38\ (=d)$. If through C, the center of the pond, HI and FG be drawn parallel to the ſides of the rectangle, and there be put $AB = x$, and $AD = y$; then will $AF = \frac{x}{2} - \frac{aa-cc}{2x}$, and $AH = \frac{y}{2} + \frac{cc-dd}{2y}$; from whence, and the queſtion, we ſhall (becauſe $AF^2 + AH^2 = AC^2$) have the two following equations, $\frac{xx}{4} + \frac{yy}{4} + \frac{\overline{aa-cc}|^2}{4xx} + \frac{\overline{cc-dd}|^2}{4yy} = \frac{aa}{2} + \frac{dd}{2}$, and $xy = 4840 = A$. Let, now, the firſt equation be multiplied by 4, and $\frac{A}{x}$ ſubſtituted therein for y; ſo ſhall

$$xx + \frac{A^2}{xx} + \frac{\overline{aa-cc}|^2}{xx} + \frac{\overline{cc-dd}|^2}{A^2} \times xx = 2aa + 2dd;$$

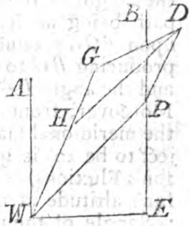
and conſequently $\overline{1 + \frac{\overline{cc-dd}|^2}{A^2}} \times x^4 - 2 \times \overline{aa+dd} \times xx = -A^2 - \overline{aa-cc}|^2$: Put $1 + \frac{\overline{cc-dd}|^2}{A^2} = g$, $aa + dd = h$, $A^2 + \overline{aa-cc}|^2 = k$, and the equation will ſtand thus, $gx^4 - 2hx^2 = -k$; whence $x = \sqrt{\frac{h + \sqrt{hh-gk}}{g}} = 94{\cdot}9961$, and $y = \frac{A}{x} = 50{\cdot}9494$.

Much after the ſame manner it was anſwered by Meſſrs *Aſh, Bamfield, Kingſton, Milbourn, Moſs, Nichols, Widd,* and ſome others.—Mr. *O'Cavanah* conſtructs this problem geometrically; but, his demonſtration being ſomewhat long, we are obliged to omit the whole, for want of room.

XII. QUESTION 387 *anſwered by Mr.* W. Bevil.

Let P and W repreſent the places of the two pillars, whoſe given heights (40 and 60) let be denoted by a and b, reſpectively; then, ſuppoſing $v =$ the co-tangent of the ſun's altitude, the lengths of their reſpective ſhades, DP and WG, will

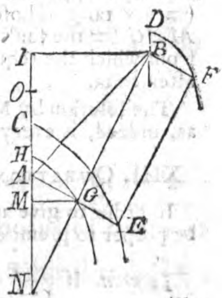
will be av and bv: Draw DH parallel to PW, and WE perpendicular to the two meridian lines WA and BPE, putting $c = DH$ ($= PW$) $= 200$, $f = b - a$, $s =$ sine $45° = EWP = AWP$, $z =$ sine of AWG (= the sun's azimuth from the south), and $u =$ its cosine; then GH ($= bv - av$) $= fv$, and $sz + su =$ cosine of $GWP = GHD$: Therefore $HD^2 + HG^2 - GH \times HD \times 2$ cos. $\angle H = GD^2 = c^2 + f^2v^2 - 2csfvz - 2scfvu$, a minimum: Put $p = \frac{f}{2cs}$,



$q = \frac{c}{2fs}$, then $q + pv^2 - vz - vu$, a minimum; let e and d = the sine and cosine of the latitude, n = the sine of the sun's declination, and x and y the sine and cosine of his altitude, then we have $q + \frac{py^2}{x^2} - \frac{y}{x}\sqrt{\overline{1 - \frac{ex-n}{dy}}\big|^2} - \frac{y}{x} \times \frac{ex-n}{dy}$; which, by reduction, (putting $-dp + dq - e = r$, $ee + dd = 1$, and $dd - nn = h$) becomes $\frac{dp + rx^2 + nx - x\sqrt{h - x^2 + 2enx}}{dx^2}$. This, thrown into fluxions and reduced, gives $\overline{2dp + nx} \times \sqrt{h - x^2 + 2enx} = hx + enx^2$; whence $x = \cdot 39359048 =$ sine of 23° 10′ 41″, the sun's altitude; and from thence the time of the day is found 5 h. 27 m. 3 sec. in the afternoon.

The same answered by Mr. T. Moss.

Let A be the place of the southermost pillar, and B that of the northermost; and suppose AE and BF to be any contemporary positions of the two shadows, taken as parallel: Then, if BG be made parallel to FE, it will be equal to FE, and GE likewise $= BF$. But AE is to BF as the height of the pillar A, is to that of the pillar B; whence AG must be to AE in the constant ratio of the difference of the said heights to the altitude of the pillar at A: So that the path HG of the point G will be exactly similar to that of the point E, and is moreover the very same as would be described by



 the

the shadow of an object at A, whose height is the excess of the height of the pillar at A above the pillar at B: Which path being an hyperbola, suppose O to be its center, and upon AOC produced let fall the perpendiculars GM and BI, producing BG to meet IA in N. Then, AB being $= 200$, and the angle $IAB = IBA$, we have $AI (= BI) = 141.42$: Moreover, from the sun's meridian altitude, the length of the meridional shadow AH (supposing the height of the object to be 20) is given $= 10.656$: And (by art. 467 of Simpson's Fluxions) it will be as, the rectangle of the sines of the sun's altitude at noon and depression at midnight, is to the rectangle of the radius and the sine of twice the sun's declination, so is (20) the height of the object projecting the shadow, to (64.072) the transverse axis of the described hyperbola: Also, as the square root of the former of the said rectangles, is to the cosine of the sun's declination, so is the height of the object, to (38.402) the semi-conjugate axis of the hyperbola. Put now a $(= 32.036) = OH$, $b = 38.401$, c $(= 98.728) = OI$, and d $(= 141.42) = BI$; and let $OM = az$. Then, by the property of the hyperbola, $MG = \frac{b}{a}\sqrt{aazz - aa} = b\sqrt{zz-1}$, and $MN = \frac{bb}{aa} \times az = \frac{bbz}{a}$ (since it is evident that BG, to be the shortest possible, must fall upon the curve at right angles): Hence, because of the sim. triangles, we have $\frac{MN}{MG} = \frac{NI}{BI}$; that is, $\frac{bz}{a\sqrt{zz-1}} =$

$$\frac{\overline{c + a + \frac{bb}{a}} \times z}{d}, \text{ or } \frac{z}{\sqrt{zz-1}} = \frac{ac}{bd} + \overline{\frac{aa}{bd} + \frac{b}{d}} \times z; \text{ which,}$$

in numb. becomes $\frac{z}{\sqrt{zz-1}} = 0.58237 + 0.46052z$; whence $z = 1.56204$; OM $(= az) = 50.04$; $AM = 7.348$; MG $(= b \times$ tang. whose secant is $z) = 46.083$; and the angle MAG (= the sun's azimuth from the north) $= 80^\circ\ 56'\ 27''$; from which the required time is found to be 5h. 26m. 58 sec. after noon.

The solution by Mr. *T. Peart* is both concise and elegant, as, indeed, is every thing sent us by this author.

XIII. QUESTION 388 *answered by Mr.* E. Rollinson.

In order to give a general solution to this problem, it will be proper to premise the following

Lemma. If $\frac{\dot{p}}{1-mpp} + \frac{\dot{q}}{1-mqq} + \frac{\dot{r}}{1-mrr} + \frac{\dot{s}}{1-mss}$, &c.

&c. $= \frac{\dot{x}}{1-mxx}$; *wherein* m *is constant, and* p, q, r, s, *&c. variable; then, if the sum of all the quantities* p, q, r, s, *&c. be denoted by A, the sum of all their rectangles by* B, *the sum of all their solids by* C, *&c. I say, that*

$$x = \frac{A+mC+m^2E+m^3G+m^4I, \&c.}{1+mB+m^2D+m^3F+m^4H, \&c.}.$$ For, by taking the fluent, we have hyp. log. $\frac{1+m^{\frac{1}{2}}p}{1-m^{\frac{1}{2}}p}$ + hyp. log. $\frac{1+m^{\frac{1}{2}}q}{1-m^{\frac{1}{2}}q}$ +, &c. = hyp. log. $\frac{1+m^{\frac{1}{2}}x}{1-m^{\frac{1}{2}}x}$; and consequently $\frac{1+m^{\frac{1}{2}}p}{1-m^{\frac{1}{2}}p} \times \frac{1+m^{\frac{1}{2}}q}{1-m^{\frac{1}{2}}q}$, &c. $= \frac{1+m^{\frac{1}{2}}x}{1-m^{\frac{1}{2}}x}$. Put this value of $\frac{1+m^{\frac{1}{2}}x}{1-m^{\frac{1}{2}}x} = \mathcal{Q}$; whence x will be found $= \frac{\mathcal{Q}-1}{m^{\frac{1}{2}} \times \overline{\mathcal{Q}+1}} =$

$$\frac{\overline{1+m^{\frac{1}{2}}p}\,.\,\overline{1+m^{\frac{1}{2}}q}\,.\,\overline{1+m^{\frac{1}{2}}r}, \&c. - \overline{1-m^{\frac{1}{2}}p}\,.\,\overline{1-m^{\frac{1}{2}}q}\,.\,\overline{1-m^{\frac{1}{2}}r}, \&c.}{m^{\frac{1}{2}}.\,\overline{1+m^{\frac{1}{2}}p}\,.\,\overline{1+m^{\frac{1}{2}}q}\,.\,\overline{1+m^{\frac{1}{2}}r}, \&c. + m^{\frac{1}{2}}.\,\overline{1-m^{\frac{1}{2}}p}\,.\,\overline{1-m^{\frac{1}{2}}q}\,.\,\overline{1-m^{\frac{1}{2}}r}, \&c.}$$

(by substituting the value of $\mathcal{Q}$): But, by multiplication, $\overline{1+m^{\frac{1}{2}}p}\,.\,\overline{1+m^{\frac{1}{2}}q}\,.\,\overline{1+m^{\frac{1}{2}}r}$, &c. $= 1+m^{\frac{1}{2}} \times \overline{p+q+r}$, &c. $+ m \times \overline{pq+pr}$, &c. $= 1+m^{\frac{1}{2}}A + mB + m^{\frac{3}{2}}C$, &c. &c. Hence our equation becomes $x = \frac{A+mC+m^2E+m^3G, \&c.}{1+mB+m^2D+m^3F, \&c.}$ $\mathcal{Q}.E.D.$

If $m = -1$, the given equation will become $\frac{\dot{p}}{1+pp} + \frac{\dot{p}}{1+qq} + \frac{\dot{q}}{1+rr}$, &c. $= \frac{\dot{x}}{1+xx}$; and the value of $x = \frac{A-C+E-G, \&c.}{1-B+D-F, \&c.}$.

Now, to apply this to the question proposed, let the arcs P, $\mathcal{Q}$, R, &c. and their tangents p, q, r, &c. be considered as in a flowing state; and let x be the required tangent of $P+\mathcal{Q}+R$, &c. Then, it being known that $\dot{P} = \frac{\dot{p}}{1+pp}$,

$\dot{\mathcal{Q}} =$

$\dot{Q} = \frac{\dot{q}}{1+qq}$, &c. we thence have $\frac{\dot{p}}{1+pp} + \frac{\dot{q}}{1+qq} + \frac{\dot{r}}{1+rr}$, &c. $= \frac{\dot{x}}{1+xx}$; and consequently $x = \frac{A - C + E - G + I, \&c.}{1 - B + D - F + H, \&c.}$, by the preceding corollary; A being the sum of all the tangents p, q, r, s, &c. B the sum of all their rectangles, C the sum of all their solids, &c. &c. *W. W. R.*

Corollary. If all the arcs P, Q, R, &c. are equal, and their number be denoted by n; then will $A = np$, $B = n . \frac{n-1}{2} p^2$, $C = n . \frac{n-1}{2} . \frac{n-2}{3} p^3$, &c. and therefore $x =$

$$\frac{np - n . \frac{n-1}{2} . \frac{n-2}{3} p^3 + n . \frac{n-1}{2} . \frac{n-2}{3} . \frac{n-3}{4} . \frac{n-4}{5} p^5, \&c.}{1 - n . \frac{n-1}{2} p^2 + n . \frac{n-1}{2} . \frac{n-2}{3} . \frac{n-3}{4} p^4, \&c.}$$

The same answered by Mr. W. Bevil.

It is proved, by the writers on trigonometry, that *the tangent of the sum of two arcs (the radius being unity) is equal to the sum of the tangents of those arcs divided by the excess of the square of the radius above their rectangle or product.*

Hence the tang. of $P + Q$ will be $\frac{p+q}{1-pq}$; and, if $P+Q$ be considered as one arc, then the tangent of $\overline{P+Q}+R$ will, by the same rule, be $= \frac{\frac{p+q}{1-pq} + r}{1 - \frac{pr+qr}{1-pq}} = \frac{p+q+r-pqr}{1-pq-pr-qr}$:

After the same manner the tangent of $P+Q+R+S$ is found to be $= \frac{p+q+r+s-pqr-pqs-prs-qrs}{1-pq-pr-ps-qr-qs-rs+pqrs}$: And thus, by carrying on the process a step or two farther, the law of continuation will appear manifest; being such, that, if the sum of all the given tangents be denoted by A, the sum of all their rectangles by B, the sum of all their solids by C, &c. then will the tangent of the sum of all the arcs be $\frac{A - C + E - G, \&c.}{1 - B + D - F, \&c.}$.

In this last manner it was answered likewise by Mr. *Hugh Brown*, Mr. *G. Burgess*, Mr. *T. Moss*, Mr. *Harland Widd*, and some others.

XIV. QUES-

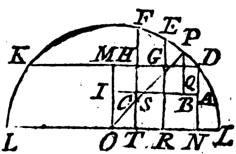
XIV. QUESTION 389 *answered by the proposer,* *Mr.* E. Rollinson.

Let R and r be the semi-diameters of the orbits of the earth and moon, P and p the periodic times in those orbits, S and s the sun's mean apparent semi-diameter and moon's mean horizontal parallax, and N and n any two numbers in the required ratio of the densities of the sun and earth, respectively. Then, the real semi-diameters of the sun and earth being in the ratio of RS to rs, their masses will be as $R^3S^3 \times N : r^3s^3 \times n$; and consequently their forces, at the distances R and r, as $\frac{R^3S^3N}{R^2} : \frac{r^3s^3n}{r^2}$, or as $RS^3N : rs^3n$.

But these (by the laws of central forces) are also as $\frac{R}{PP} : \frac{r}{pp}$; therefore, by dividing the antecedents of these equal ratios by RS^3, and the consequents by rs^3, we have as $N : n :: \frac{1}{P^2S^3} : \frac{1}{p^2s^3} :: 1 : \frac{P^2}{p^2} \times \frac{S^3}{s^3}$; which, in numbers, (taking $P = 365$ d. 5 h. 49 m. $p = 27$ d. 7 h. 43 m. $S = 16$ m. $5\frac{1}{4}$ s. and $s = 57$ m. $17\frac{1}{2}$ s.) will come out as 1 to 3·957, for the ratio of the density of the sun to that of the earth. *W. W. R.*

XV. QUESTION 390 *answered by Mr.* J. Morland.

Construction. In any right line AI set off SA, SB, and SC, equal to the three given longitudes of the moon from the sun; and make AD, BE, and CF perpendicular to AI, so as to express the given latitudes corresponding: Then, through the three points D, E, and F, let the circumference of a circle be described; and from O the center thereof, through S, draw the radius OP, and upon AI let fall the perpendicular PQ: So shall SP be the distance of the two centers, at the time of the greatest obscuration, and SQ the required difference of longitudes at that time. For, since the circumference of the circle thus described, coincides with the real curve (whatever it is) in three points (D, E, F) which are but at a small distance from one another, it must necessarily have nearly the same degree of curvature, and therefore likewise coincide with it in the intermediate spaces, very near. To derive the numerical solution from this construction, let the chord DK be parallel to AI, and let OIM be perpendicular



dicular thereto; alſo let FHC and EGB be produced to meet the diameter LL, at right angles, in T and R.

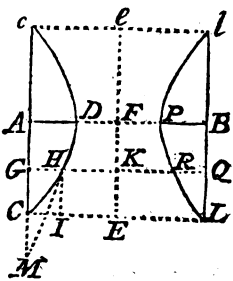
Put $DG\ (AB) = a$, $DH\ (AC) = b$, $EG\ (BE - AD) = c$, $FH\ (CF - AD) = d$, $OM = x$, and $MK\ (MD) = y$: Then, by the property of the circle, $GD \times GK = GE \times \overline{2GR + GE}$, and $HD \times HK = HF \times \overline{2HT + HF}$; that is, $a \times \overline{2y - a} = c \times \overline{2x + c}$, and $b \times \overline{2y - b} = d \times \overline{2x + d}$: Whence x is found $= \frac{a \times \overline{bb + dd} - b \times \overline{aa + cc}}{2bc - 2ad}$, and $y = \frac{c \times \overline{bb + dd} - d \times \overline{aa + aa}}{2bc - 2ad}$: From which values, thoſe of $OK\ (OP)$, OI, OS, PS, SQ, and AQ will all become known.—As to the time anſwering to this (or any other) given value of AQ, it is beſt determined from the common method of interpolating by differences: According to which, the two given intervals correſponding to AB and BC being denoted by p and q, the required interval, between the poſition Q and the firſt poſition A, will be repreſented by $\frac{\overline{pbb - qaa} \times AQ + \overline{qa - pb} \times AQ^2}{ab \times \overline{b - a}}$ (See M. de Caille's Aſtronomy, p. 60.)

The propoſer reſolves this problem by means of a parabolic curve deſcribed through three given points: And obſerves, 'That, if the equation $yy = g + hx + kx^2$ were to 'be aſſumed for the general relation of the latitude (y) to 'the difference of longitude (x), the reſult would come out 'more neat and ſimple than from any curve of the parabolic 'kind:' But adds, 'that this laſt method is not general, being 'only applicable when the moon has a conſiderable latitude 'during the whole time of the eclipſe; ſince the aſſumed 'equation (which anſwers to an ellipſis or hyperbola) becomes 'impoſſile on the moon's paſſing from one ſide of the ecliptic 'to the other.' He obſerves farther, 'That the concluſions, 'according to either of the above methods, will ſeldom be 'found to differ by more than one minute in time from thoſe 'ariſing from the common way of computation.' Which laſt he therefore thinks may be uſed as ſufficiently near, till the theory of the moon's motion is known to a greater degree of exactneſs.

Mr. *Harland Widd* ſent a ſolution to this problem.

The PRIZE QUESTION *anſwered by the Propoſer.*

Suppoſing Cc and Ll to be the lengths, and AD and BP the ſemi-breadths of two adjacent piers (or ſtarlings), let Ee, parallel to Cc, biſect AB at right angles, in F; and let IH, parallel to Ee, be the direction of a particle of water impinging



pinging upon the surface CD in H; also let HM be a tangent at H, intersecting AC produced in M.

Call AC, a; AD, b; CE, c; CG, x; and GH, y; and let the tangents of the angles ECD and ADC (to the radius 1) be denoted by p and q respectively. The celerity wherewith the stream passes any section HR being inversely as the breadth, the velocity of the particle acting at H will therefore be as $\frac{1}{c-y}$, and its force (by mechanics) as $\frac{1}{c-y} \times \frac{GH^2}{MH^2} = \frac{1}{c-y} \times \frac{\dot{y}\dot{y}}{\dot{x}\dot{x}+\dot{y}\dot{y}}$; which, drawn into $\dot{y}$, gives $\frac{1}{c-y} \times \frac{\dot{y}^3}{\dot{x}\dot{x}+\dot{y}\dot{y}}$ for the fluxion of the resistance upon CH. But (by art. 408 of Simpson's Fluxions) it appears that, if S be assumed to denote any quantity expressed in term of y and given coefficients, the fluent of $S \times \frac{\overline{\dot{x}\dot{x}+\dot{y}\dot{y}}^{\,n}}{\dot{y}^{2n-1}}$ (corresponding to any given value of x) will be a max. or min. when the relation of x and y is such that the value of $\frac{S\dot{x} \times \overline{\dot{x}\dot{x}+\dot{y}\dot{y}}^{\,n-1}}{\dot{y}^{2n-1}}$ is every-where the same. Therefore, by transforming our fluxion, $\frac{1}{c-y} \times \frac{\dot{y}^3}{\dot{x}\dot{x}+\dot{y}\dot{y}}$, to $\frac{1}{c-y} \times \frac{\overline{\dot{x}\dot{x}+\dot{y}\dot{y}}^{\,-1}}{\dot{y}^{-3}}$, and then comparing it with that above, we shall get $\frac{1}{c-y} \times \frac{\dot{x}\dot{y}^3}{\overline{\dot{x}\dot{x}+\dot{y}\dot{y}}^{\,2}} = d$, (a constant quantity). In order to the resolution of this equation, put $\dot{x} = v\dot{y}$ (v being the tang. of the angle MHG) then, by substitution, &c. $\frac{1}{c-y} \times \frac{v}{\overline{vv+1}^{\,2}} = d$; and consequently $d \times \overline{c-y} = \frac{v}{\overline{1+vv}^{\,2}}$: Which, in fluxions, gives $-d\dot{y} = \frac{\dot{v} - 3v^2\dot{v}}{\overline{1+vv}^{\,3}}$, and $dv\dot{y} (= d\dot{x}) = \frac{3v^3\dot{v} - v\dot{v}}{\overline{1+vv}^{\,3}}$; whereof the

the corrected fluent will be $dx = \frac{1+3pp}{2.\overline{1+pp}|^2} - \frac{1+3vv}{2.\overline{1+vv}|^2}$.

To find d, take $y=0$; in which circumstance, v being $=p$, the equation $\frac{1}{c-y} \times \frac{v}{\overline{vv+1}|^2} = d$ becomes $\frac{p}{c.\overline{1+pp}|^2}$ $=d$; which value being substituted for d, our two equations, after proper reduction, will become $y = c - \frac{cv \times \overline{1+pp}|^2}{p \times \overline{1+vv}|^2}$, and $x = c \times \frac{1+3pp}{2p} - c \times \frac{\overline{1+3vv} \times \overline{1+pp}|^2}{2p \times \overline{1+vv}|^2}$.

These equations give the general relation of x, y, and v; but to apply them to any particular case proposed, something further remains to be done, since the value of p (the tangent of the angle ECH) is not given, but must be found from the known values of CE, CA, and AD. In order to this, suppose H to coincide with D; then, x becoming $=a$, $y=b$, $v=q$, if these values be substituted in the aforesaid equations, we shall, after due reduction, have $\frac{\overline{1+pp}|^2}{\overline{1+qq}|^2} = \frac{c-b}{c}$ $\times \frac{p}{q}$, and $a = c \times \frac{1+3pp}{2p} - \overline{c-b} \times \frac{1+3qq}{2q}$.

Put $r = \frac{2a}{c} + \frac{c-b}{c} \times \frac{1+3qq}{q}$, and then $p = \frac{\sqrt{\overline{rr-12}}+r}{6}$; from which equations the values of p and q may be found. Thus, for example, if a be supposed $=12$, $b=5$, and $c=8$; then p will come out $=2$, and $q=3$: So that the angles ECH and ADC are here 63° 26′ and 71° 34′, respectively.

Corollary. If c be supposed exceeding great, or, which comes to the same, if every particle of the fluid impinges with the same velocity, then $\frac{y}{c}$ will vanish, and the equation $\frac{y}{c} = 1 - \frac{v \times \overline{1+pp}|^2}{p \times \overline{1+vv}|^2}$ will become $\frac{v \times \overline{1+pp}|^2}{p \times \overline{1+vv}|^2} = 1$, and consequently $v=p$; therefore the angle CHI being everywhere the same, CD will, in this case, become a right line: From whence it appears, that the less DF is in respect of CE, the greater must be the curvature of the surface upon which the water acts.

*** *This was the only true solution received.*

End of the Second Volume.